LIE ALGEBRAS COURSE SUMMARY

CRYSTAL HOYT

(1) Algebras, **Lie algebras**, ideals, subalgebras, homomorphisms, derivations, 1 and 2 dimensional Lie algebras

(2) Representations, adjoint action, nilpotent Lie algebras, **Engel's Theorem**

(3) Simple 3-dimensional Lie algebras, solvable Lie algebras, radical, semisimple, **Lie's Theorem**

(4) **Cartan's Criterion**, bilinear form, Killing form, **Criterion for semisimplicity**, simple ideals in a semisimple Lie algebra

(5) **Modules**, Representations of a 1-dimensional Lie algebra, simple modules of a solvable Lie algebra are 1-dimensional, **Schur's Lemma**, Casimir element

(6) **Weyl's Theorem**, diagonalizable, $\mathfrak{sl}_2(\mathbb{F})$ is simple, **Representations of $\mathfrak{sl}_2(\mathbb{C})$**

(7) $\mathfrak{sl}_n$ is simple, **Cartan subalgebras**, **root space decomposition** for semisimple Lie algebras, Killing form on root spaces, Euclidean space

(8) Root space decomposition (cont.), Euclidean space, **Abstract root systems**, angle between two roots, **Weyl group**

(9) **Base**, simple roots, order on roots, height of root, Weyl group: W, Weyl chambers, roots of a Weyl chamber, length of $w \in W$, W **acts simply transitively on the set of bases**

(10) Irreducible root systems, a simple Lie algebra has an irreducible root system, **Cartan matrix**, Coxeter group, **Dynkin diagrams**, **classification of Dynkin diagrams**, **classification of irreducible root systems**

(11) Graded and filtered algebras, tensor and symmetric algebras, **universal enveloping algebra**, **PBW Theorem**

(12) Borel subalgebras, inner automorphisms, free Lie algebras, generators and relations, **Serre's Theorem**, **classification of complex semisimple Lie algebras**

(13) Root lattice, weight lattice, fundamental weights, weights, highest weight, induced modules, **Verma modules**

(14) **Finite dimensional modules**, characters, **Weyl Character Formula**