EXERCISE 3 IN COMMUTATIVE ALGEBRA AND ALGEBRAIC GEOMETRY - INTRODUCTION TO CATEGORY THEORY

DMITRY GOUREVITCH

(1) (P) Show that an equivalence of categories is fully faithful and essentially surjective. Show that it is both left and right adjoint to its inverse functor.

(2) (P) Find the left and the right adjoint to the forgetful functor Topol. Spaces $\to$ Sets.

(3) (P) Any partially ordered set defines a category, which has a unique morphism between a and b if $a \leq b$, and no morphisms otherwise. Note that a functor between two such categories is just a monotone function. Find the left and right adjoint functors.

(4) (a) (P) Give a definition of direct sum (coproduct) using the Yoneda embedding.
  (b) Show that direct sum of sets is disjoint union
  (c) Show that the natural map between direct sum and direct product of two abelian groups is an isomorphism, while the natural map between direct sum and direct product of infinitely many abelian groups is not.

(5) (P) Give an example of an abelian category which has zero object, direct sums, direct products, the natural map between them is an isomorpism, but the semigroup $Mor(A, B)$ is not always a group.

URL: `http://www.wisdom.weizmann.ac.il/~dimagur/AlgGeo.html`

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