EXERCISE 6 IN COMMUTATIVE ALGEBRA AND ALGEBRAIC GEOMETRY II

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(1) (P) $M \otimes_A \operatorname{Hom}(M, A) \cong A \iff M$ is locally isomorphic to A.

(2) (P) ν^0 is left adjoint to ν_0 and $\nu^\star$ is left adjoint to $\nu_\star$.

(3) (P)
  (a) If X is affine then any invertible sheaf is very ample
  (b) α_i is very ample on $X_i \Rightarrow \pi_1 \alpha_1 \otimes \pi_2 \alpha_2$ is very ample in $X_1 \times X_2$
  (c) If $\mathcal{L}$ is any inv. sheaf on a projective variety X, then $\mathcal{L}(m)$ is very ample for $m \gg 0$

(4) (P)
  (a) $\mathcal{O}(m)$ on $\mathbb{P}^n$ is very ample. Thus it defines $\mathbb{P}^n \to \mathbb{P}^N$. Find this map.
  (b) $\mathbb{P}^n, \mathcal{O}(1)$ and $\mathbb{P}^k, \mathcal{O}(1)$ define a very ample sheaf $\mathcal{O}(1) \otimes \mathcal{O}(1)$ on $\mathbb{P}^n \times \mathbb{P}^k$. It defines $\mathbb{P}^n \times \mathbb{P}^k \to \mathbb{P}^{nk+n+k}$. Find this map.