EXERCISE 6 IN REPRESENTATION THEORY

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Fourier transform for finite groups.

Let C be a finite commutative group, $n = |C|$. We will denote by $\widehat{C}$ the dual group of characters $C \to \mathbb{C}$. We define Fourier transform $\mathcal{F} : F(C) \to F(\widehat{C})$ by $\mathcal{F}(u)(\psi) = \sum u(g)\psi(g)$.

(1) (P) Show that if we define an L^2-structure on spaces of functions by $||u||^2 = 1/n \sum |u(g)|^2$ then the operator $\mathcal{F}$ satisfies the Plancherel formula $||\mathcal{F}(u)||^2 = n||u||^2$.

(2) (P) Using the Plancherel formula prove the following Theorem(Gauss). Fix a non-trivial multiplicative character χ and a nontrivial additive character ψ for the finite field $\mathbb{F}_q$ and consider the Gauss sum $\Gamma = \sum \chi(g)\psi(g)$, where the sum is taken over $g \in F^\times$. Then $|\Gamma| = q^{1/2}$.

URL: http://www.wisdom.weizmann.ac.il/~dimagur/CompRepTheo.html

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