Theory of Automorphic representations.

Joseph Bernstein **Fall 2018**

Course description.

This is a second part of a year long course on the theory of Automorphic forms and Automorphic Representations. This theory is now one of the focal points of Mathematics. It has very many applications in different areas of Number Theory, Physics, Combinatorics and so on.

This theory is in the process of constant development and it is not easy to formulate what are the goals of the theory. I will try to describe basic structures of the theory and main ideas developed in it. I will also try to illustrate them by examples of applications.

In Fall semester I will discuss analytic aspects of the theory. They are related to L-functions that can be associated to automorphic representations and with Automorphic Periods on these representations. My main goal is to describe

(i) How to introduce the L-functions related to Automorphic Representations

(ii) How to get information about automorphic Representations using Automorphic Periods.

(iii) How to relate L-functions with Automorphic periods.

My main motivation is to show how the study of automorphic periods allows to prove some good analytic properties of automorphic L-functions.

In particular, I am planning to discuss some results about convexity and subconvexity bounds for automorphic periods and corresponding L-functions.

In the process of the course I will often use some non-trivial results about representations of real and p-adic reductive groups. Mostly I will try to formulate precise statements of these results and give references, but will not discuss their proofs.

Books. In my exposition I will use many sources. Here are some of them.

Book "Introduction to Langlands program"

Book " Automorphic Forms and Representations of Adele Groups" by S. Gelbart

Paper "Perspectives of the Analytic Theory of L-functions" by H. Iwaniec and P. Sarnak

Prerequisits. Good knowledge of Representation Theory.

Basic knowledge of the theory of algebraic groups.

Knowledge of Complex Analysis.

Some knowledge of representation theory of real and p-adic reductive groups will be quite useful.

Syllabus of second part of the course (Fall semester 2018).

1. Generalities on Automorphic Representations. Global fields – Arithmetic and Geometric cases.

2. Theory of Eisenstein series for automorphic representations. Meromorphic continuation and Functional Equation for Eisenstein Series.

3. Langlands L-functions associated to automorphic representations.
Langlands conjectures.

4. Functoriality conjecture and Base change lifting.

5. Automorphic periods. Product formulas. Mutiplicity one results (local and global).

6. Relations between Automorphic periods and L-functions.

7. General theory of L-functions.
General Riemann Conjecture and Lindelof conjecture.

8. Convexity and subconvexity bounds for L-functions and Automorphic Periods.

9. Introduction to Trace Formula and Relative Trace Formula for Automorphic Representations.