Class Field Theory.

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Course description.

Let K be a global field (e.g. a finite extension of $\mathbf{Q}$). One of the main problem in Number Theory (and in fact in Mathematics) is to find a way to describe the absolute Galois group $\Gamma_K := Gal(\bar{K}/K)$.

The problem is that this group is defined only up to conjugationn – it depends on a choice of an algebraic closure of K. So a correct way to formulate this problem is to describe representations of the group Γ_K.

Class Field Theory (CFT) deals with one dimensional representations, i.e. it describes the abelian group Γ_K^{ab}. In terms of field extensions CFT allows to describe the finite abelian extensions of the field K.

It turns out that this description is closely related to solving analogues problem for local fields describing the completion of the field K at different places. This is the topic of local CFT.

In first part of my course I will mostly concentrate on different approaches to local CFT. I will discuss several known approaches how to define (characterize, construct) the isomorphism of local CFT and how to prove that it is an isomorphism. In my opinion this is a very fascinating problem that is solved, but not really understood.

Later in the course I will describe the relation betwee local and global CFT. I will describe analytic tools from L-function theory and use them to formulate and prove global versions of CFT.

Prerequisits. Knowledge of basic facts about global and local fields. Representations of finite groups. Some complex analysis.

Sillabus.

1. Relations beteen local and global Galois groups.
2. Examples of explicit constructions in Class field theory.
3. Global characterization of the Artin map.
4. Neikrich's local characterization of Artin map.
5. Lubin-Tate comstruction
6. Galois cohomologiy and CFT.
7. L-functions, densities of primes.
8. Statement and proof of global CFT.