Let $X$ be a smooth algebraic variety and $Z \subset X$ a (Zariski) closed subvariety. Let $i : Z \hookrightarrow X$ denote the embedding.

(1) Let $X$ be a smooth affine variety, and $Z \subset X$ be a closed smooth subvariety. As in the lecture, define a functor $i' : \mathcal{M}(\mathcal{D}_X) \to \mathcal{M}(\mathcal{D}_Z)$ by

$$i'(\mathcal{F}) := \text{Hom}_{\mathcal{D}_X}(\mathcal{D}_Z, \mathcal{F}).$$

Show that

(i) For affine $X$, $i'(\mathcal{M})$ is isomorphic as an $\mathcal{O}_Z$-module to the subspace $\text{Ann}_{\mathcal{M}}I(Z)$ of $\mathcal{M}$ consisting of elements annihilated by the ideal $I(Z)$.

(ii) $i'\iota_0 N \simeq N$ for any $N \in \mathcal{M}(\mathcal{D}_Z)$.

(iii) $\iota_0$ is left adjoint to $i'$.

(2) (P) Let $V$ be a vector space, and $M$ be a $\mathcal{D}_V$-module supported at $\{0\}$. Let $E := \sum x_i \partial_i \in \mathcal{D}(V)$ be the Euler operator. Show that all the eigenvalues of $E$ on $M$ are negative.

URL: http://www.wisdom.weizmann.ac.il/~dimagur/DmodI_3.html

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