SYLLABUS FOR THE COURSE "REPRESENTATION THEORY OF FINITE AND COMPACT GROUPS", FALL SEMESTER 2013

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The speed and the details level of the exposition of some basic topics will depend on the preliminary knowledge of the audience. The only absolute prerequisite is good knowledge of linear algebra. Other topics that can help are: representation theory of finite groups, basic functional analyses, basic topology and differential topology, Lie algebras. Each of these topics will be briefly reviewed if majority of the audience knows it, or taught slower otherwise.

(1) Representations of a finite group G (depending on the audience).
   (a) Groups, actions of groups on sets, natural constructions
   (b) Basic definitions: representation of a group G, morphisms of representations.
   (c) Irreducible representations. Schurs lemmas.
   (d) Natural constructions with representations.
   (e) Complete reducibility.
   (f) Intertwining numbers and their properties.
   (g) Decomposition of the regular representation.
   (h) Group algebra and its structure.
   (i) Burnside theorem and its corollaries.
   (j) Characters, Orthogonality relations. Character rings.
   (k) Brauers theorem (optional)
   (l) Restriction and induction
   (m) Mackeys theory (optional)
   (n)
   (o) Representations of finite abelian groups. Fourier transform.

(2) Some results about representations of topological groups.
   (a) Representations of commutative groups and Fourier transform.
   (b) Basic results about representations of the compact group $G = SO(3)$.

(3) Representations of general compact groups
   (a) Basic definitions and properties
   (b) Peter-Weyl theorem

(4) Lie groups and Lie algebras

(5) Representations of compact Lie groups via representations of Lie algebras

(6) Representations of general Lie groups (if time permits)
   (a) Lie groups and Lie algebras
   (b) The space of smooth vectors, Garding theorem on density, Dixmier-Malliavin theorem, the action of the Lie algebra
   (c) Cocompact subgroups, smooth induction
   (d) SL(2,R)
   (e) Very brief introduction to algebraic groups

URL: http://www.wisdom.weizmann.ac.il/~dimagur/IntRepTheo.html

Date: August 4, 2013.