SYLLABUS FOR THE COURSE "INTRODUCTION TO REPRESENTATION THEORY", FALL SEMESTER 2011

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The speed and the details level of the exposition of some basic topics will depend on the preliminary knowledge of the audience.

(1) Groups.
   (a) Basic definitions and properties (depending on the audience).
   (b) Actions of groups on sets. $G$-sets. Natural constructions with $G$-sets.
   (c) Counting principle and applications.

(2) Representations of a finite group $G$.
   (a) Basic definitions: representation of a group $G$, morphisms of representations.
   (b) Irreducible representations. Schurs lemmas.
   (c) Natural constructions with representations.
   (d) Complete reducibility. Application to the description of endomorphism algebras.

(3) Basic results about representations of finite groups.
   (a) Intertwining numbers and their properties.
   (b) Decomposition of the regular representation.
   (c) Group algebra and its structure.
   (d) Burnside theorem and its corollaries.

(4) Character theory.
   (a) Definition of a character.
   (b) Orthogonality relations. Character rings.
   (c) Brauers theorem

(5) Frobenius reciprocity and Mackey theory.
   (a) General notions from category theory. Restriction and induction functors.
   (b) Explicit construction of induction functor using equivariant sheaves.
   (c) Frobenius formula for the character of the induced representation.
   (d) Mackeys theory.

(6) Representations of abelian groups. Fourier transform.

(7) Representations of semi-direct products.

(8) Gelfand Pairs

(9) Representations of symmetric groups.

(10) Representations of the Heisenberg group. Weil representation of the group $G = \text{SL}(2,\mathbb{F}_q)$.

(11) (if time permits) Representations of the group $G = \text{SL}(2, F_q)$.

(12) Some results about representations of topological groups.
   (a) Representations of commutative groups and Fourier transform.

Date: September 21, 2011.
(b) Basic results about representations of the compact group \( G = SO(3) \).

(13) Representations of compact groups, Peter-Weyl theorem

(14) (if time permits) Representations of Lie groups

(a) Lie groups and Lie algebras

(b) The space of smooth vectors, Garding theorem on density, Dixmier-Malliavin theorem, the action of the Lie algebra

(c) Cocompact subgroups, smooth induction

(d) Very brief introduction to algebraic groups