

**SYLLABUS FOR THE COURSE "INTRODUCTION TO REPRESENTATION
THEORY", FALL SEMESTER 2011**

DMITRY GOUREVITCH

The speed and the details level of the exposition of some basic topics will depend on the preliminary knowledge of the audience.

- (1) Groups.
 - (a) Basic definitions and properties (depending on the audience).
 - (b) Actions of groups on sets. G -sets. Natural constructions with G -sets.
 - (c) Counting principle and applications.
- (2) Representations of a finite group G .
 - (a) Basic definitions: representation of a group G , morphisms of representations.
 - (b) Irreducible representations. Schurs lemmas.
 - (c) Natural constructions with representations.
 - (d) Complete reducibility. Application to the description of endomorphism algebras.
- (3) Basic results about representations of finite groups.
 - (a) Intertwining numbers and their properties.
 - (b) Decomposition of the regular representation.
 - (c) Group algebra and its structure.
 - (d) Burnside theorem and its corollaries.
- (4) Character theory.
 - (a) Definition of a character.
 - (b) Orthogonality relations. Character rings.
 - (c) Brauers theorem
- (5) Frobenius reciprocity and Mackey theory.
 - (a) General notions from category theory. Restriction and induction functors.
 - (b) Explicit construction of induction functor using equivariant sheaves.
 - (c) Frobenius formula for the character of the induced representation.
 - (d) Mackeys theory.
- (6) Representations of abelian groups. Fourier transform.
- (7) Representations of semi-direct products.
- (8) Gelfand Pairs
- (9) Representations of symmetric groups.
- (10) Representations of the Heisenberg group. Weil representation of the group $G = \mathrm{SL}(2, \mathbb{F}_q)$.
- (11) (if time permits) Representations of the group $G = \mathrm{SL}(2, F_q)$.
- (12) Some results about representations of topological groups.
 - (a) Representations of commutative groups and Fourier transform.

- (b) Basic results about representations of the compact group $G = SO(3)$.
- (13) Representations of compact groups, Peter-Weyl theorem
- (14) (if time permits) Representations of Lie groups
 - (a) Lie groups and Lie algebras
 - (b) The space of smooth vectors, Garding theorem on density, Dixmier-Malliavin theorem, the action of the Lie algebra
 - (c) Cocompact subgroups, smooth induction
 - (d) Very brief introduction to algebraic groups

URL: <http://www.wisdom.weizmann.ac.il/~dimagur/IntRepTheo.html>