Fast Matched Filter in Linear Time and Group Representation: What? Why? How?

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Abstract: In the *discrete radar* problem we design a function (waveform) $S(t)$ in the Hilbert space $\mathcal{H} = \mathbb{C}(\mathbb{Z}/p)$ of complex valued functions on $\mathbb{Z}/p = \{0, ..., p-1\}$, the integers modulo a prime number $p >> 0$. We transmit the function $S(t)$ using the radar to the object that we want to detect. The wave $S(t)$ hits the object, and is reflected back via the echo wave $E(t) \in \mathcal{H}$, which has the form

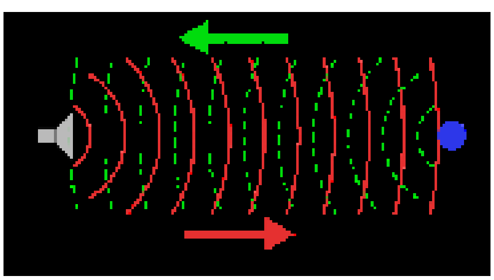
$$E(t) = e^{\frac{2\pi i}{p}\omega_0 t} \cdot S(t + \tau_0) + \Omega(t),$$

where $\Omega(t) \in \mathcal{H}$ is a white noise, and $\tau_0, \omega_0 \in \mathbb{Z}/p$ encode the distance from, and velocity of, the object.

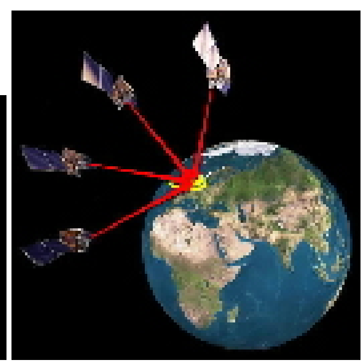
Problem (discrete radar problem) Extract τ_0, ω_0 from E and S.

In my lecture I first introduce the classical matched filter (MF) algorithm that suggests the 'traditional' way (using fast Fourier transform) to solve the discrete radar problem in order of $p^2 \cdot \log(p)$ operations. I will then explain how to use techniques from group representation theory to design (construct) waveforms $S(t)$ which enable us to introduce a fast matched filter (FMF) algorithm, that we call the "flag algorithm", which solves the discrete radar problem in a much faster way of order of $p \cdot \log(p)$ operations.

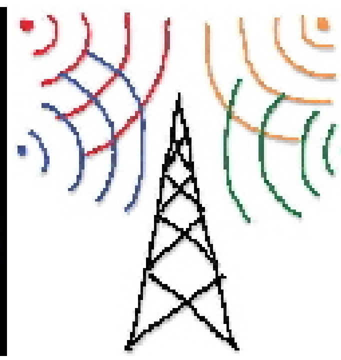
Time permits, I will demonstrate applications to *global positioning system (GPS)*, and *mobile communication*.



Radar



GPS



Mobile Communication

This is a joint work with A. Fish (Mathematics, Madison), R. Hadani (Mathematics, Austin), A. Sayeed (Electrical Engineering, Madison), and O. Schwartz (Computer Science, Berkeley).

I will assume knowledge of elementary linear algebra.