Handout 2: Approximation Algorithms

March 21, 2011

http://www.wisdom.weizmann.ac.il/∼dinuri/courses/11-BoundaryPNP/

All topics covered today are explained in detail in the first chapter of the book *The Design of Approximation Algorithms* by David Williamson and David Shmoys. An electronic almost final version of the book is available for free on the web (subject to some copyright restrictions that you should follow if you download the book).

We shall introduce the notions of approximation algorithms, approximation ratio, PTAS.

Through the problems of *set cover* and its special case *vertex cover*, we will illustrate an important algorithmic paradigm – that of the use of linear programming techniques for approximation algorithms. In doing so, we shall encounter concepts such as integer programming (IP), linear programming (LP), LP-relaxations, rounding, LP-duality, primal-dual algorithms, complementary slackness, dual fitting, randomized rounding. For lack of time, we shall not provide an in depth coverage of the theory of linear programming. Topics not covered include various forms of linear programs (such as standard and canonical forms), geometric interpretations (in terms of high dimensional polytopes and polyhedrons), algorithms for linear programming (Simplex, Ellipsoid, interior point methods), basic feasible solutions, total unimodularity.

Recommended homework. Read Chapter 1 of the book by Williamson and Shmoys.

Open questions. Approximate versions of coloring – one is allowed to use more colors than the minimum required, but do so in polynomial time. Specifically, is there any polynomial time algorithm that would color a 3-colorable graph using 100 colors? $\log n$ colors? $n^{1/10}$ colors? Is there an algorithm running in subexponential time $2^{o(n)}$ that can color 3-colorable graphs with 100 colors?