Problem Set # 4

Due: April 18th, 2011 *Lecturer: Irit Dinur*

Questions:

1. We denote by q-CSP$_\Sigma$ the constraint satisfaction problem with q-ary constraints over variables that take values in an alphabet Σ. Further, the (α, β)-gap version of this problem is the problem of deciding whether in a given instance there is an assignment satisfying at least α of the constraints (YES), or whether every assignment satisfies at most β of them (NO).

   Every q-CSP can be "converted" into a 2-CSP by introducing a variable for every constraint, and constraints for checking consistency. Formulate this reduction, and prove that if $(1, \alpha)$-gap-q-CSP$_\Sigma$ is NP-hard then $(1, \alpha')$-gap-2-CSP$_{\Sigma'}$ is NP-hard. What are the values of α', Σ' that you obtain? Can α' ever be smaller than 1/2 ?

2. Assume the exponential time hypothesis (ETH), i.e. that there is some universal constant $c > 0$ such that every algorithm for solving 3-SAT instances on n variables must run in time at least 2^{cn} in the worst case.

   Assuming the PCP theorem, prove that every algorithm for approximating 3-SAT to within any approximation factor $\alpha < 1$ must run in time at least $f(n)$, where n is the number of variables. What is the largest value of $f(n)$ for which your proof holds? How might the PCP theorem be improved to deduce that there is no sub-exponential time algorithm for approximating 3-SAT? (this is not known, and not necessarily believed)

3. Håstad's theorem says that for every $\varepsilon > 0$, $(1 - \varepsilon, 1/2 + \varepsilon)$-gap-3-LIN is NP-hard, where LIN specifies predicates of the form $x + y + z = 0$ or $x + y + z = 1$ modulo 2. Prove (using this theorem), that MAX3SAT is NP-hard to approximate to within any factor larger than 7/8.