R3SAT

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We consider the distribution $F_{n,m}$ on 3CNF formulas with n variables and m clauses, where each clause is chosen independently at random among all clauses with three literals that correspond to distinct variables. We call $d = m/n$ the density of the formula. Our goal is to design algorithms for 3SAT (that for every input formula correctly determine whether the formula is satisfiable or not) that for most formulas from $F_{n,m}$ run in polynomial time. (By *most formulas* we mean that for a formula chosen at random from $F_{n,m}$ the event happens with probability at least 1/2.) Our ability to do so will depend on the density d. An algorithm as above will be called a *heuristic* for R3SAT.

In class we saw a heuristic for R3SAT for $d > cn^{3/2} \log \log n$. The presentation was patterned after an approach of [5, 3]. A related earlier heuristic for $d > cn^{3/2}$ is presented in [6]. The theorem about the existence of small even covers is proved in [4].

The concept of R3SAT-hardness and basic results appear in [2]. The replacement (in many cases) of the R3SAT assumption by the assumption that NP does not have subexponential algorithms is from [7]. Additional hardness of approximation results along these lines are presented in [1], among other places.

Homework

1. For a regular graph G, consider the adjacency matrix A, and another version of this matrix, denoted here by B and defined as follows. The diagonal of B is all 0, and for $i \neq j$, $B_{ij} = n - d - 1$ if $(i, j) \in E$ and $B_{ij} = -d$ if $(i, j) \notin E$. Observe that every row of B sums up to 0. How are the eigenvectors of A and B related to each other? How are the eigenvalues related?

   Remark. For a given regular (or nearly regular) graph G, it is often more convenient to consider a matrix such as B rather than the adjacency matrix A. Specifically, in theoretical analysis of the eigenvalues of random graphs, it is much easier to use the trace method on B then on A.

2. Recall that a vertex cover is the vertex complement of an independent set. Hence for the following problems it may be easier to think of reductions to maximum independent set, and then see what they imply for vertex cover. Prove that it is R3SAT-hard to approximate vertex cover within a ratio of $7/6 - \epsilon$. (Hint, reduce from a random 3AND instance with m clauses to a graph with m vertices.)

References

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[4] Uriel Feige: Small linear dependencies for binary vectors of low weight. In Building Bridges Between Mathematics and Computer Science, Bolyai Society Mathematical Studies, 19, Springer. Editors: Martin Grotschel and Gyula Katona. Pages 283–307, 2008.

[5] Uriel Feige, Jeong Han Kim, Eran Ofek: Witnesses for non-satisfiability of dense random 3CNF formulas. FOCS 2006: 497–508.

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