Final assignment

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You may consult relevant references, but should write your answers on your own (no copy-paste from other sources). It is advisable to include drawings in your answers, wherever applicable. (No need to spend much time on producing electronic drawings - they can be drawn by hand.) Please hand in your exam on or before July 22, 2011.

1. The *sparsity* of a cut induced by a set S of vertices in an unweighted graph $G = (V, E)$ is the ratio between the number of edges $|E(S, V \setminus S)|$ crossing the cut and the product $|S| \cdot |V \setminus S|$ of sizes. Show that for some $\epsilon > 0$ it is R3SAT-hard (in the sense of [3]) to approximate *sparsest cut* within a ratio better than $1 + \epsilon$. In designing the reduction and proving its correctness, you may consult [1] (or versions of that paper available of the home pages of its authors).

2. For a d-regular unweighted graph $G = (V, E)$, let $1 = \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ be its normalized eigenvalues, i.e. the eigenvalues of the matrix whose ij-th entry is $1/d$ if there is an edge from i to j and zero otherwise.

   Prove that for some constant $c > 0$ and every $\epsilon, \beta > 0$ there is a polynomial-time algorithm that when given as input a d-regular graph G over n vertices with at least $n^{c\beta}$ normalized eigenvalues above $1 - \epsilon$ it outputs a set S satisfying

   (a) S is small: The size of S is at most n/n^{β}.

   (b) S is non-expanding: The expansion of S is at most

   $$\frac{|E(S, V \setminus S)|}{d|S|} \leq \sqrt{\epsilon/\beta}.$$

   You may consult [2].

References

[1] Christoph Ambuhl, Monaldo Mastrolilli, Ola Svensson: Inapproximability Results for Sparsest Cut, Optimal Linear Arrangement, and Precedence Constrained Scheduling. FOCS 2007: 329–337.

[2] Sanjeev Arora, Boaz Barak, David Steurer: Subexponential Algorithms for Unique Games and Related Problems. FOCS 2010: 563-572

[3] Uriel Feige: Relations between average case complexity and approximation complexity. STOC 2002: 534–543.