MANIFOLDS: FALL 2014
EXERCISE 1

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Errata: Two smooth atlases on a manifold are called compatible if their union is still a smooth atlas. This is an equivalence relation (prove it!), and there exists a maximal smooth atlas compatible with the given one. In general, the extension to the maximal atlas is be assumed without further comment (in particular, it was implicitly assumed in the definition of a submanifold).

Problem 1. *Let M be $\{(x,y)|x \in \mathbb{R} \quad y \in \{0,1\}\}$ with points $(x,0)$, $(x,1)$ glued together for $x > 0$. What is the basis of natural topology of M? Why M is not a manifold? The same for $x \geq 0$.*

Problem 2.
1. *Let $Gr(2,4)$ be the set of all 2-dimensional planes passing throught the origin in $\mathbb{R}^4$. Make it a manifold (i.e. produce an atlas).*
2. *Let $M \subset Gr(2,4)$ be the set of all 2-dimensional planes in $\mathbb{R}^4$ intersecting a given 2-dimensional plane non-trivially (i.e. the intersections has positive dimension). Is M a submanifold of $Gr(2,4)$?*

Problem 3. *Define $v_1 = (x_1, y_1), v_2 = (x_2, y_2) \in \mathbb{R}^2$ to be equivalent if*
1. *$v_1 = v_2 + (m,n)$ for some $m,n \in \mathbb{Z}$. Prove that the set of equivalence classes is diffeomorphic to a torus $\mathbb{S}^1 \times \mathbb{S}^1$.*
2. *$v_1 = v_2 + (m,n)$ or $v_1 = -v_2 + (m,n)$ for some $m,n \in \mathbb{Z}$. Show that the set of equivalence classes is homeomorphic to a sphere $\mathbb{S}^2$.*

Problem 4. *1.1.4 by Guillemin, Pollack*

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