MANIFOLDS: FALL 2014
EXERCISE 3

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Abstract. Chapters 1.3-1.4 by Guillemin-Pollack, partition of unity, embedding of compact manifolds to $\mathbb{R}^N$.

Problem 1. *1.4.2, 1.4.7*

Problem 2. *Let* $P_n = \{p(x) \in \mathbb{R}[x], \deg p = n\}$ *be the affine space of polynomials of degree* n *in one variable, and consider the mapping* $\phi : P_2 \times P_2 \to P_4$, $\phi(p,q) = pq$. *Describe those pairs* (p,q) *for which* $d\phi_{(p,q)}$ *is not surjective. What are the possible number of preimages of* $r \in P_4$*?*

Problem 3. *Let* X *be a compact manifold, and let* $\{U_i\}_{i=1}^n$ *be an open cover of* X. *Prove that there exists a partition of unity* θ_i *such that* $\operatorname{supp} \theta_i \subset U_i$.

Problem 4. *Let* $Z \subset X$ *be a closed submanifold of a compact manifold* X *of codimension* 1. *Construct a smooth function on* X *vanishing exactly on* Z.

Problem 5. *Let* $\phi : \mathbb{S}^1 \to \mathbb{R}^3$ *be the immersion* $\phi(x) = (\cos x, \sin 2x, \cos 3x)$. *For a given* $\epsilon > 0$ *find a small perturbation* $\tilde{\phi}$ *of* ϕ *which is an embedding and* $\|\tilde{\phi}(x) - \phi(x)\| < \epsilon$ *for all* $x \in \mathbb{S}^1$.

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