MANIFOLDS: FALL 2014
EXERCISE 4

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Abstract. Chapters 1.4 by Guillemin-Pollack, Sard theorem

Problem 1. *1.4.6, 1.4.8, 1.4.11, 1.4.13*

Problem 2. *Let $X \subset \mathbb{R}^N$ be a submanifold of $\mathbb{R}^N$, $\dim X = n$.*

(1) *Assume that for some vector $\vec{v} \in \mathbb{R}^n$ $\forall x \in X : \vec{v} \notin T_x X$, and let $\pi : \mathbb{R}^N \to \vec{v}^\perp$ be the projection along $\vec{v}$ to the hyperplane orthogonal to $\vec{v}$. Show that the restriction π to X is a immersion.*
(2) *Assume that, in addition, $\forall x \in X, t \in \mathbb{R} \quad x + t\vec{v} \notin X$. Then the restriction π to X is an embedding.*
(3) *Show that one can always find such $\vec{v}$ if $N > 2n$ or $N > 2n+1$ respectively.*

Problem 3. *Let $f : \mathbb{R} \to \mathbb{R}$ be a smooth function, and y be its regular value. Is it true that $f^{-1}(U) \cong f^{-1}(y) \times U$ for a sufficiently small neighborhood U of y?*

Problem 4. *Let P_3 be the space of monic cubic polynomials of degree 3, and let $\Sigma = \{(p,t) | p \in P_3, t \in \mathbb{R}, p(t) = 0\} \subset P_3 \times \mathbb{R} \cong \mathbb{R}^4$. Is Σ a submanifold of $\mathbb{R}^4$? Let $\pi : \Sigma \to P_3$ be the projection. Find the critical values of π.*

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