MANIFOLDS: FALL 2014
EXERCISE 5

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ABSTRACT. Sard theorem

Problem 1. *Does there exists a smooth function* $f : \mathbb{R} \to \mathbb{R}$ *such that* $f^{-1}(a)$ *is uncountable for all* $a \in \mathbb{R}$*?*

Problem 2. *Let* $\Sigma = \{(p, v) | p \in \mathbb{R}P^1, v \in p\}$*, where* $\mathbb{R}P^1$ *is the set of lines in* $\mathbb{R}^2$ *passing through the origin.*

(1) *Show that* Σ *with its natural structure of smooth manifold is diffeomorphic to a Mobius band.*
(2) *Give a formula (in suitable charts) of the* blow-up *mapping* $\pi : (p, v) \to v$*. Is it a submersion? Find its singular locus.*
(3) *Write an equation of* $\pi^{-1}(\{y^2 = x^3\})$ *in a natural chart.*

Problem 3. *1.7.5. Show that the set of critical* points *of a smooth function* $f : \mathbb{R} \to \mathbb{R}$ *can be an arbitrary closed subset of* $\mathbb{R}$*.*

Problem 4. *Show that any smooth map* $f : \mathbb{S}^l \to \mathbb{S}^k$*,* $k > l$*, can be extended to a continuous mapping* $F : B^{l+1} \to \mathbb{S}^k$*, where* B^{l+1} *is a standard* $(l + 1)$*-dimensional ball,* $\mathbb{S}^l$ *is its boundary and* $F|_{\mathbb{S}^l} = f$*. In particular, any smooth loop in* $\mathbb{S}^k$*,* $k > 1$*, is homotopic to zero.*

Date: December 1, 2014.