

**MANIFOLDS: FALL 2014**  
**EXERCISE 6**

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**Problem 1.** *Let*

$$F : (\phi, \psi) \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos \phi & 0 & -\sin \phi \\ 0 & 1 & 0 \\ \sin \phi & 0 & \cos \phi \end{pmatrix} \begin{pmatrix} 2 + \cos \psi \\ \sin \psi \\ 0 \end{pmatrix}$$

*be a parameterization of the torus  $\mathbb{T}^2 \subset \mathbb{R}^3$  by  $[0, 2\pi) \times [0, 2\pi)$ . Draw the sets  $F^{-1}(\{z \leq c\})$  for various  $c$ .*

**Problem 2.** *1.7.9, 1.7.19*

**Problem 3.** *Give an example of a smooth (i.e. smoothly dependent on  $s$ ) family of smooth function  $f_s : \mathbb{R} \rightarrow \mathbb{R}$ ,  $s \in \mathbb{R}$  such that any close smooth family  $\tilde{f}_s$  (in a sense that  $\|\tilde{f}_s - f_s\|_{C^2} < \epsilon$  for some sufficiently small  $\epsilon > 0$ ) contains a function with a non-Morse critical point.*