MANIFOLDS: FALL 2014
EXERCISE 7

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Problem 1. *2.1.10, 2.1.11*

Remark 1. *Here* $\vec{n}(z)$ *should denote a tangent vector pointing inside, i.e. tangent to a curve* $\gamma : [0, \epsilon) \to X$, $\gamma(0) = z$ *and not tangent to* ∂X. *To define the* normal *vector one would need a scalar product on* $T_z X$, *which can be e.g. inherited from an embedding of* X *into* $\mathbb{R}^n$.

Problem 2. *2.2.1, 2.2.6, 2.2.7,*

Problem 3. *Let* $f : B^n \to B^n$ *be a smooth mapping.*

- *Assume that the restriction* $f|_{\mathbb{S}^{n-1}}$ *is identity. Show that* f *is onto.*
- *In particular, if* $v(x)$ *is a vector field on* $\mathbb{R}^n$ *such that* $v(x) = x$ *for* $x \in \mathbb{S}^{n-1}$, *then* v *has a singular point inside* B^n.
- *Assume that the restriction* $f|_{\mathbb{S}^{n-1}}$ *maps* $\mathbb{S}^{n-1}$ *into* $\mathbb{S}^{n-1}$ *and has no fixed points . Then* f *is onto.*

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