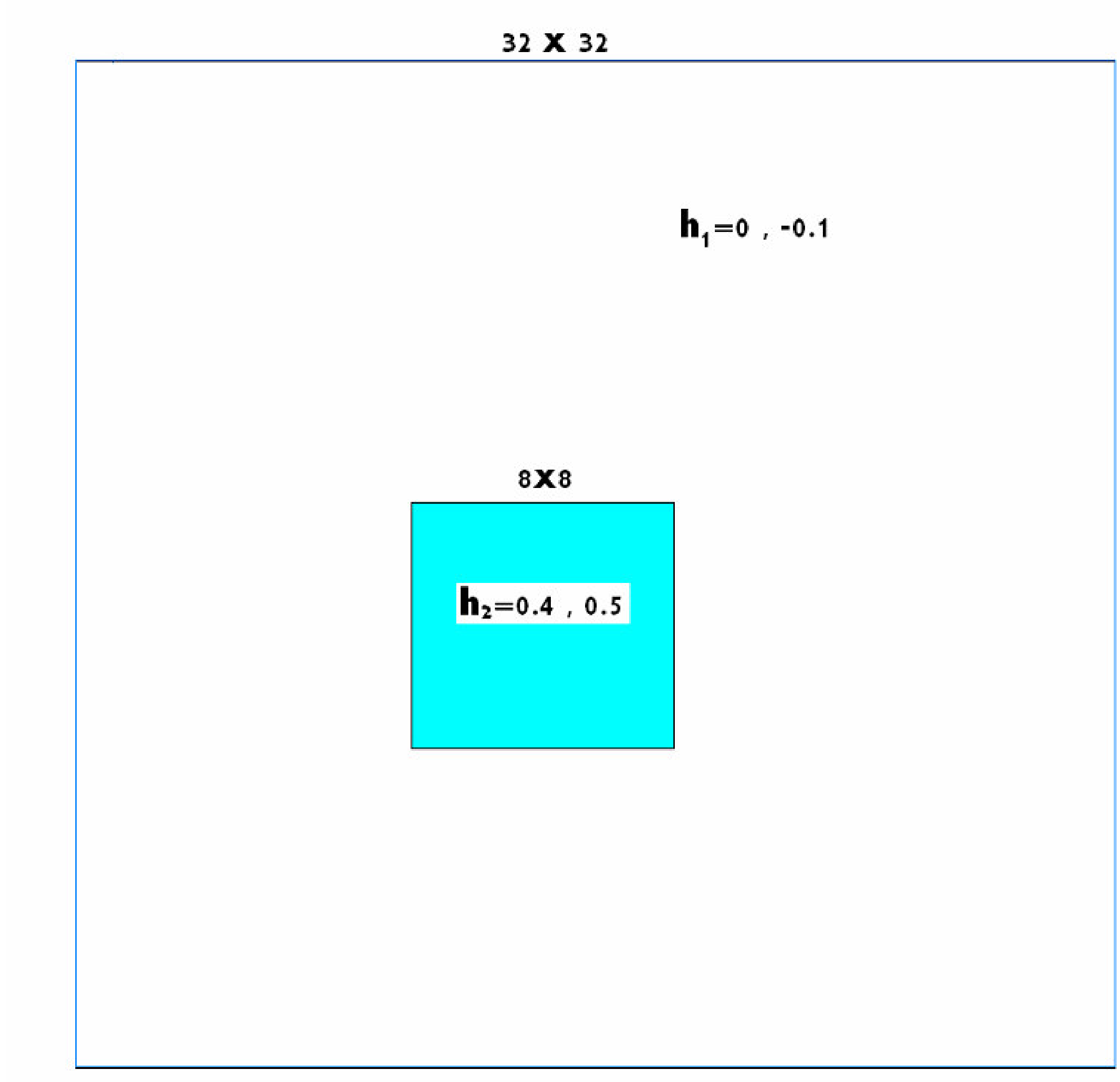
Hard Optimization Problems: Practical Approach

Problems for Lectures 1-3



1. 2D Ising spins exercise

- Minimize $-\sum_{<i,j>} s_i s_j - \sum_i h_i s_i$
- Periodic boundary condition
- Initialize randomly: $s_i = \pm 1$ with probability .5

1) Go over the grid in lexicographic order, for each spin choose 1 or -1 whichever minimizes the energy (choose with probability ½ when the two possibilities have the same energy) until no changes are observed.
2) Repeat 3 times for each of the 4 possibilities of ($\mathbf{h_1},\mathbf{h_2}$).
3) Is the global minimum achievable?
4) What local minima do you observe?

2. Pointwise relaxation for P=1

- Minimize $E(x) = \sum_{ij} a_{ij} | x_i - x_j |$
- Pick a variable x_i, fix all $x_j \quad j \neq i$ at $\tilde{x}_j$

➢ Minimize $E(x_i) = \sum_j a_{ij} | x_i - \tilde{x}_j |$

➢ Find the optimal location for x_i

3. Steepest descent exercise

For $E(x,y) = 2x^2 + y^2 + 100\sin(\frac{x}{10}) + 150\cos(\frac{y}{3})$

at $(x,y) = (12,15)$

Find the steepest descent direction

Compare its analytical and numerical calculations
Choose 2 small steps in this direction
Draw a parabola through the 3 points
Find the minimum of the parabola
Verify reduction in the energy
Find a step that increases the energy

4. Permutation's invariants

1) Prove that $\sum_{i=1}^{n} \tilde{x}_i^{\,m} v_i \quad , \quad m = 1,2$ are invariant under permutation.
2) Is it also true for m=3?

5. Lagrange multipliers inequality constraints

minimize x^2+y^2
subject to $x+2y<1$ **and** $1/2-y<0$
starting at $(1,1/4)$

1) Find the minimum
2) Calculate the Lagrange multipliers
3) Which constraint is binding, explain