

Hard Optimization Problems: Practical Approach

Problems for Lecture 6

Exc#6: Window relaxation for the graph drawing problem

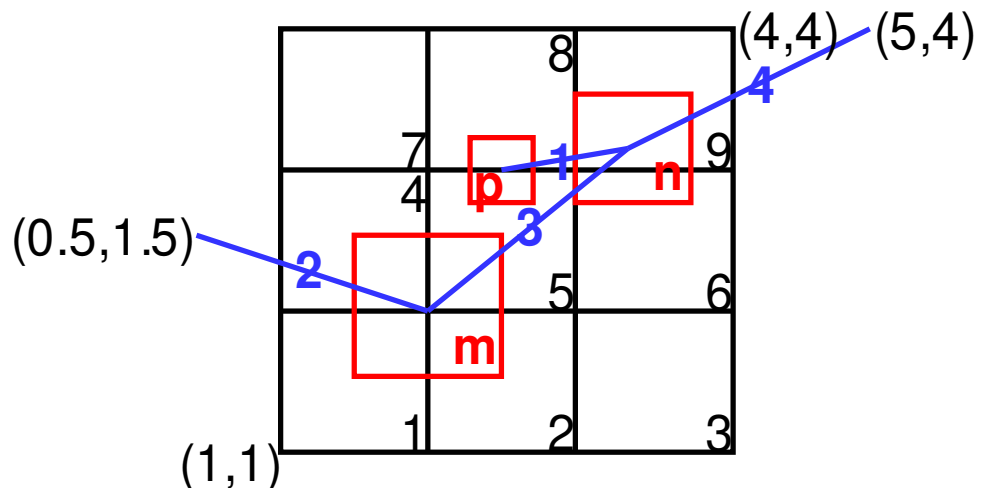
Consider the following window W of 3×3 squares containing the nodes m, n and p :

m is of size 1×1 located at $(2,2)$;

n is of size 0.8×0.8 located at $(3.4, 3.2)$;

p is of size 0.5×0.5 located at $(2.5, 3)$.

Find a correction to the locations of m, n and p such that the quadratic energy is minimized subject to inequality constraint demands that the area of nodes at each square ≤ 0.3



- a. Calculate the current amount of nodes' area present in each of the 9 squares
- b. Calculate a_{mkx} : the change (per unit length) in the amount of nodes' area induced by a small change in the x direction of node m to square k , $k=1, \dots, 9$. Similarly calculate a_{mky} , a_{nkx} , a_{nky} , a_{pkx} and a_{pky}
- c. Write the quadratic energy E as a function of the corrections to the variables in W
- d. Calculate the current value of E
- e. Write the 9 inequalities constraints associated with each square
- f. Choose the active set of constraints and write the Lagrangian
- g. Calculate the resulting system of equations and solve it
- h. Does the solution seem to be reasonable?
- i. Choose .25 of the solution, does E decrease at that point?
- j. Write the linear programming formulation

Exc#7: SA for the 2D Ising (see Exc#1)

Consider the following cases:

1. For $\mathbf{h}_1 = \mathbf{h}_2 = 0$ set a stripe of width 3, 6 or 12 with opposite sign
2. For $\mathbf{h}_1 = -0.1$, $\mathbf{h}_2 = 0.4$ set -1 at \mathbf{h}_1 and +1 at \mathbf{h}_2
3. Repeat 2. with 2 squares of 8×8 plus spins with $\mathbf{h}_2 = 0.4$ located apart from each other

Calculate T_0 to allow 10% flips of a spin surrounded by 4 neighbors of the same sign

Use faster / slower cooling scheduling

- a. What was the starting T_0 , E in each case
- b. How was T_0 decreased, how many sweeps were employed
- c. What was the final configuration, was the global minimum achievable? If not try different T_0
- d. Is it harder to flip a wider stripe?
- e. Is it harder to flip 2 squares than just one?