Hard Optimization Problems: Practical Approach

Problems for Lecture 8

Exc#9: Error calculations

1. Use Taylor expansion to calculate the error when *U''(x)* is approximated by

$$\frac{U(x-h)-2U(x)+U(x+h)}{h^2}$$

2. Find *a,b,c,d* and *e* such that

$$aU(x-2h)+bU(x-h)+cU(x)+dU(x+h)+eU(x+2h)=U''(x)+O(h^4)$$

This is a higher order approximation for *U''(x)* than the one in exercise 1.

Exc#10: Gauss Seidel relaxation

Solve the 1D Laplace equation U''(x)=0 by Gauss Seidel relaxation.

Start with the approximations 1. U_i = random(0,1) , 2. $U_i = \sin(\pi x)$, where $U_0 = U_N = 0$ for N=32.

Plot the L2 norm of the error and of the residual versus the number of iterations k=1,...,100, where the L2 norm of a vector v is $\| v \|_2 = [\frac{1}{n}\sum_{i=1}^{n} v_i^2]^{1/2}$

and the residual of LU=F is R=F-LU

Do you see a difference in the asymptotic behavior between the 2 norms?

Which case converges faster 1. or 2. , explain