Algorithmic Game Theory - handout 11 and 12

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We discussed the maximum welfare problem with submodular bidders and fractionally subadditive bidders. We presented greedy algorithms and algorithms based on the configuration LP. Some related references are listed below.

No need to hand in the following assignment.

Homework.

1. Given a universe M of m items, the indicator vector $\mathbf{x}$ of a set S is a vector $\mathbf{x} \in \{0,1\}^m$, in which $\mathbf{x}(i) = 1$ iff $i \in S$. A set function $f : \{0,1\}^m \longrightarrow R$ assigns a value to each set. The *Lovasz extension* f^L of f extends its domain to the convex set $[0,1]^m$ (allowing fractional values to the coordinates). Its value is defined as the following expectation: $f^L(\mathbf{x}) = E_\lambda[f(\mathbf{x}_\lambda)]$, where λ is chosen uniformly in the range $[0,1]$, and $\mathbf{x}(i) = 1$ if $\mathbf{x}_\lambda(i) \geq \lambda$ and $\mathbf{x}(i) = 0$ otherwise. Observe that f^L is equal to f on integer points. Show that if f is submodular then the corresponding f^L is a convex function. Namely, the region $f(\mathbf{x}) \leq t$ is convex.

2. In the max-coverage problem one is given a universe U of n items, a nonnegative value v_i for each item $i \in U$, a collection $\mathcal{S}$ of subsets of U, and a parameter k. The goal is to select k subsets from $\mathcal{S}$ so that the sum of values of items covered by the union of the selected subsets is maximized. Write an integer program expressing this problem. Relax the integer program to a linear program. Present a randomized rounding procedure for the linear program and prove that its expected approximation ratio is at least $1 - 1/e$.

   Remarks. There is also a greedy algorithm that approximates max coverage with a ratio of $1 - 1/e$. Achieving an approximation ratio better than $1 - 1/e$ for this problem is NP-hard.

References

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[2] Subhash Khot, Richard Lipton, Evangelos Markakis, and Aranyak Mehta. Inapproximability results for combinatorial auctions with submodular utility functions. In WINE, 2005.

[3] Benny Lehmann, Daniel J. Lehmann, Noam Nisan: Combinatorial auctions with decreasing marginal utilities. Games and Economic Behavior 55(2): 270–296 (2006).

[4] Jan Vondrak: Optimal approximation for the submodular welfare problem in the value oracle model. STOC 2008: 67–74.