Handout1, November 5, 2003

Hand in the homework assignment in two weeks time, by November 19, 2003. You may use either Hebrew or English.

We sketched in class how diagonalization can be used to prove hierarchy theorems. The purpose of this homework assignment is to make sure that students know how to handle the details that were missing in our description.

Prove that $SPACE(f(n)) \neq SPACE(g(n))$, were $g(n) > f(n)$ is some convenient function of your choice. (For example $g(n) = f(n) \log f(n)$.)

Among the details that need to be discussed are the following:

1. Does the proof hold for every possible function $f(n)$? If not, which conditions need to hold regarding the function $f(n)$?

2. What is the exact model of computation that is used in the proof? (For example: How many tapes does the Turing machine have? What is the size of alphabet?)

3. Make sure that you are using the same model of computation both for SPACE(f(n)) and for SPACE(g(n)). This may require to explain how one simulates a Turing maching with a large alphabet size or several tapes by a Turing machine with smaller alphabet size or smaller number of tapes.

4. Does the theorem apply when $f(n) < n$? In particular, it is desirable to be able to deduce that $SPACE(\log n) \neq SPACE((\log n)^2)$.

You are free to consult any books, notes etc., and also to discuss the assignment with other students. However, once you know and understand the answer, write it down yourself.