Complete languages

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Definitions:

A *log-space transducer* is a deterministic Turing machine with a read-only input tape, a write-only output tape, and a log-space work tape.

Language A is *logspace reducible* to language B, denoted as $A \leq_L B$, if there is a log-space computable function (by a log-space transducer) reducing A to B.

Languages:

- $3SAT = \{\phi \mid \phi$ is a satifiable 3CNF Boolean formula$\}$. A 3CNF formula (CNF means "Conjunctive Normal Form") is a collection of clauses, where each clause contains at most 3 literals (a Boolean variable or its complement), and a truth assignment to the variables satisfies the formula if in every clause at least one literal is satisfied.
- $PATH = \{(G, u, v) \mid$ there is a directed path from vertex u to vertex v in digraph G$\}$. Also known as STCON.
- $GG = \{(G, v) \mid$ player 1 has a winning strategy on (G,v)$\}$. *Generalized geography* (GG) is a game in which players take turns in choosing edges of a simple path that starts in vertex v of digraph G, and the first who cannot move loses.

In class we show that 3SAT is NP-complete under polynomial time reductions, and that PATH is NL-complete under logspace reductions.

Homework:

1. Show that if $A \leq_L B$, and $B \leq_L C$, then $A \leq_L C$.
2. GG is PSPACE-complete under logspace reduction (you are not asked to prove this). Show that $GG \notin NL$.
3. Assume that someone shows that $NL \neq P$. Does the fact that 3SAT is NP-complete under polynomial time reductions then imply that $3SAT \notin NL$? Explain your answer.
4. Show that 3SAT is NP-complete under logspace reductions.