PCP Course, Take Home Exam

In the course we proved the PCP theorem, up to the point that we could show that max-3SAT is NP-hard to approximate within a ratio better than 7/8. However, some parts we did not cover. These include:

1. Full analysis of the total degree test.
2. Proof of the parallel repetition theorem.
3. Fourier analysis for max-3LIN.
4. Achieving perfect completeness in the hardness of approximation for max-3SAT.

The purpose of the take home exam is to fill in one of these gaps, namely, item 3 above. The issue of Fourier analysis appears in many tight hardness of approximation results (and other questions in mathematics and CS), hence the importance of gaining familiarity with it.

Consider Hastad's PCP showing that max-3LIN is NP-hard to approximate within a ratio better than 2 (e.g., as given in Hastad's paper, or in the lecture notes). You may assume without proof all properties stated for the outer PCP system. Give a full detailed proof of the properties of the inner PCP system. Namely, show that if the outer PCP system accepts with probability at most δ, then the combined outer+inner PCP system accepts with probability at most $1/2 + \epsilon$ (where ϵ tends to 0 as δ tends to 0).

Please hand in by August 1, 2003. (If you need an extension, let me know in advance.) You may (and should) use any information available on the subject of the exam, including lecture notes, Hastad's paper (available on Johan Hastad's home page), or other papers explaining the result (such as Chapter 4 of the PhD thesis of Jonas Holmerin). Hastad's paper may be difficult to read, but note that many parts of it are not needed in order to answer the question. You may also discuss the question with friends. The goal is that you understand the answer, and it is less important here whether you figured it out yourself, read it somewhere, or had someone explain it to you. However, once you understand the answer, write it down yourself, in your own words. Its best to submit the answer electronically as a ps file (but without cutting and pasting from someone else!). Handwritten answers are also fine, if the handwriting is clear and easily readable.

Good luck!