Algorithms – handout 8

Matchings continued

January 9, 2002

We shall show the algorithm for finding maximum cardinality matchings in general graphs. This will lead to the Gallai-Edmonds decomposition of a graph.

We shall then show an algorithm for finding minimum weight perfect matchings in complete bipartite graphs. In doing so, we shall prove the following theorem of Egevary (from 1931).

Theorem: Let $G(U \cup V, E)$ be a complete bipartite graph with $|U| = |V|$, and with edge weights $w : E \longrightarrow R$. A cost function $c : U \cup V \longrightarrow R$ for the vertices is *feasible* if for every edge (u, v), $c(u) + c(v) \leq w((u, v))$. Then the weight of the minimum weight perfect matching is equal to the maximum of $\sum_{v \in U \cup V} c(v)$ over all feasible cost functions.

Homework:

1. Vizing's theorem says that the edges of every simple graph (no parallel edges or self loops) can be partitioned into $\Delta + 1$ matchings, where Δ denotes the maximum degree. Show that for bipartite graphs, Δ matchings suffice. (Hint: use last week's homework.)

2. Show that the problem of finding a maximum weight matching can be reduced in polynomial time to the problem of finding a minimum weight perfect matching in complete graphs. Moreover, show that the reduction can be made to preserve bipartiteness. (Hint: change the original graph by adding new vertices and edges, and by modifying weights of edges.)