Some remarks on final exam.

1. SDP for theta function. It is not wise to have separate variables for y_{ij} and y_{ji}, and the constraint $y_{ij} = y_{ji}$, because then to solve the SDP one would need to add slackness to this constraint, ruining the symmetry. Instead, there should be just one variable y_{ij} appearing in two symmetric locations in the matrix.

2. Bounded ball. It is not enough to say that one adds slackness to the constraints. One should also analyse the effect of this on the radius of the bounded ball (especially, given that there is also the semidefiniteness constraint). An ideal answer would give the radius of the bounded ball explicitly. Alternatively, one may use a bounded "box" of nonnegligible volume, and say what its sidelength is (and hence its volume).

3. To show that a graph is perfect (e.g., the graph obtained by duplicating vertices of a perfect graph), one can either show that for all its subgraphs, the clique size and chromatic number are the same, or use Berge's conjecture (which now is a theorem). However, to correctly use Berge's conjecture, one should show that both the graph *and* its complement do not contain chordless odd cycles (showing just one of them does not suffice).

4. The Ellipsoid algorithm is a method to find feasible solutions. If one wishes to use it in order to find optimal solutions, one needs to explain exactly how it is done. (This may effect the question of whether for the primal LP with clique constraints, the Ellipsoid algorithm really finds sufficiently many "essential" cliques so as to make the solution also optimal with respect to those cliques found.)

5. If the set of "essential" cliques only guarantees a near optimal solution, then the dual of the resulting LP does not have value exactly equal to $\alpha(G)$. (For this reason one needs to take a clique that the dual solution gives maximum weight, rather than any clique of positive weight.)

6. To establish the fact the the clique that received maximum weight in the dual has nonnegligible weight, one can use the fact that in LP, given a solution, one can move in polytime to a basic feasible solution of at most the same value. In this BFS, the weight of the maximum weight clique can be bounded as a function of the number of contraints rather than the number of variables. (In our context, the number of variables (cliques) in the dual may depend on the accuracy with which we solve the primal, and this may lead to cyclic arguments in trying to get good enough accuracy.)

7. Given the above, it may be wise to use the ellipsoid algorithm for the primal LP to search for (and not find) a solution of value at least $\alpha(G) + 1/2n$, and consider the essential cliques found in this process (which is stopped when good enough precision is reached).

8. In a perfect graph, not every maximum clique intersects every maximum independent set. (E.g., a path with 4 vertices.)