Algorithms – handout 1

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This is an advanced course on algorithms. It will mainly deal with linear programming and related topics. Students are assumed to be familiar with algorithms, data structures, probability theory and linear algebra, at an undergraduate level.

Homework assignments are very important to the process of learning, and also effect the final grade to a large extent. Try to do the best you can. They should be handed in within two weeks.

The first topic in the course is *matchings*. We shall show how to find maximum cardinality matchings in bipartite graphs, and more generally, in arbitrary graphs (Edmonds' algorithm). The algorithm uses the notion of alternating paths. Similar to greedy algorithms, it constructs a sequence of partial solutions, each larger than the previous one. However, unlike greedy algorithms, when extending from one solution to another, it retracts some of the decisions of the former solution.

The proofs of optimality for our algorithms will involve exibiting a lower bound to the size of the matching. In the bipartite case, this lower bound is the size of the minimum vertex cover. In bipartite graphs, maximum matchings and minimum vertex covers may be viewed as *duals* of each other. Duality theory plays an important role in this course, and is a central notion for linear programming.

Homework:

1. Show that a bipartite graph $G(V,E)$ has a perfect matching iff for every set H of vertices, $|N(H)| \geq |H|$, where $N(H)$ is the set of neighbors of H. (H and $N(H)$ need not be disjoint.)

2. Show that every bipartite d-regular graph is the disjoint union of d perfect matchings.

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