Handout 3

LP, Algebraic formulation

March 24, 2004

Topics covered: formulating problems as a linear program, transformation to *standard* form, basic solutions of linear programs.

Homework

1. Consider the problem of minimizing the ratio

$$\frac{c^t x}{f^t x}$$

of two linear functions, subject to all the following constraints:

- $Ax \geq b$,
- $f^t x \geq 1$,
- $c^t x \geq -8$,
- $c^t x \leq 8$.

Show how linear programming can be used as a subroutine so as to find the optimal solution within any degree of accuracy (the running time may depend on the degree of accuracy required). (Hint: consider the problem of deciding whether the objective function is at most a given value.)

2. Recall that a degenerate solution to an LP in standard form is one in which the number of nonzero variables is less than m. Recall also that a basic feasible solution is determined by a basis of m linearly independent columns for the constraint matrix.

   (a) Show that if two bases give the same solution, then this bfs is degenerate. (This was shown in class, but write down the proof.)

   (b) Show that the converse of the above is not always true. That is, write down an LP in standard form that has a degenerate feasible solution, and there is only one basis that gives this degenerate solution.