# Take home exam

Algorithms Course

10 July, 2002

A polynomial time algorithm for finding a minimum coloring of perfect graphs was presented in class. The presentation was rather sketchy. You are required to write a detailed explanation of this algorithm (or of a variation of the algorithm, if you wish). In doing so, you may use your own notes from class, the lecture notes on the web, any books, and so on. But please do this work alone (do not use other students). You may hand in this exam either in English or in Hebrew, either printed or in CLEAR handwriting. Please hand in by August 1, 2002.

Among other things, the explanation should include the following:

1. How does one formulate a semidefinite program for the chromatic number in perfect graphs, and why is the value of the program equal to the chromatic number in this case.

2. How can the ellipsoid algorithm be used in order to (approximately) solve this semidefinite program. In particular, how does one find a bounding ball, a bounded ball, separating hyperplanes for infeasible points.

3. How does one use approximate solutions to the semidefinite program in order to find a maximum clique in a perfect graph. How does one find a clique of maximum weight (or near maximum weight) in a vertex-weighted perfect graph.

4. How does one find an independent set that intersects all cliques of maximum size. If an LP is used for this, explain how it is solved, and why it is the case that the independent set that is found intersects all maximum cliques.

The explanations should be at a level that other students with general knowledge about algorithms and linear programming (but no specific knowledge about perfect graphs and semidefinite programming) can follow, understand and verify.