Algorithms – part 2

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This part of the algorithms course will deal mostly with Linear Programming and related topics. There are many good books on Linear Programming, that cover large parts of the course. Other parts are based on recent conference/journal papers. Students who have not taken the first part of the course are welcome to join.

First lecture: formulating problems as a linear program, transformation to *standard* form, basic solutions of linear programs.

Homework

1. A factory produces natural paint in three colors, red, green and yellow. Red paint sells at 3$ a gallon, green paint sells at 4$ a gallon, and yellow paint sells at 5$ a gallon. To produce a gallon of red paint it takes 7 kilos of tomatos, 2 kilos of bananas, and 1 kilo of apples. To produce a gallon of green paint it takes 2 kilos of tomatos, 1 kilo of bananas, and 8 kilo of apples. To produce a gallon of yellow paint it takes 3 kilos of tomatos, 7 kilos of bananas, and no apples. The factory has 200 kilos of tomatos, 190 kilos of bannanas, and 180 kilos of apples.

   Write a linear program in standard form that expresses the mix of products that maximizes the profit for the factory, subject to the above constraints. (Fractions of gallons sell for the corresponding fraction of their price.)

   Find two different basic feasible solutions for your Linear Program.

2. Consider the problem of minimizing the ratio
   $$\frac{c^t x}{f^t x}$$
   of two linear functions, subject to $Ax \leq b$ and $f^t x > 0$. Suppose that we further know that the value of the objective function is between two values L and H. Show how linear programming can be used as a subroutine so as to find the optimal solution within any degree of accuracy (the running time may depend on the degree of accuracy required). (Hint: consider the problem of deciding whether the objective function is at most a given value.)

Grades for the previous semester: will be based on the final exam and 10 best (out of 12) homework assignments. If you had a good reason for not handing in some of the homework assignments please let me know. (Especially if you handed in less than 10 homework assignments.)