Geometry of linear programs

Uri Feige

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Some lecture notes will be avaliable on my home page soon.

In Lecture 1 we presented linear programs from a linear algebra point of view. The use of a geometric representation helps build intuition about linear programming. This is easiest to visualize in two dimensions (when there are only two variables, or only two constraints), and still manageable in three dimensions. Much of the low dimension intuition is also true in higher dimensions, and can be formalized and applied for linear programs with an arbitrary number of dimensions.

Homework: A *vertex* of a polyhedron is a face of dimension 0. An *extreme point* in a polyhedron is a point that is not the convex combination of any other two points in the polytope. A *basic feasible solution* for a system of linear inequalities (the set of feasible solutions is the polyhedron) is a feasible solution for which n linearly independent constraints are tight (nonnegativity constraints, if they exist, are treated like any other contraint).

The following lemma will be proved in class for linear programs in standard form. As homework, write down a full proof for the case that the linear program is in general form.

Lemma: For a polyhedron P and a point $x \in P$, the following three statements are equivalent.

1. x is an extreme point of P.
2. x is a vertex of P.
3. x is a basic feasible solution.