Algebraic   Geometry   for   Theoretical   Computer   Science
Assignment 10
Lecturer: Gil Cohen   *Hand in date: January 15, 2015*

Instructions: Please write your solutions in LaTeX / Word or exquisite handwriting. Submission can be done individually or in pairs.

Let p be a prime power, and let $q = p^2$. In class we saw the Hermitian function field $\mathbb{F}_q(x, y)/\mathbb{F}_q$, with the defining equation $y^p + y = x^{p+1}$. We now consider the algebraic extension $\mathbb{F}_q(x, y, z)/\mathbb{F}_q$ of the Hermitian function field, with the defining equation $z^p + z = y^{p+1}$. We denote this function field by $H^2/\mathbb{F}_q$.

1. How many rational places $H^2/\mathbb{F}_q$ has?

2. Show that the function x has a unique pole at $H^2/\mathbb{F}_q$, and that it lies over P_∞ – the unique pole of x at $\mathbb{F}_q(x)$. We denote this unique pole by P'_∞.

3. What is the Weierstrass semi-group of P'_∞?

4. What is $\dim(r \cdot P'_\infty)$? (an answer up to a constant multiplicative factor will do).

5. What are the parameters of the Goppa code one obtains by taking $\mathcal{L}(r \cdot P'_\infty)$ as the space of functions to be evaluated, and the rational places from the first item (not including P'_∞) as the set of evaluation places?

6. What are the parameters of the small-bias set one gets by applying the general transformation we saw in class to this code?