Instructions: Please write your solutions in \LaTeX / Word or exquisite handwriting. Submission can be done individually or in pairs.

1. Let $\mathcal{O}$ be a valuating ring of the rational function field $K(X)/K$. Assume that $x \notin \mathcal{O}$. Show that in this case, $\mathcal{O} = \mathcal{O}_\infty$. This completes the proof done in class, and shows that we have accounted for all places of the rational function field.

2. Let $A, A'$ be two divisors of $F/K$ such that $A \sim A'$. Show that $\mathcal{L}(A)$ and $\mathcal{L}(A')$ are isomorphic as vector spaces over $K$.

3. Let $F/K$ be a function field. Show that $\mathcal{L}(0) = K$.

4. Let $A$ be a divisor of a function field. Show that if $A < 0$ then $\mathcal{L}(A) = \{0\}$.

5. Let $A$ be a degree zero divisor of a function field. Prove that the following assertions are equivalent.

   (a) $A$ is principle.

   (b) $\ell(A) \geq 1$.

   (c) $\ell(A) = 1$. 