

Growth Competition and Hyperbolicity

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Consider a competition between growing clusters of red and blue on a graph. Start with a red vertex and a blue vertex. Every turn the blue grow, coloring blue all its neighbors that are not already colored red. A vertex that gets colored stay colored forever. The red is lazy and grows only at even times. If both clusters aim at the same vertex the blue win. It is easy to observe that on Euclidean lattices the blue will surround the red after some finite time (identify the shape).

A while ago (PAMS 2002) we observed that on hyperbolic graphs there are ways to place the blue and red which guarantee that the red will grow forever.

I forgot to ask the following question: assume G is a Cayley graph of an infinite group, does the weak (red) can survive, (grow forever from some starting vertex), if and only if G is Gromov hyperbolic?

What drives the survival of the weak in a hyperbolic graph, is that geodesic diverge exponentially, in principle slightly weaker uniform divergence can still imply survival of the reds.