How is large growth archived?

11.9.07

Could it be that any Cayley graph of a finitely generated group of super polynomial volume growth either has infinite asymptotic dimension, in the sense that the group contains discrete hypercubes of arbitrarily dimension, (were contains means there is a bi-Lipschitz embedding of a cube after rescaling the length of the edges), or it contains a stretched binary tree as a subgraph? Were by stretched we mean each edge in the binary tree can be replaced by a path of length up to k for some fixed $k \in \mathbb{N}$.

If true it will be a pleasing result in the sense that we will learn what "mechanisms" creates large growth. A counter example might be even more enlightening.

A related problem is to show that all Cayley graph of a finitely generated group of super polynomial volume growth contains bi-Lipschitz embedding of rescaled simplexes of any dimension.