Introduction to Computational Complexity - Assignment 2

To be submitted by 9/3/2008

1. Prove that $\mathcal{ZPP} = \mathcal{RP} \cap \text{co}\mathcal{RP}$, or complete the outline given in class.

2. Prove that $\mathcal{PP}$ is computationally equivalent to $\#\mathcal{P}$, where two classes are computationally equivalent if each problem in one class is polynomial-time reducible to some problem in the other class.

3. Assuming that each problem in $\#\mathcal{P}$ can be approximated to within some constant factor, show how to obtain an approximation that is within any $(1 \pm (1/\text{poly}))$ factor.
That is, assume that there exists some constant $c \in (0,1)$ such that for any $R \in \mathcal{PC}$ there exists an algorithm A such that for any x it holds that $A(x) \in [c \cdot |R(x)|, |R(x)|/c]$. Show that, under this assumption, for any $R \in \mathcal{PC}$ and any positive polynomial p there exists an algorithm A' such that $A'(x) \in [(1 - 1/p(|x|)) \cdot |R(x)|, (1 + 1/p(|x|)) \cdot |R(x)|]$.
Hint: For any $R \in \mathcal{PC}$ and any polynomial t, consider the relation R' such that

$$R'(x) \stackrel{\text{def}}{=} \{(y_1, ..., y_{t(|x|)}) : (\forall i \in [t(|x|)])\ y_i \in R(x)\}$$

for some appropriate function $t(n)$.