Complexity Theoretic Aspects of Property Testing

ODED GOLDREICH

Some complexity theorists may view property testers as PCPs of Proximity without the proof part. In general, property testing is concerned with approximate decisions, where the task is distinguishing between objects having a predetermined property and objects that are "far" from having this property. A potential tester is a randomized algorithm that queries the (representation of the) tested object at locations of its choice.

On the relation between adaptive and non-adaptive query complexity of graph properties in the adjacency matrix model. For any fixed property Π, let q denote the query complexity of (general, i.e., adaptive) testing of Π, and Q denote the corresponding non-adaptive query complexity (i.e., which refers to non-adaptive testers of Π). Following is a list of known and conjectured results, where $\widetilde{\Omega}$ and $\widetilde{\Theta}$ denote bounds with a slackness of a polylogarithmic factor.

- Theorem (see [3]): For any graph property in the adjacency matrix model, it holds that $Q = O(q^2)$.
- Theorem in [2]: There exist graph properties in the adjacency matrix model such that $Q = \widetilde{\Theta}(q)$. Actually, $Q = O(q)$ and even $Q = q$ are known too.
- Theorem in [2]: There exists a graph property in the adjacency matrix model such that $Q = \widetilde{\Theta}(q^{4/3})$.
- Theorem in [2]: There exists a graph property in the adjacency matrix model such that $Q = \widetilde{\Omega}(q^{3/2})$.
- Conjecture in [2]: For every integer $t > 2$, there exists a graph property in the adjacency matrix model such that $Q = \widetilde{\Theta}(q^{2-(2/t)})$. This conjecture is supported by a theorem that establish the same relation relation for a promise problem.

All existential results are proved using natural graph properties.

Hierarchy Theorems for Property Testing. Such results are proved for three central models of property testing: the general model of generic function, the model of bounded-degree graph properties, and the model of dense graph properties (in the adjacency matrix model). From a technical perspective, the treatment of the latter is most interesting, since it raises and resolves various natural questions regarding graph blow-up. For details, see [1].

REFERENCES

[1] O. Goldreich, M. Krivelevich, I. Newman, and E. Rozenberg, *Hierarchy Theorems for Property Testing* in the proceedings of 13th RANDOM, Springer LNCS, Vol. 5687, pages 504-519, 2009.

[2] O. Goldreich and D. Ron, *Algorithmic Aspects of Property Testing in the Dense Graphs Model* in the proceedings of 13th RANDOM, Springer LNCS, Vol. 5687, pages 520–533, 2009.

[3] O. Goldreich and L. Trevisan, *Three Theorems regarding Testing Graph Properties* Random Structures and Algorithms, Vol. 23 (1), pages 23–57, August 2003.