Comments on the paper "Three Theorems regarding Testing Graph Properties"

Oded Goldreich
Department of Computer Science
Weizmann Institute of Science
Rehovot, ISRAEL.
oded@wisdom.weizmann.ac.il

Luca Trevisan
EECS – Computer Science Div.
UC-Berkeley
Berkeley, CA 94720, USA.
luca@eecs.berkeley.edu

July 16, 2002

Simpler proof of Claim 3.1 (suggested by Amir Shpilka). The key observation is that the set of final graphs is very sparse.[1] Specifically, each basic graph gives rise to at most $N!$ secondary graphs, and each of the latter gives rise to at most $2^{\frac{2}{3}\binom{N}{2}} < 2^{N^2/3}$ final graphs (because each secondary graph misses at most $\frac{2}{3}\binom{N}{2}$ edges). Thus, the number of final graphs is at most

$$2^{2t+o(t)} \cdot (N!) \cdot 2^{N^2/3} = 2^{((2/1000)+o(1)+(1/3))\cdot 2\cdot\binom{N}{2}} < 2^{0.7\cdot\binom{N}{2}}$$

Thus, for sufficiently small $\epsilon > 0$ (e.g., $\epsilon = 0.1$ will do), less than 10% of the graphs are ϵ-close to some final graph. ■

Stronger statements of Theorem 2 and Lemma 4.4 and a simpler proof of the latter (suggested by an anonymous reviewer). In Theorem 2, we may have $q'_\Pi(N,\epsilon) = O(q_\Pi(N,\epsilon)^2)$ rather than $q'_\Pi(N,\epsilon) = O(q_\Pi(N,\epsilon)^4)$. The improvement is due to a new version of Lemma 4.4 that, assuming that the tester has error 1/6 (rather than 1/3), derived a tester with a deterministic decision without increasing the query complexity. The idea is that the new tester will inspect the induced subgraph and accept the input graph if and only if the probability associated with the induced subgraph (by the original tester) is at least 1/2. Clearly, this only doubles the acceptance probability, and so graphs that are ϵ-far from π are accepted with probability at most $2\cdot(1/6) = 1/3$. On the other hand, for each graph that has property Π, the probability that the original tester sees as subgraph associated with probability less than 1/2 is at most 1/3, and so the new tester accepts such a graph with probability at least 2/3. Finally, observe that error reduction (from 1/3 to 1/6) should be performed before passing to an isomorphism-oblivious decision (because it is not clear how to perform error reduction in a way that preserves isomorphism-oblivious decisions). ■

[1] This observation has escaped us due to the way we have developed our proof.