Nesting Hybrids

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Adaptive vs. Selective Security

adaptive attack by A on $\{\Pi_n\}_{n\in\mathbb{N}}$

- A queries $\Pi_n(.)$
- A chooses challenge $x^* \in \{0,1\}^n$
- A must break $\Pi_n(.)$ on input x^*

Adaptive vs. Selective Security

selective attack by A on $\{\Pi_n\}_{n\in\mathbb{N}}$

- A chooses challenge $x^* \in \{0,1\}^n$
- A queries $\Pi_n(.)$
- A must break $\Pi_n(.)$ on input x^*

Adaptive vs. Selective Security

Lemma (Security Leveraging)

If A breaks adaptive security with advantage $\epsilon \Rightarrow$ can use A to break selective security with advantage $\epsilon/2^n$.

selective attack using adaptive A

- guess random challenge $x' \in \{0,1\}^n$
- A queries $\Pi_n(.)$
- A chooses challenge x^*
- if $x' \neq x^*$ give up
- A must break $\Pi_n(.)$ on input x^*

Nested Hybrids in a Nutshell

proving adaptive security via leveraging

1. adaptive $\Pi_n \rightarrow$ selective Π_n (losing factor 2^n).
2. selective $\Pi_n \rightarrow \Phi$ (hybrid argument looses poly(n)).

Nested Hybrids in a Nutshell

proving adaptive security via leveraging

1. adaptive $\Pi_n \rightarrow$ selective Π_n (losing factor 2^n).
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nesting hybrids

1. adaptive $\Pi_n \rightarrow$ adaptive* Π_n (losing small factor α)
2. adaptive* $\Pi_n \rightarrow$ adaptive $\Pi_{n/2}$ (hybrid losing factor β)
3. iterate 1 and 2 $\log(n)$ times:
   adaptive $\Pi_n \rightarrow$ adaptive Π_1 losing $(\alpha\beta)^{\log(n)}$
4. adaptive $\Pi_1 \rightarrow \Phi$ lossless.

Applications

	old	new
GGM Constrained PRF[1] loss in reduction to PRG $n =$ input length, $q = \#$ queries	2^n	$q^{\log n}$
Generalized Selective Decryption[2] loss in reduction to ENC caveat : on trees $n = \# keys$	2^n	$2^{\log^2 n}$

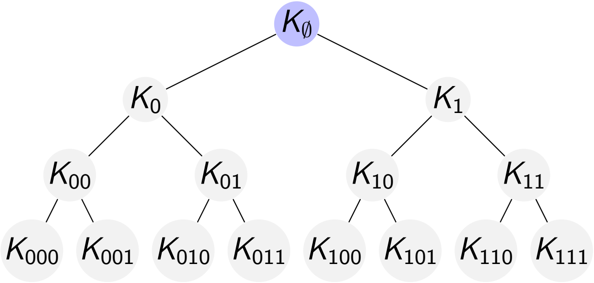
[1] *Functional Signatures and Pseudorandom Functions.* Elette Boyle, Shafi Goldwasser, Ioana Ivan **eprint.iacr.org/2013/401**
Constrained Pseudorandom Functions and Their Applications. Dan Boneh and Brent Waters **Asiacrypt 2013**
Delegatable Pseudorandom Functions and Applications. A.Kiayias, S.Papadopoulos, N.Triandopoulos, T.Zacharias. **CCS 2013**

[2] *Tackling Adaptive Corruptions in Multicast Encryption Protocols.* Saurabh Panjwani **TCC 2007**

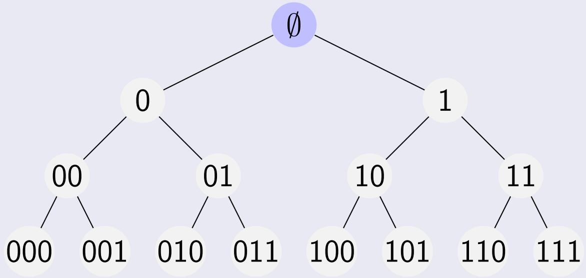
Goldreich-Goldwasser-Micali 84

GGM PRF $\mathsf{F}_K(x) = K_x$

- PRG $\mathsf{G} : \{0,1\}^n \to \{0,1\}^{2n}$
- $K = K_\emptyset \leftarrow \{0,1\}^n$
- $K_{x\|0} \| K_{x\|1} = \mathsf{G}(K_x)$



GGM Hybrid Argument

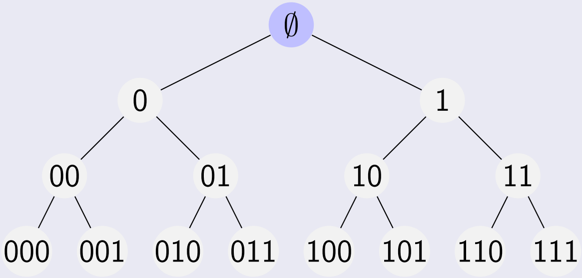


- $\mathsf{Adv}(H_0, H_{qn}) = \epsilon$ $\quad$ $q = \#$queries , n=input length.

$\Rightarrow$ $\mathsf{Adv}(H_i, H_{i+1}) \geq \epsilon/qn$

$\Rightarrow$ $\mathsf{Adv}(\mathsf{G}(U_\lambda), U_{2\lambda}) \geq \epsilon/qn$

GGM Hybrid Argument



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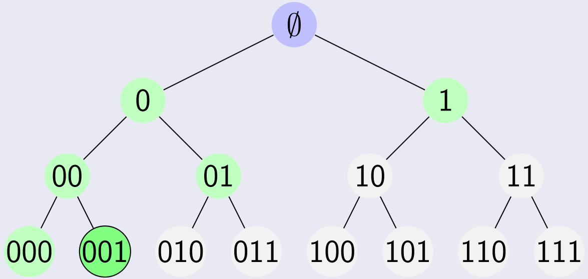

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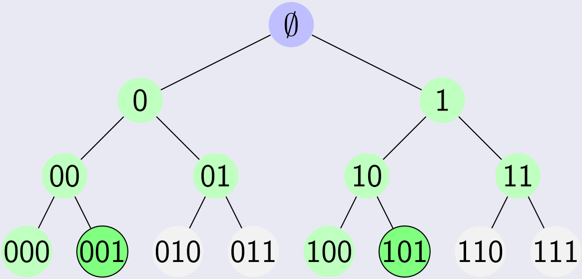
GGM Hybrid Argument

Hybrid H_0 (the real game)



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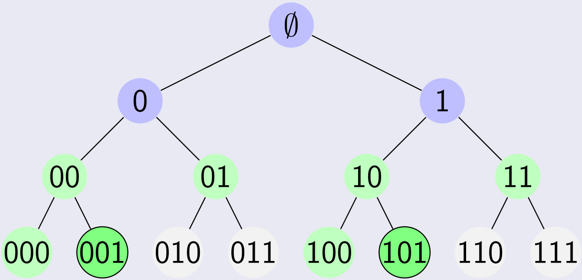
GGM Hybrid Argument

Hybrid H_1



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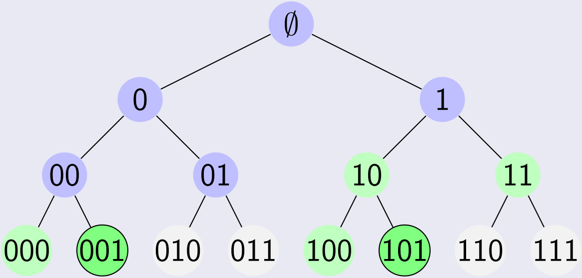
GGM Hybrid Argument

Hybrid H_2



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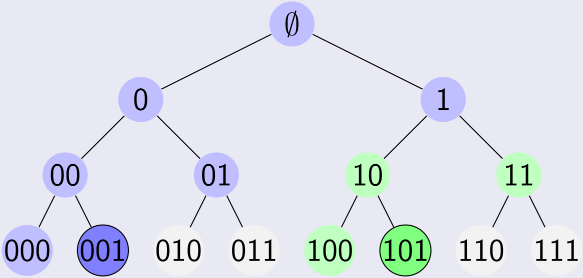
GGM Hybrid Argument

Hybrid H_3



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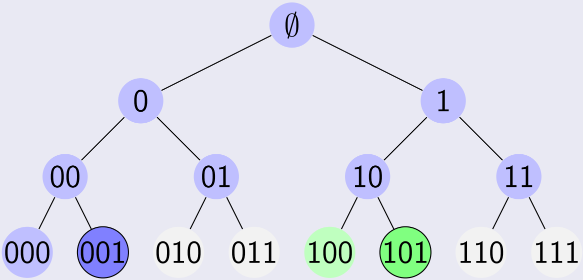
GGM Hybrid Argument

Hybrid H_4



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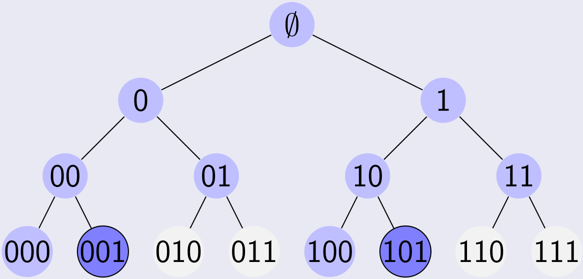
GGM Hybrid Argument

Hybrid H_5 (the random game)



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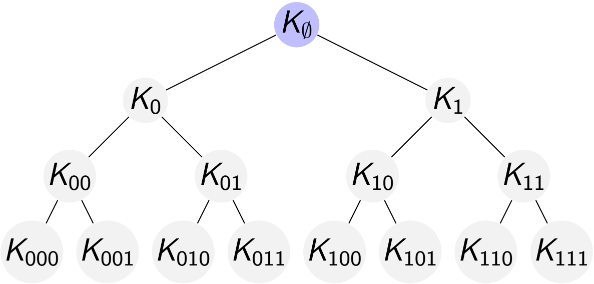
Constrained/Delegatable/Functional GGM PRF



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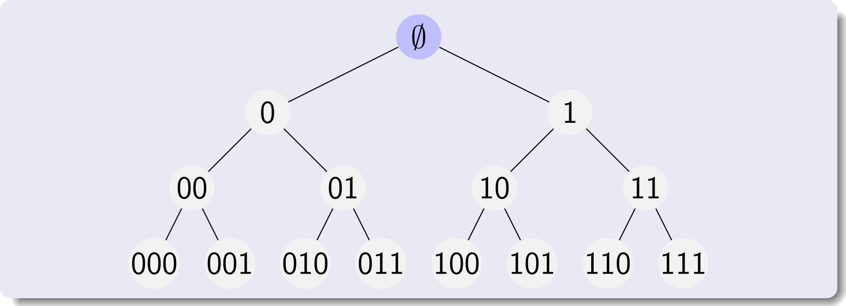

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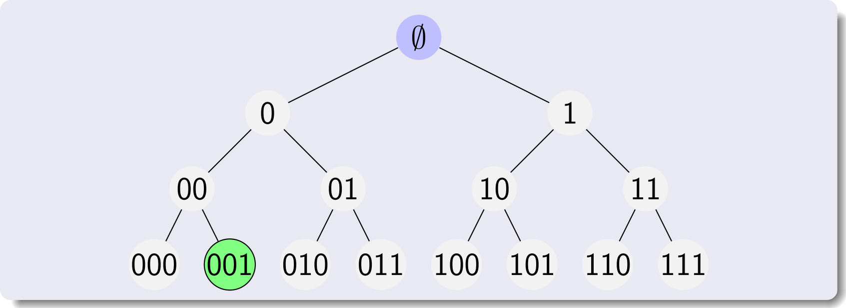

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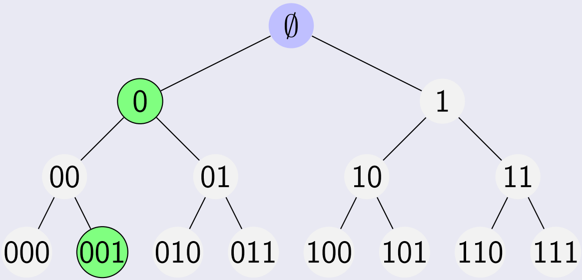
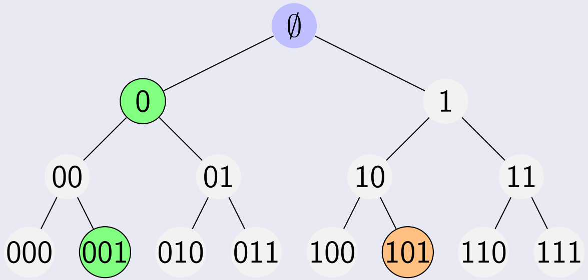


$K_{x\|y}$ trivially distinguishable from random given K_x.

Constrained/Delegatable/Functional GGM PRF



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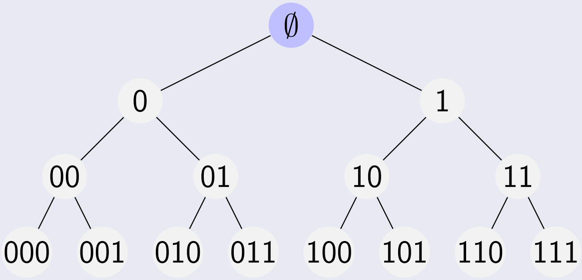
Security game for constrained PRFs

- choose x^* where no prefix of x^* was queried.
- distinguish K_{x^*} from random.

Proving Selective Security



- $\mathsf{Adv}(H_0, H_6) = \epsilon$

$\Rightarrow$ $\mathsf{Adv}(H_i, H_{i+1}) \geq \epsilon/6$

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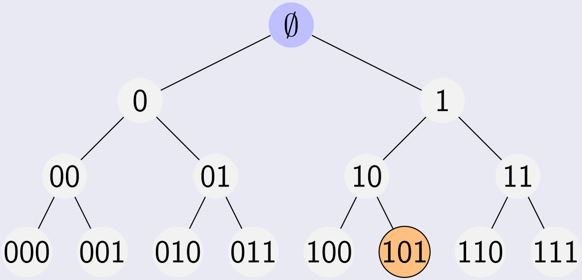
Proving Selective Security

Choose challenge



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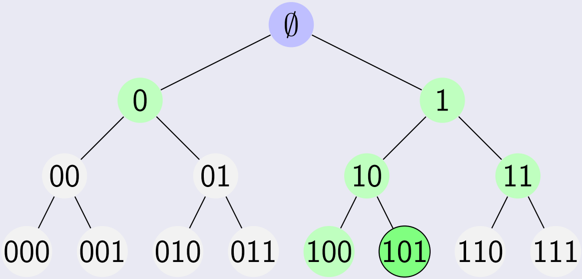
Proving Selective Security

Hybrid H_0 (real game)



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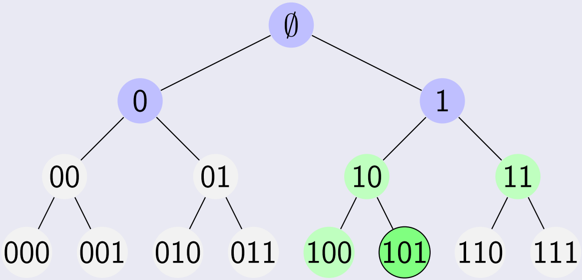
Proving Selective Security

Hybrid H_1



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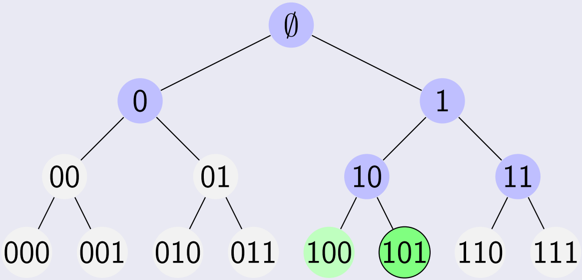
Proving Selective Security

Hybrid H_2



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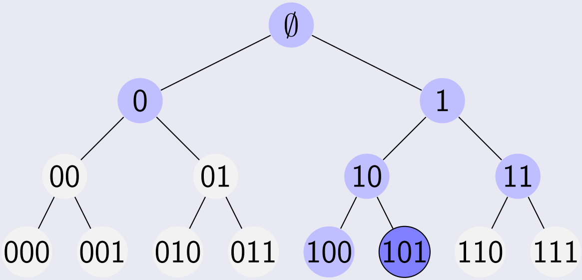
Proving Selective Security

Hybrid H_3



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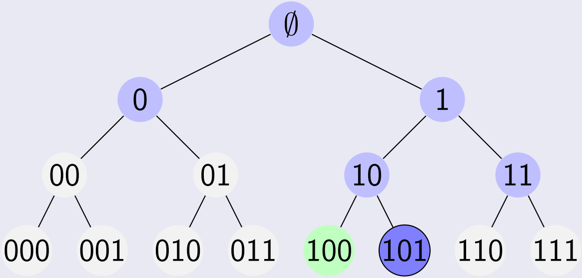
Proving Selective Security

Hybrid H_4



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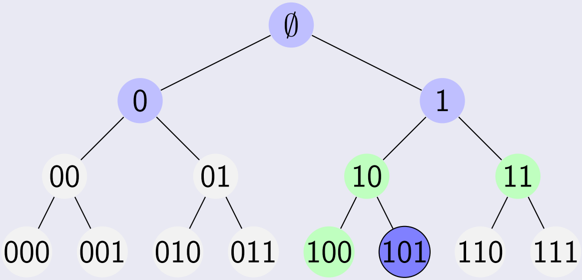
Proving Selective Security

Hybrid H_5



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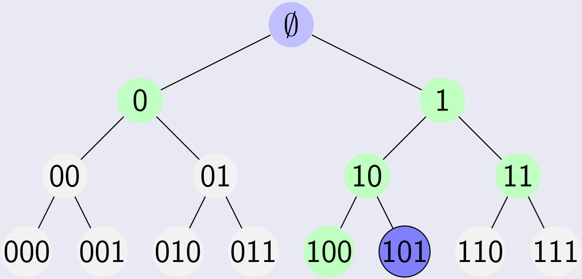
Proving Selective Security

Hybrid H_6 (random game)



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Proving Adaptive Security using Leveraging

H_0 $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8$

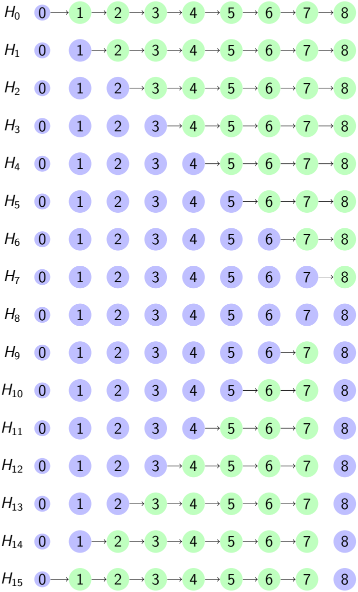
Proof

- Leveraging: Guess Challenge

$$\epsilon \rightarrow \frac{\epsilon}{2^n}$$

H_{15} $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \quad 8$

Proving Adaptive Security using Leveraging



Proof

- Leveraging: Guess Challenge
- Hybrid Argument

$$\epsilon \to \frac{\epsilon}{2^n} \to \frac{\epsilon}{2^n \cdot 2n}$$

Proving Adaptive Security by Nesting

$$0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8$$

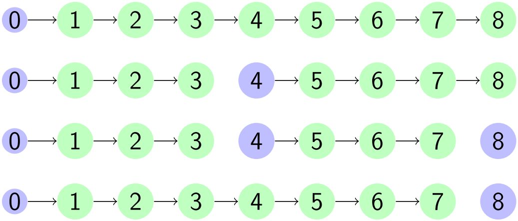
$$0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \quad 8$$

Proof

1. Guess first query that agrees with x^* on 4-prefix.

$$\epsilon \rightarrow \frac{\epsilon}{q}$$

Proving Adaptive Security by Nesting



Proof

1. Guess first query that agrees with x^* on 4-prefix.
2. Hybrid argument.

$$\epsilon \to \frac{\epsilon}{q} \to \frac{\epsilon}{3q}$$

Proving Adaptive Security by Nesting

$$0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \quad 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8$$

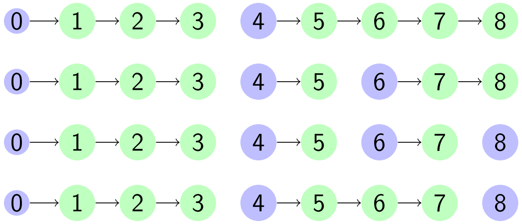
$$0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \quad 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \quad 8$$

Proof

1. Guess first query that agrees with x^* on 6-prefix.
2. Hybrid argument.

$$\epsilon \rightarrow \frac{\epsilon}{q} \rightarrow \frac{\epsilon}{3q} \rightarrow \ldots \rightarrow$$

Proving Adaptive Security by Nesting



Proof

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2. Hybrid argument.

$$\epsilon \to \frac{\epsilon}{q} \to \frac{\epsilon}{3q} \to \ldots \to$$

Proving Adaptive Security by Nesting

$$0 \to 1 \to 2 \to 3 \quad 4 \to 5 \to 6 \to 7 \to 8$$

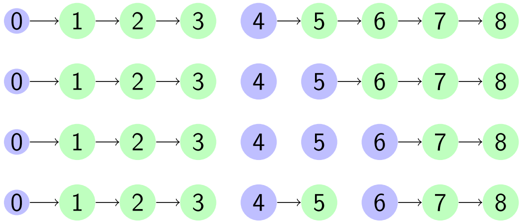
$$0 \to 1 \to 2 \to 3 \quad 4 \to 5 \quad 6 \to 7 \to 8$$

Proof

1. Guess first query that agrees with x^* on 5-prefix.
2. Hybrid argument.

$$\epsilon \to \frac{\epsilon}{q} \to \frac{\epsilon}{3q} \to \ldots \to$$

Proving Adaptive Security by Nesting



Proof

1. Guess first query that agrees with x^* on 5-prefix.
2. Hybrid argument.

$$\epsilon \to \frac{\epsilon}{q} \to \frac{\epsilon}{3q} \to \dots \to \frac{\epsilon}{(3q)^{\log n}}$$