On the Hardness of Computing the Edit Distance of Shallow Trees

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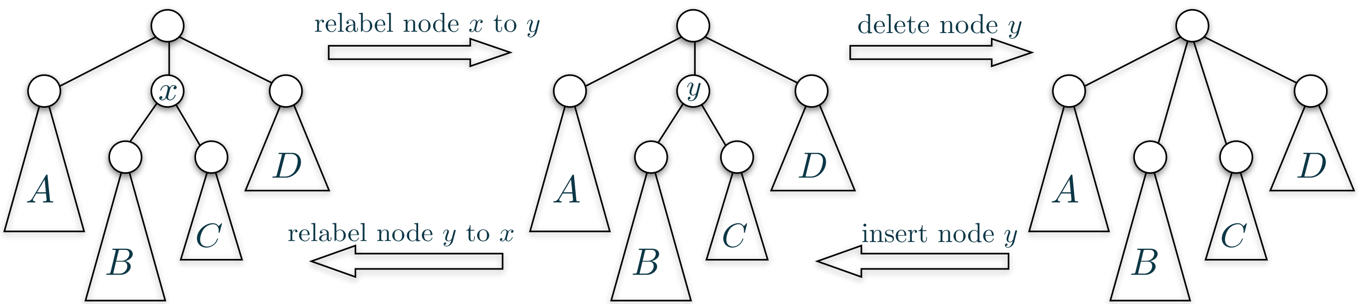

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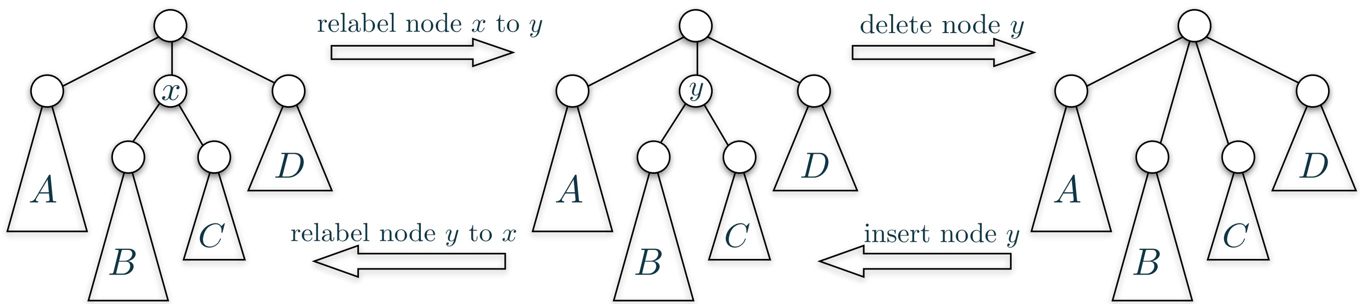

Remark: The cost function may require space quadratic in the size of the trees.

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Dynamic Programming: consider all three such options and recurse.

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Instead of reducing APSP to TED, we reduce from the equivalent NEGATIVETRIANGLE problem [Vassilevska Williams, Williams; JACM 2018]:

NegativeTriangle: Check whether a complete tripartite graph with parts of size at most n and polynomial edge-weights contains a negative triangle, that is, if there exist vertices u, v, z with $w(u, v) + w(v, z) + w(z, u) < 0$.

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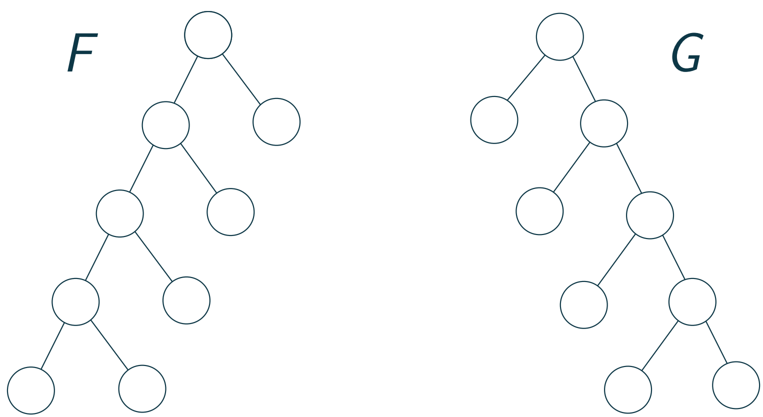

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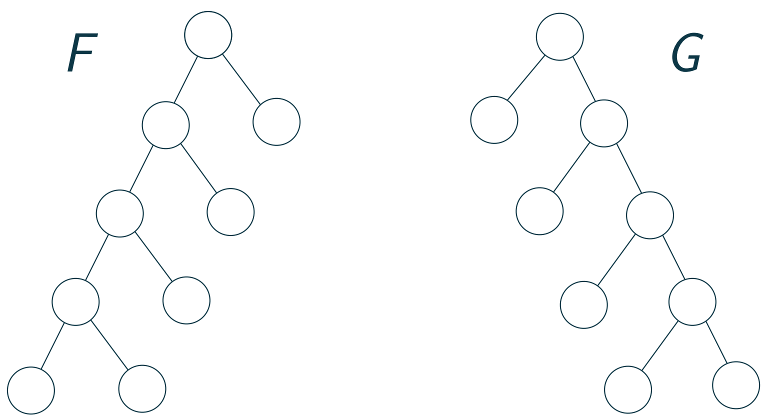

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The whole game is labelling the nodes and defining the substitution costs.

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NegativeTriangle, with $3n$ vertices

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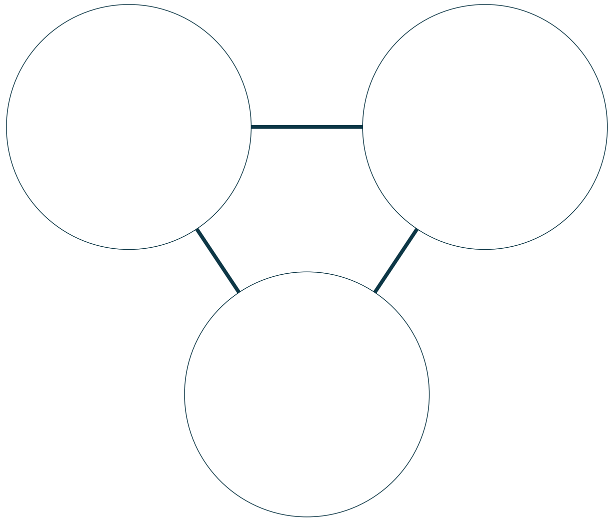
TED, over $\mathcal{O}(n^{1.5}/\sqrt{d})$-size, $\mathcal{O}(d)$-depth trees

Many Smaller NegativeTriangle Instances



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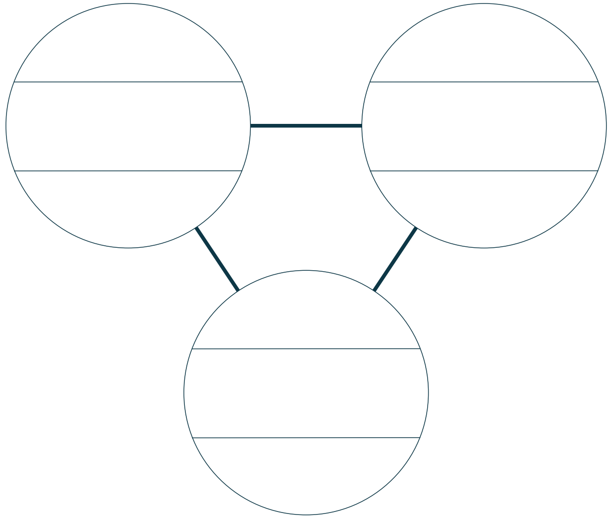

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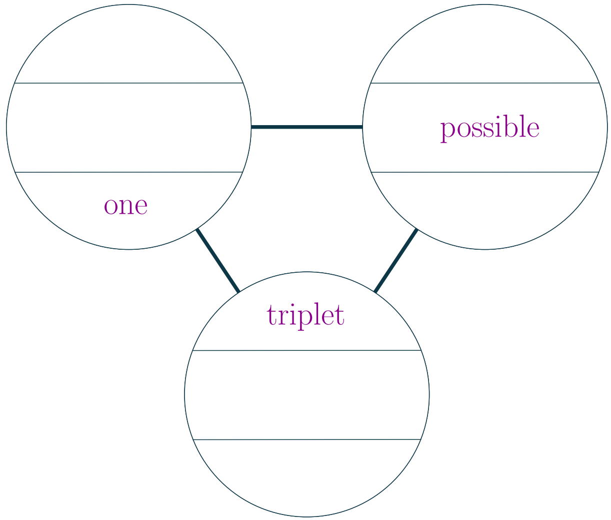

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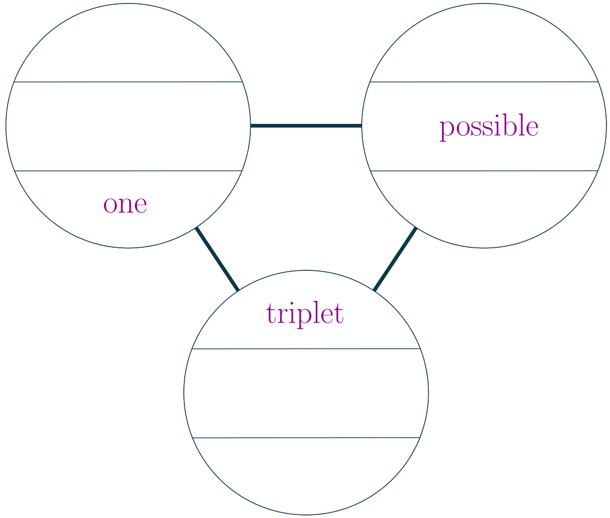

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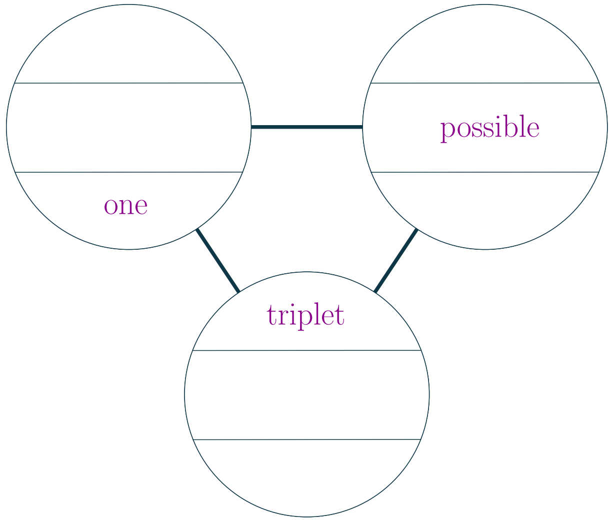

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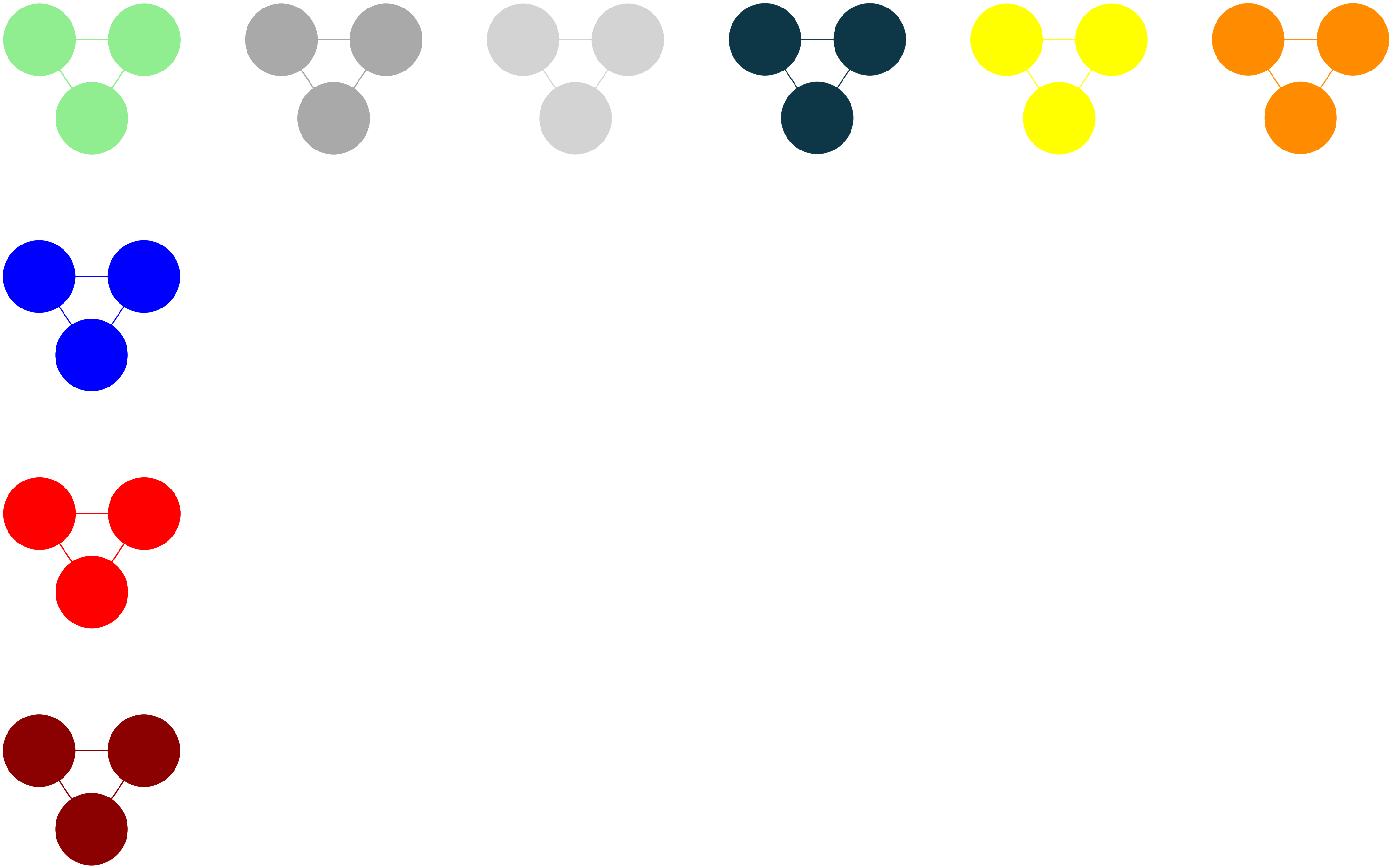
Time: $\mathcal{O}(n^2 + n^3/d)$. Recall that d is polynomial!

Many TED instances



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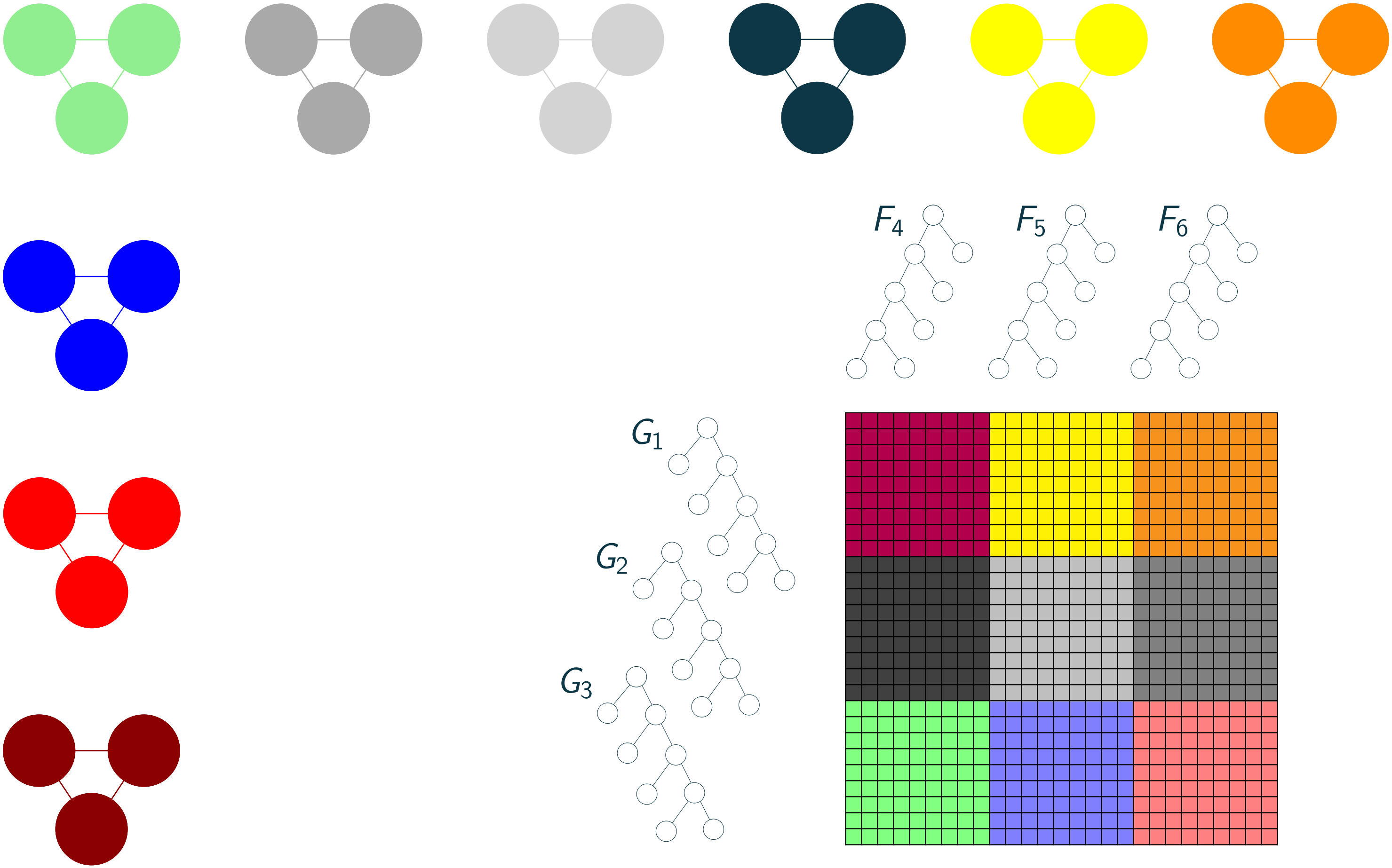

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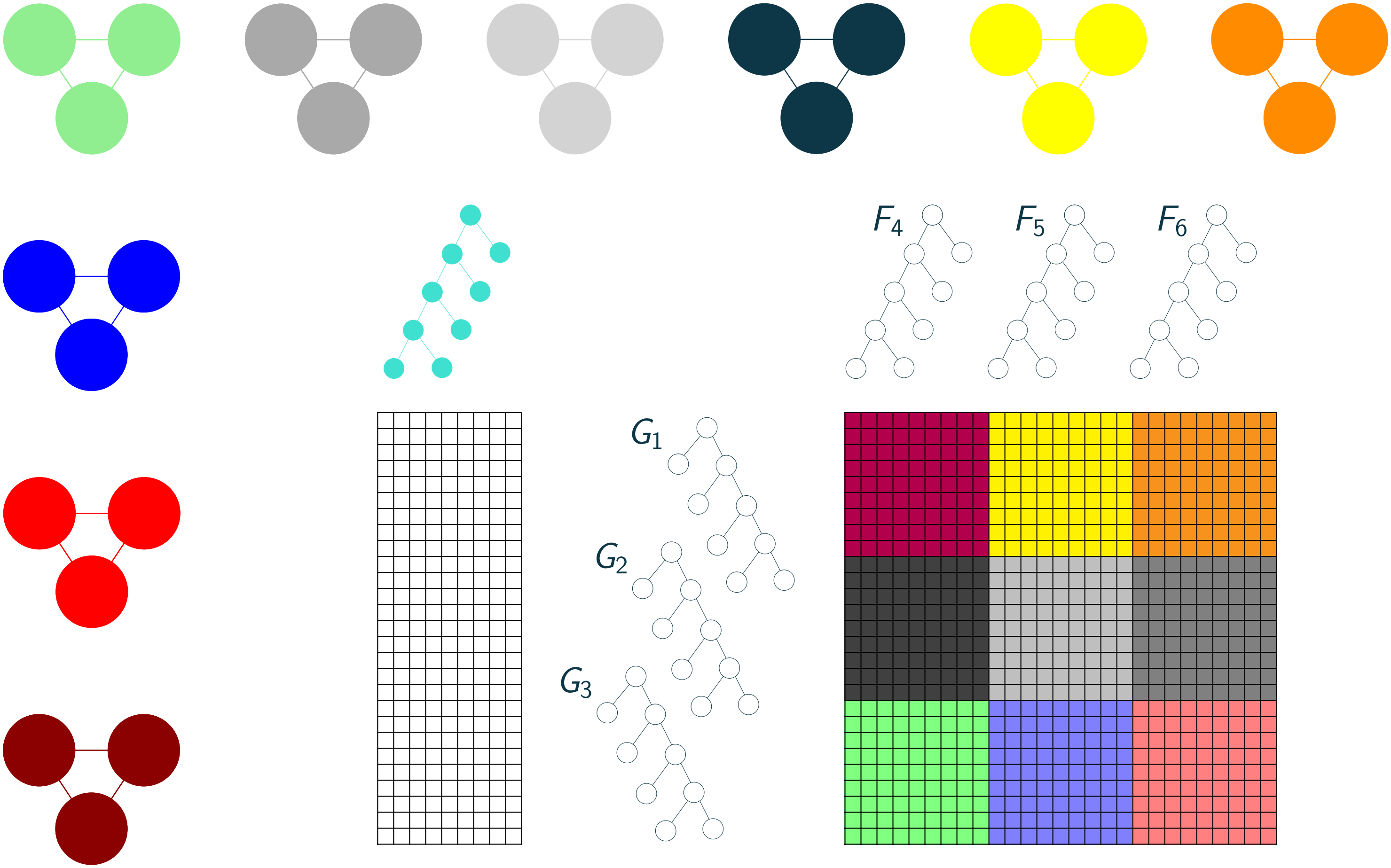

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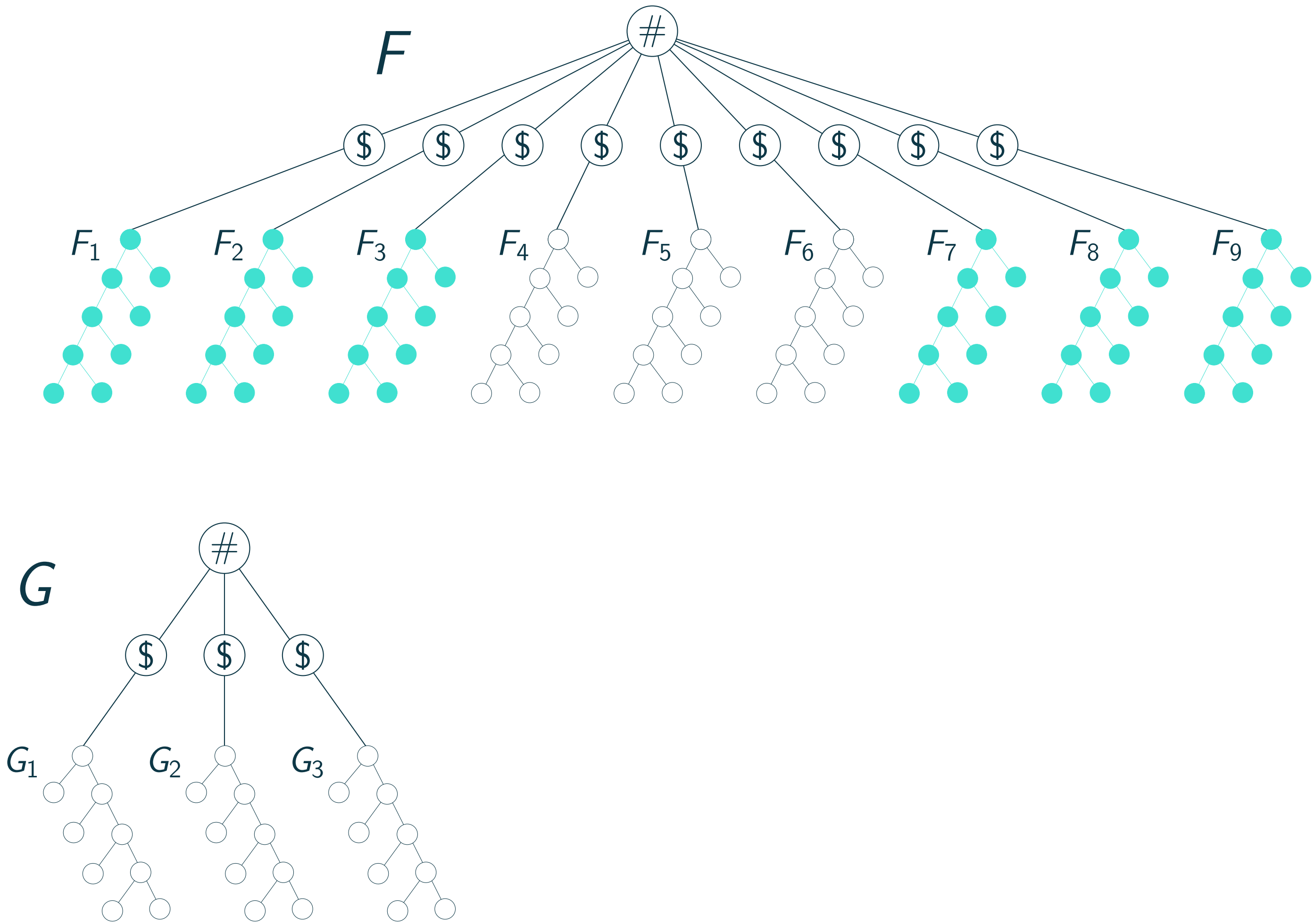

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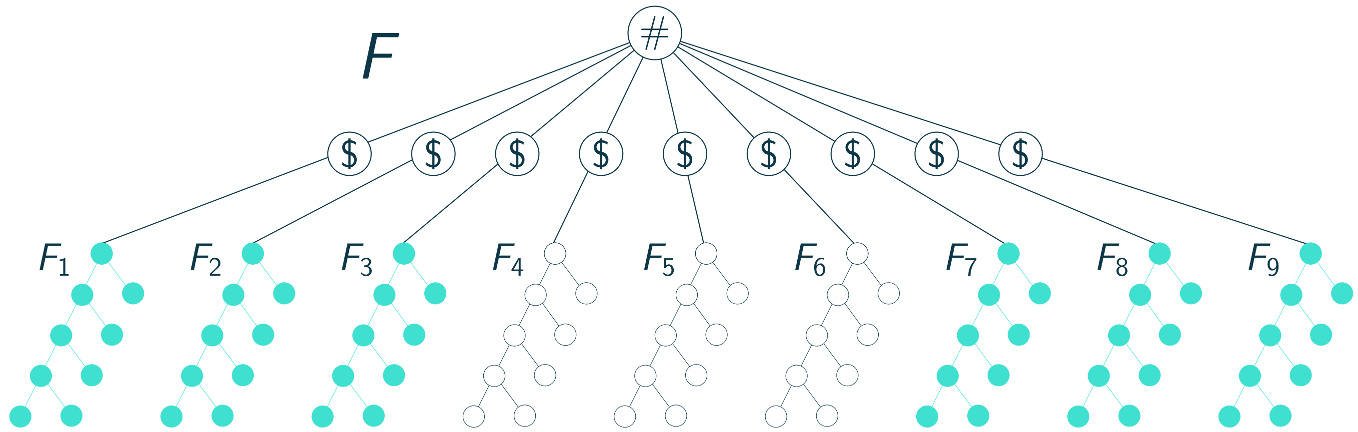

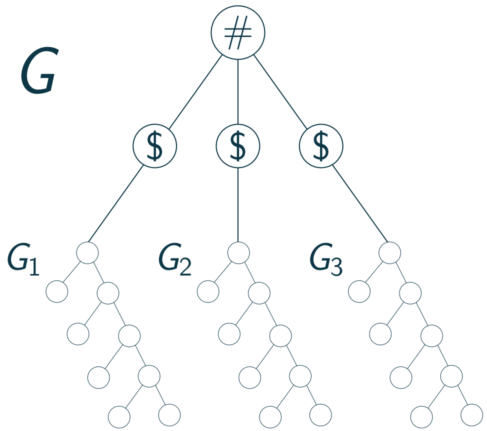

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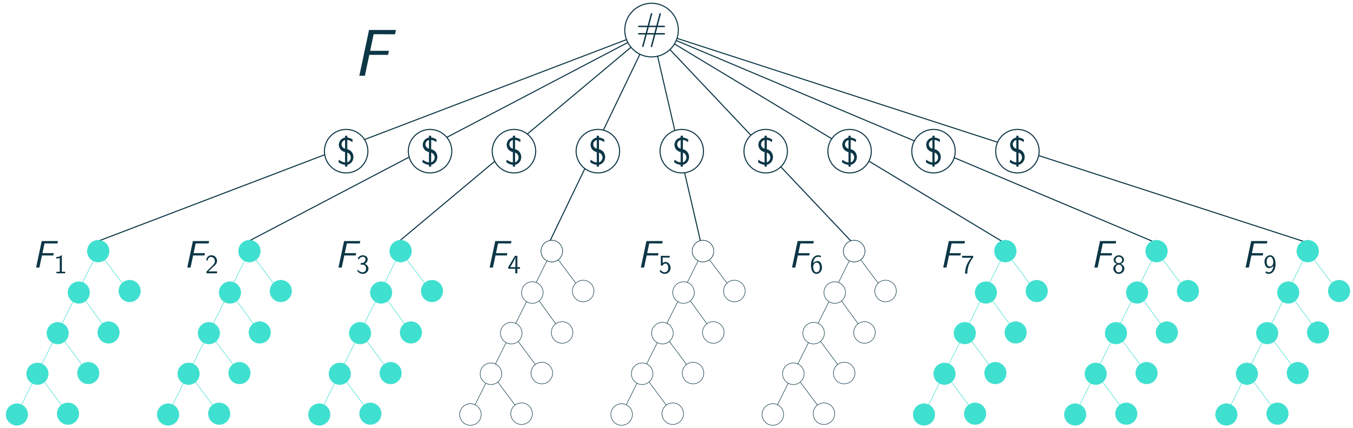

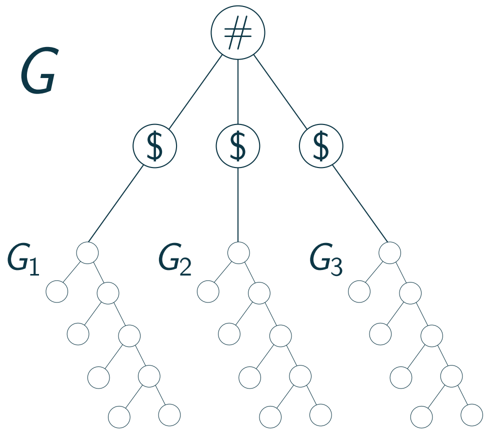

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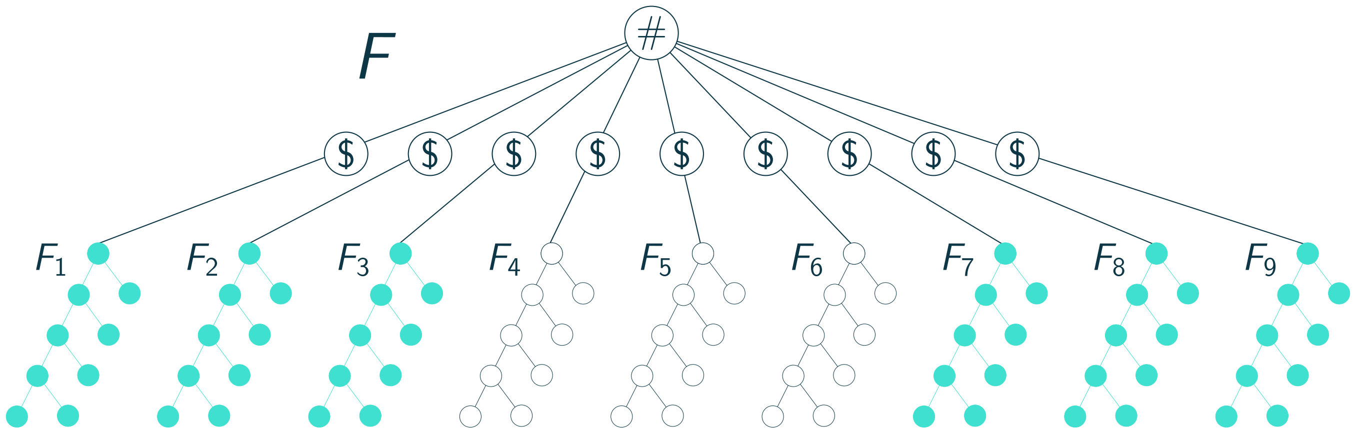

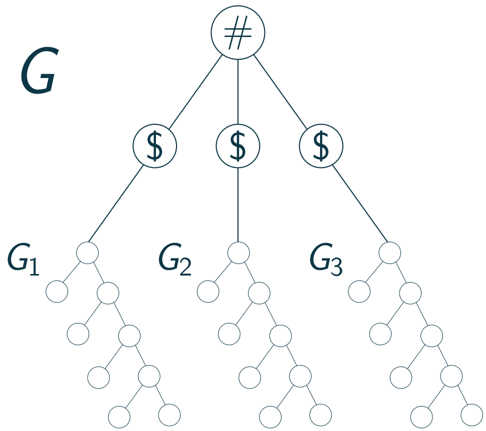

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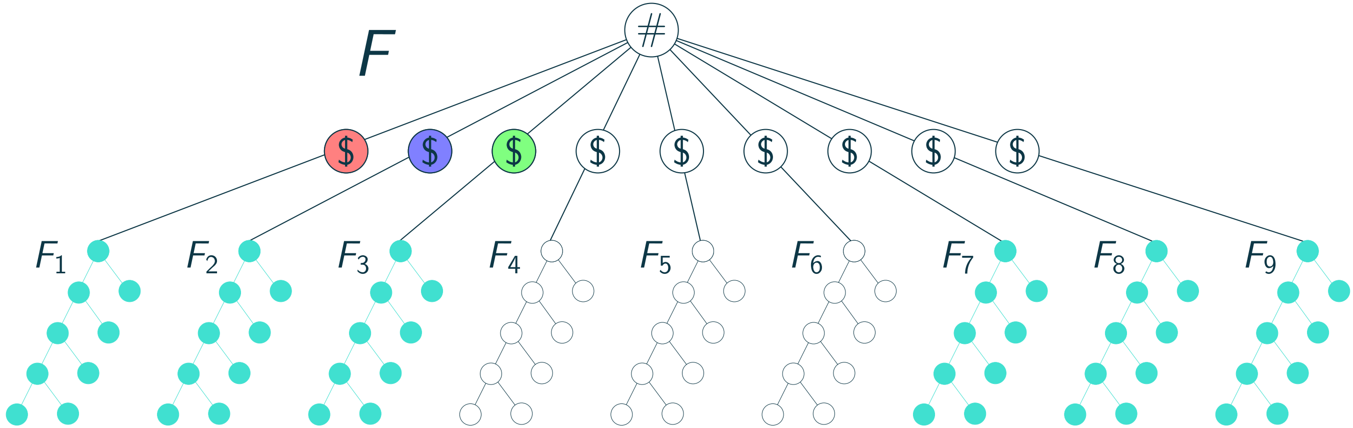

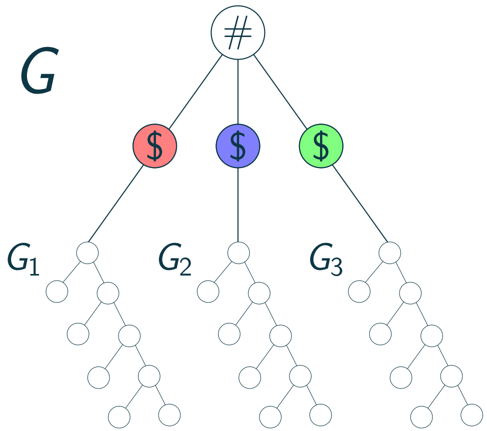

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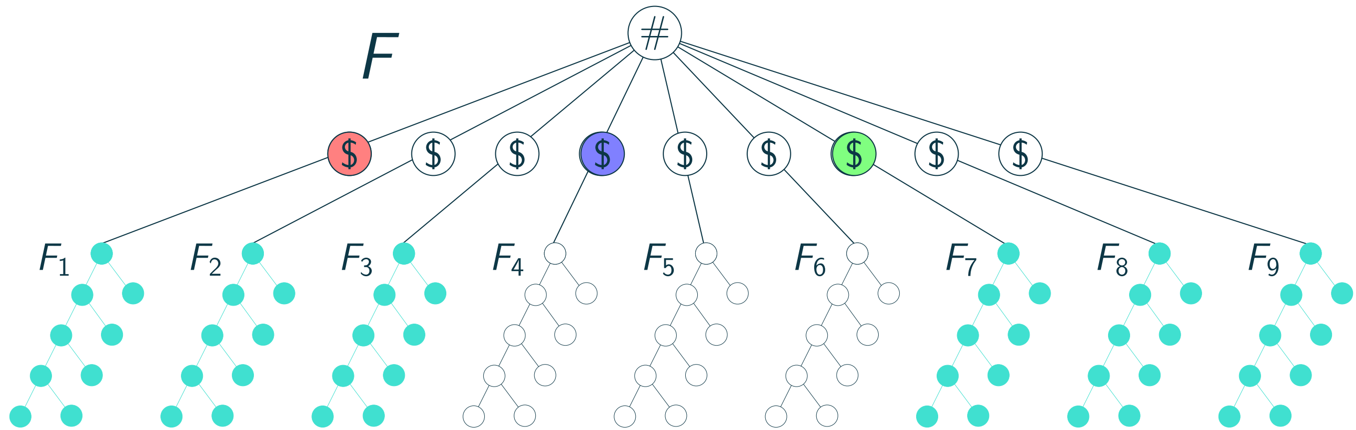

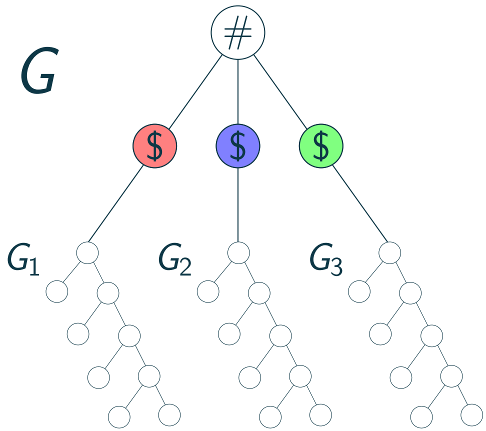

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$\mathcal{O}(n^3/d^3)$ instances of $\textsc{NegativeTriangle}$, each with $3d$ vertices

$$\downarrow$$

$\mathcal{O}(n^3/d^3)$ instances of TED, each on a pair of combs of depth $6d+1$

$$\downarrow$$

TED, over $\mathcal{O}(n^{1.5}/\sqrt{d})$-size, $\mathcal{O}(d)$-depth trees

Wrap-up

$\textsc{NegativeTriangle}$, with $3n$ vertices

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and hence a strongly subcubic algorithm for $\textsc{NegativeTriangle}$.

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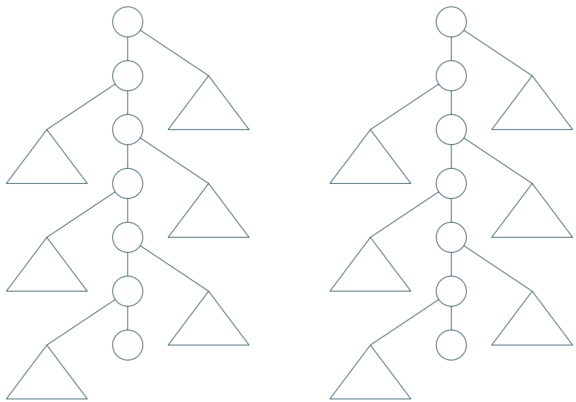

The End

There is no $o(n^2 d^2)$-time decomposition algorithm.

There is no $\mathcal{O}(n^2 d^{1-\epsilon})$-time algorithm for any constant $\epsilon > 0$ when $d = \mathrm{poly}(n)$ and $\Sigma = \Omega(n)$ under the APSP hypothesis.

Thank you for your attention! Questions?