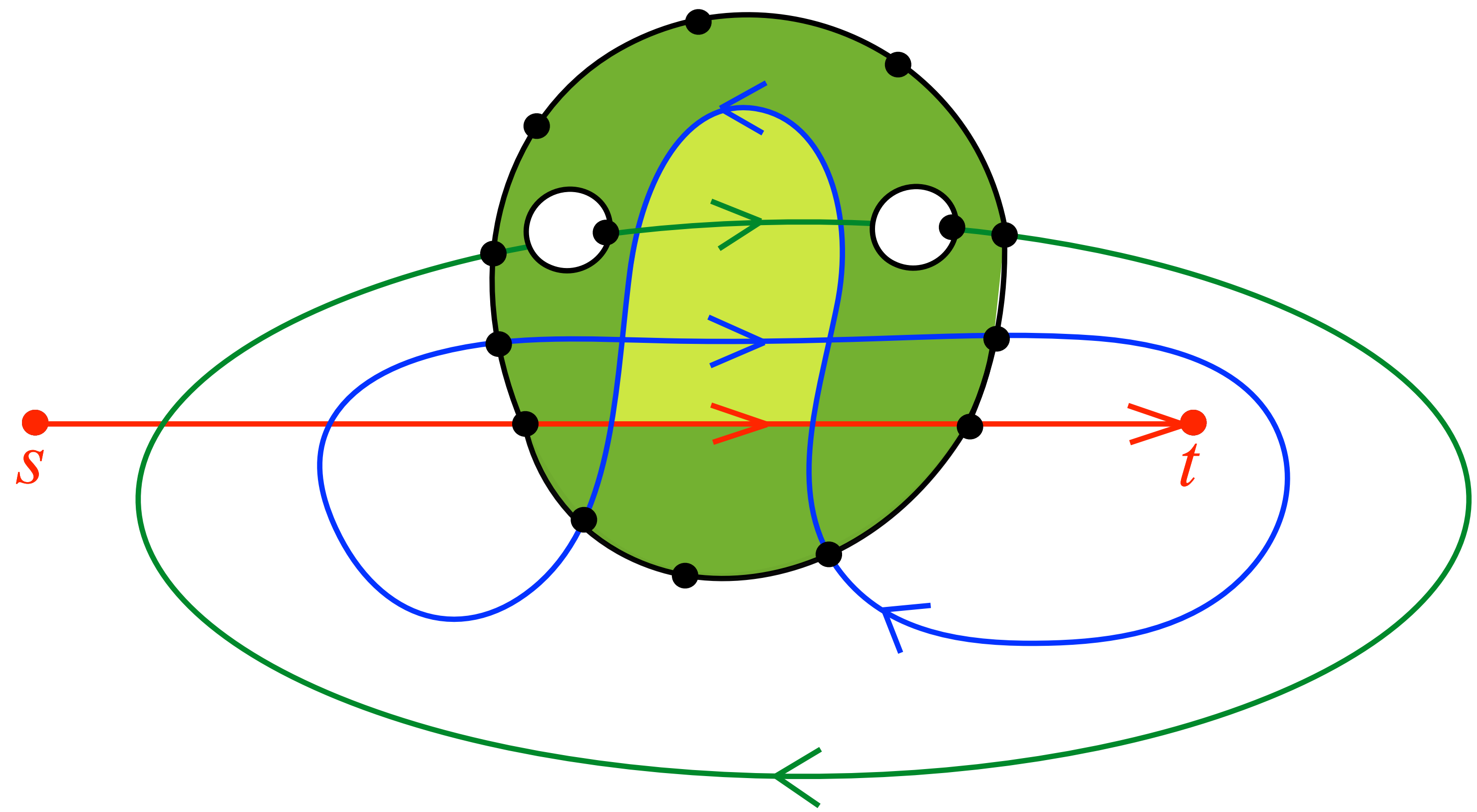
Minimum Cut of Directed Planar Graphs in O(nloglogn) Time



Shay Mozes, Kirill Nikolaev, Yahav Nussbaum, Oren Weimann

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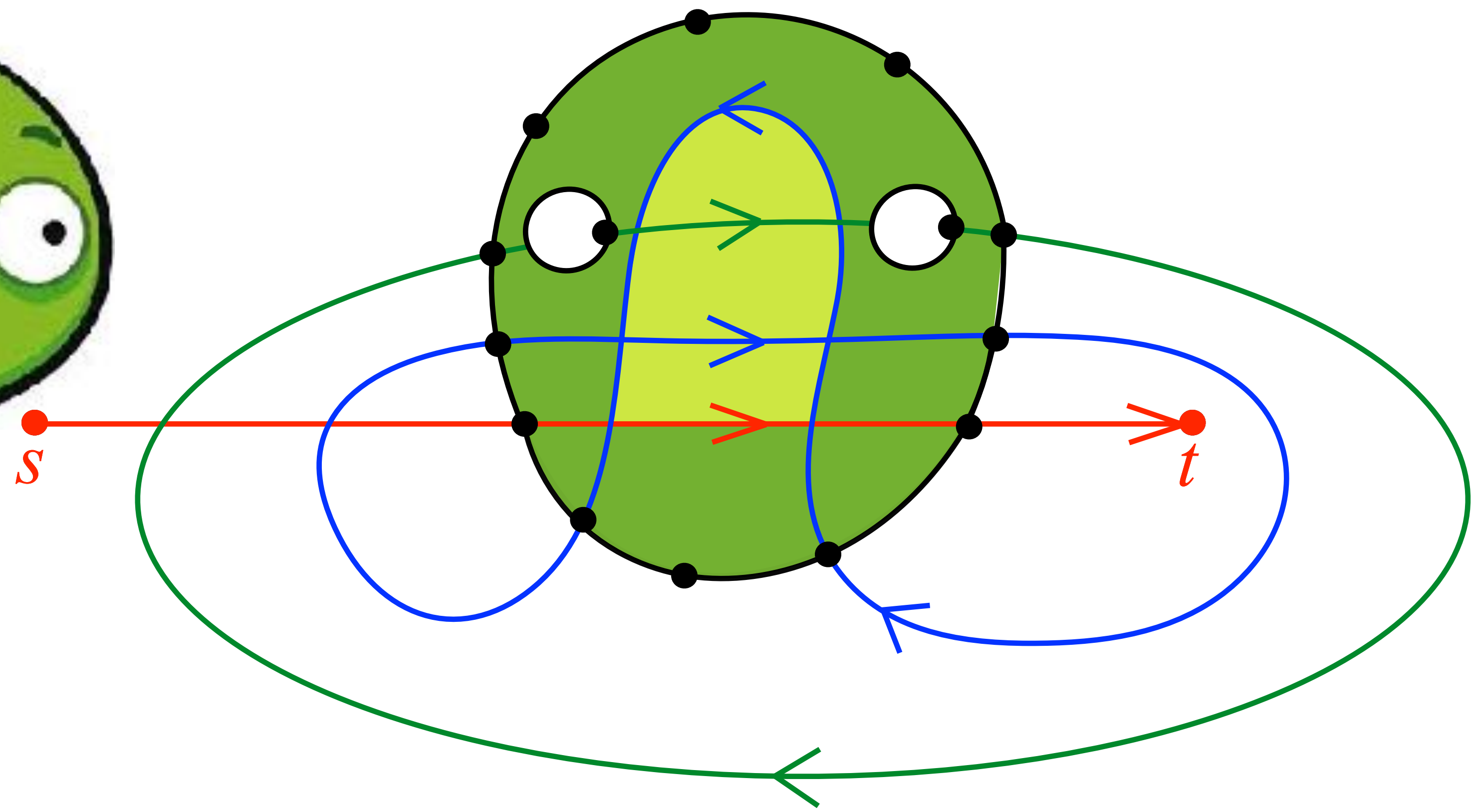

Shay Mozes, Kirill Nikolaev, Yahav Nussbaum, Oren Weimann

Minimum Cut and Minimum st-cut in Planar Graphs

- Undirected min st-cut: $O(n \log\log n)$
  [Italiano, Nussbaum, Sankowski, Wulff-Nilsen STOC 2011]

- Undirected min cut: $O(n \log\log n)$
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- Directed min st-cut: $O(n \log n)$
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Cuts and Cycles in Planar Graphs

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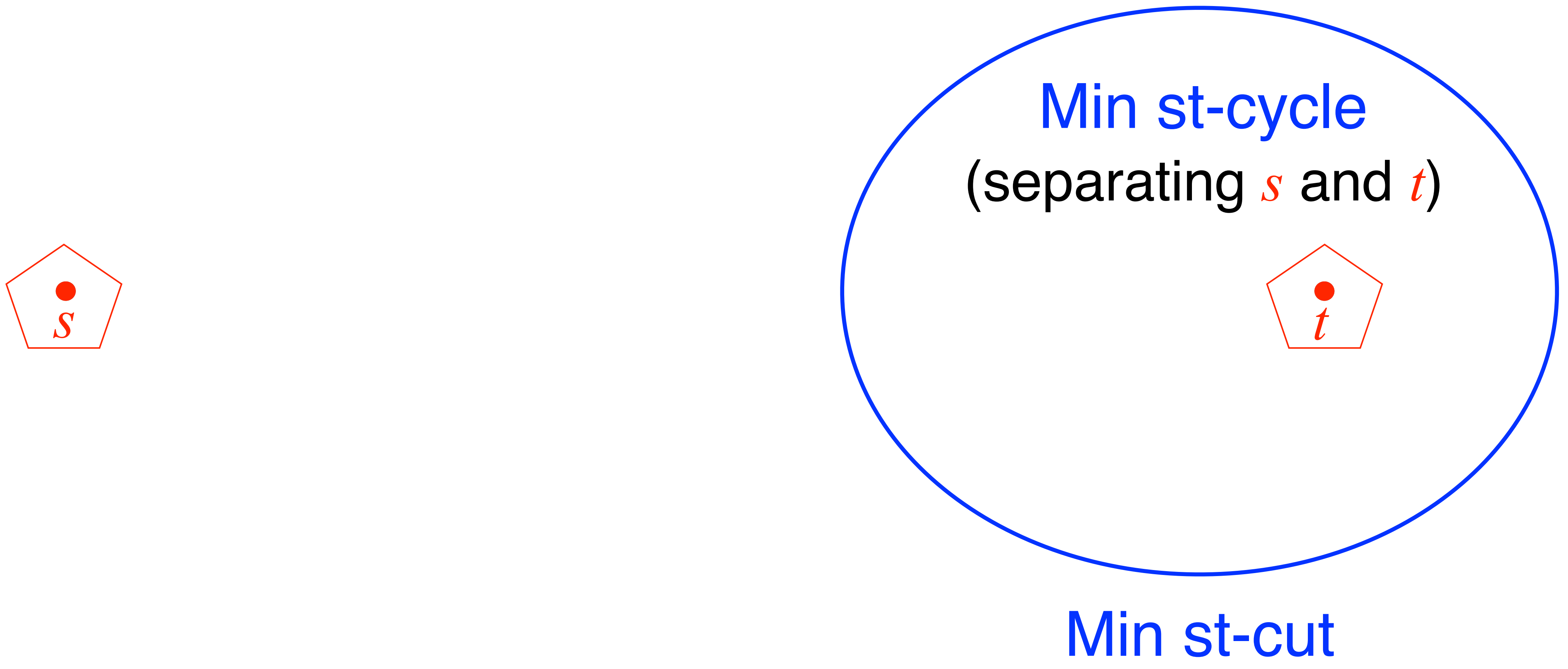

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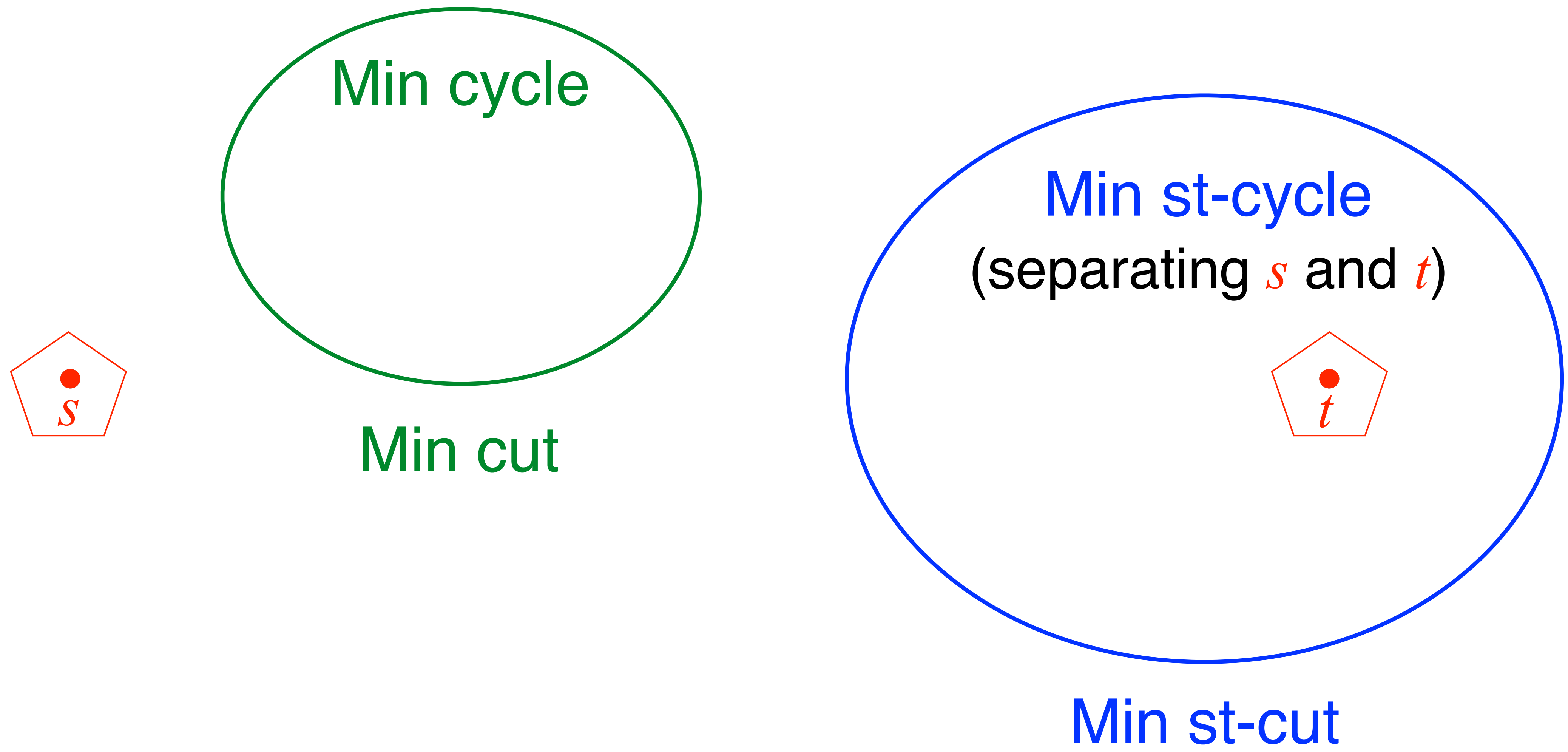

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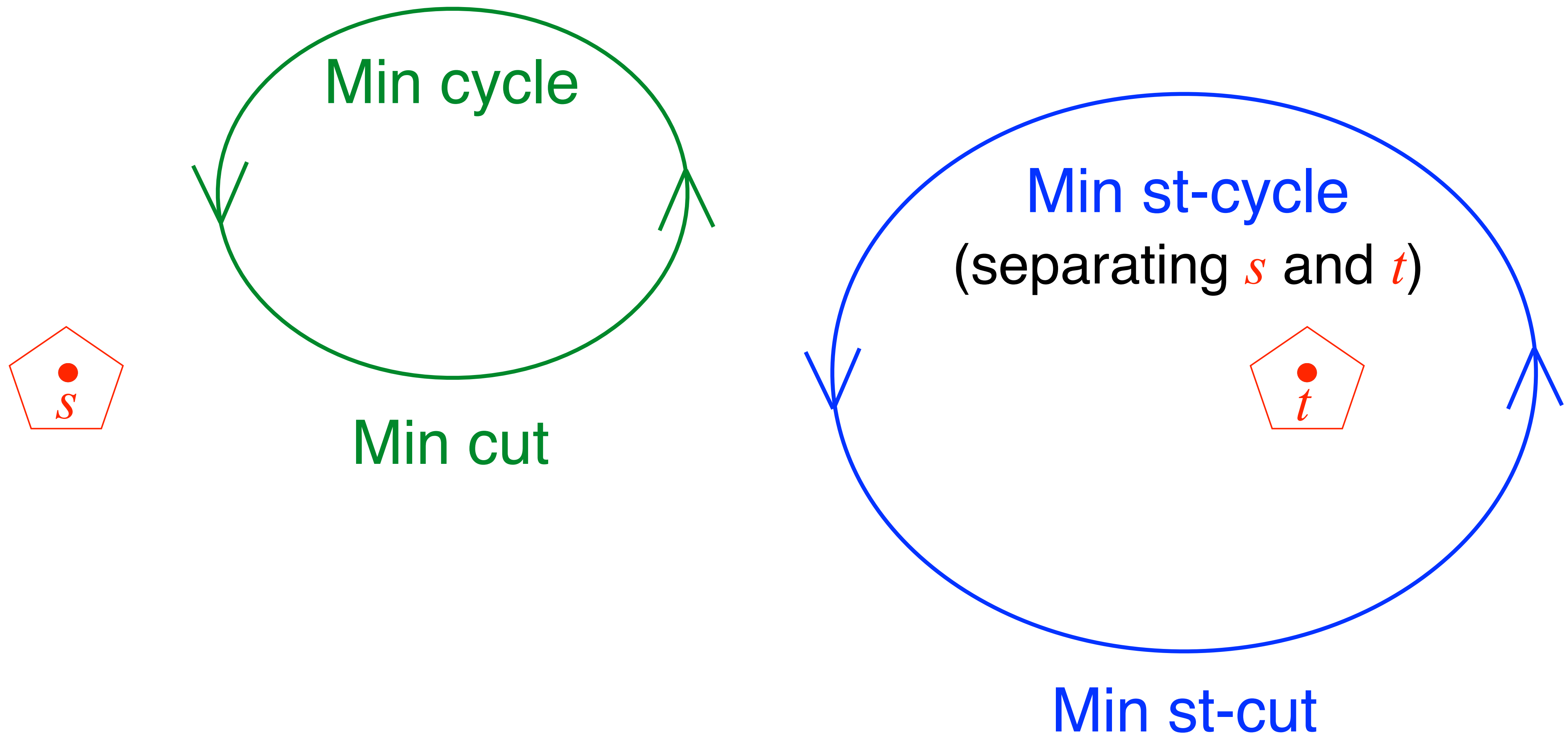

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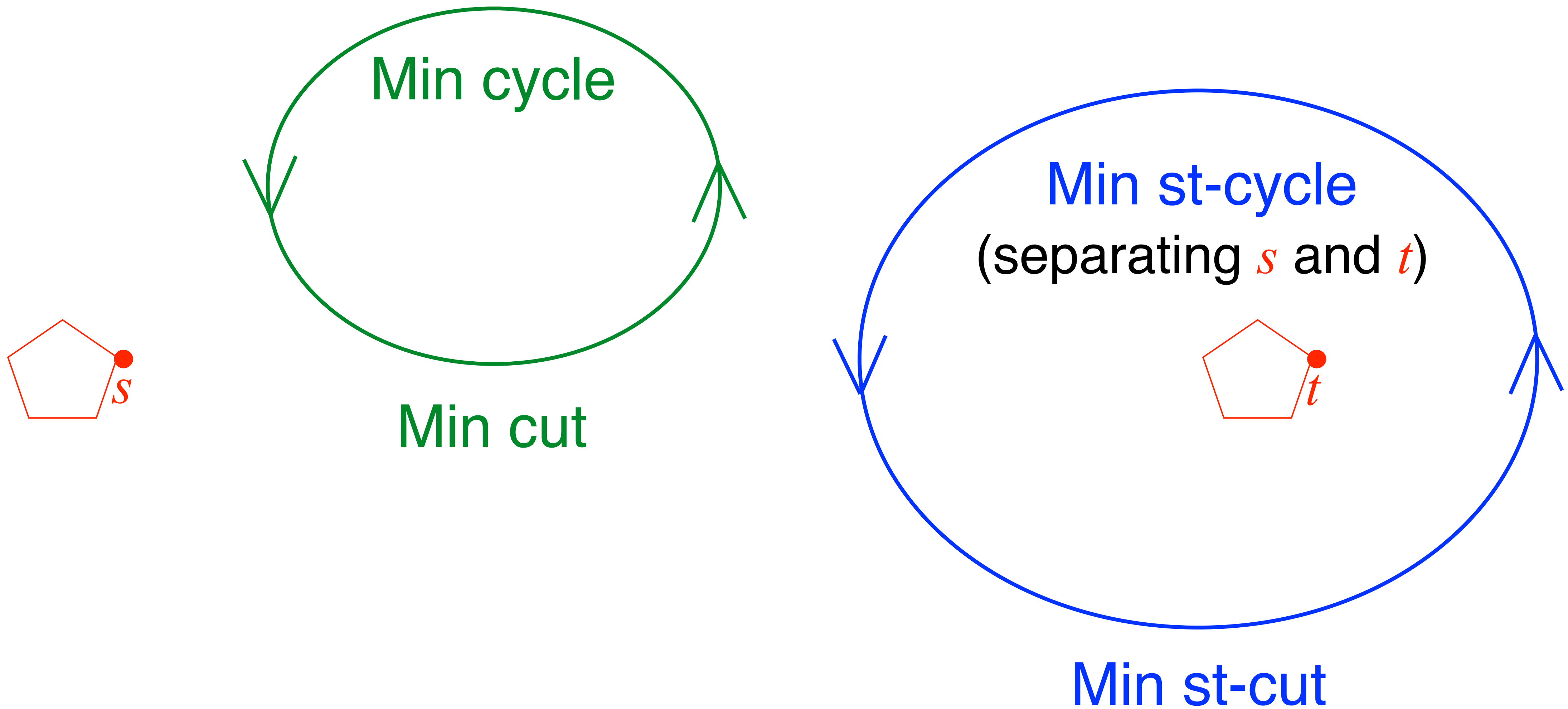

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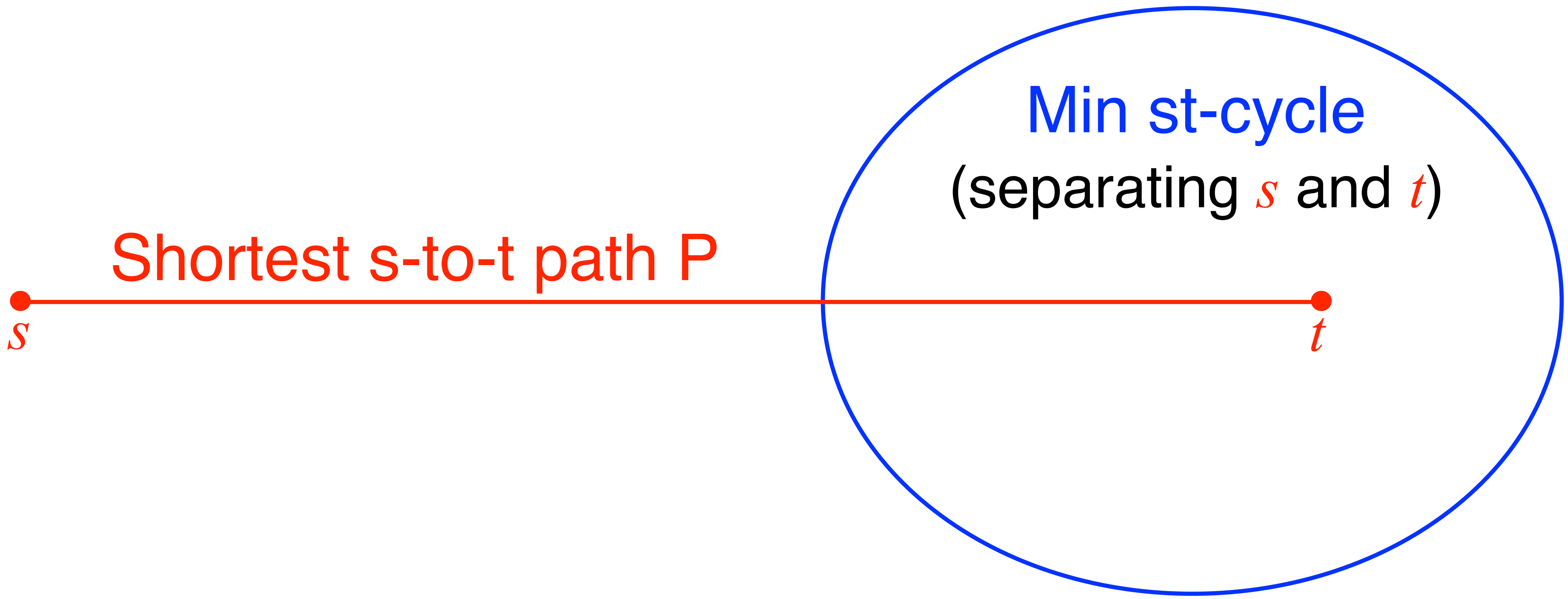
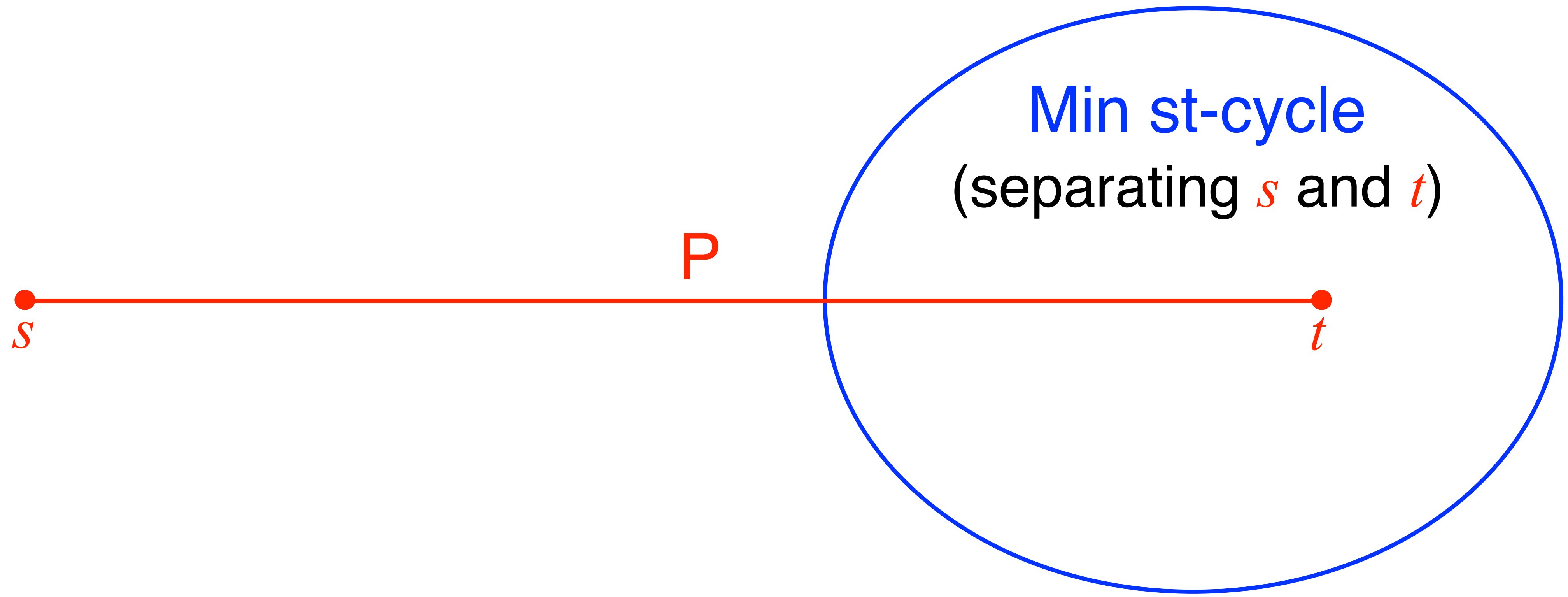


Undirected Min st-cycle Crosses P Once



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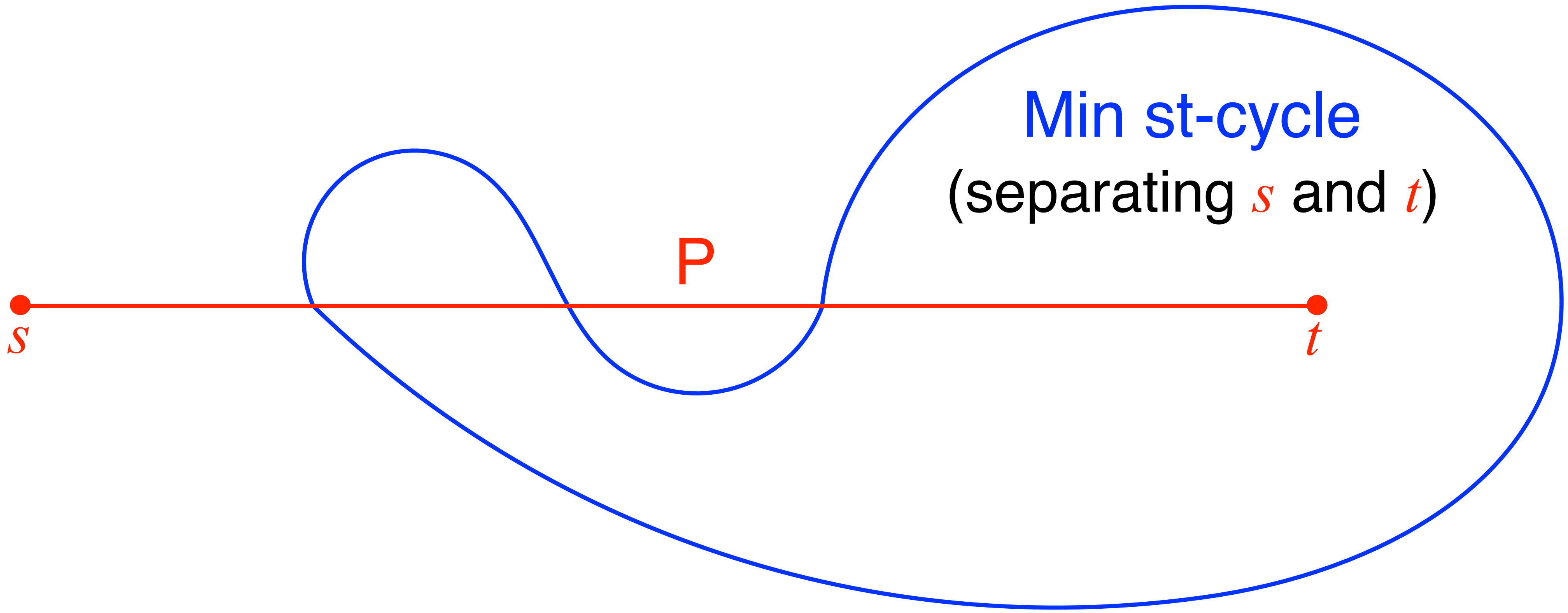

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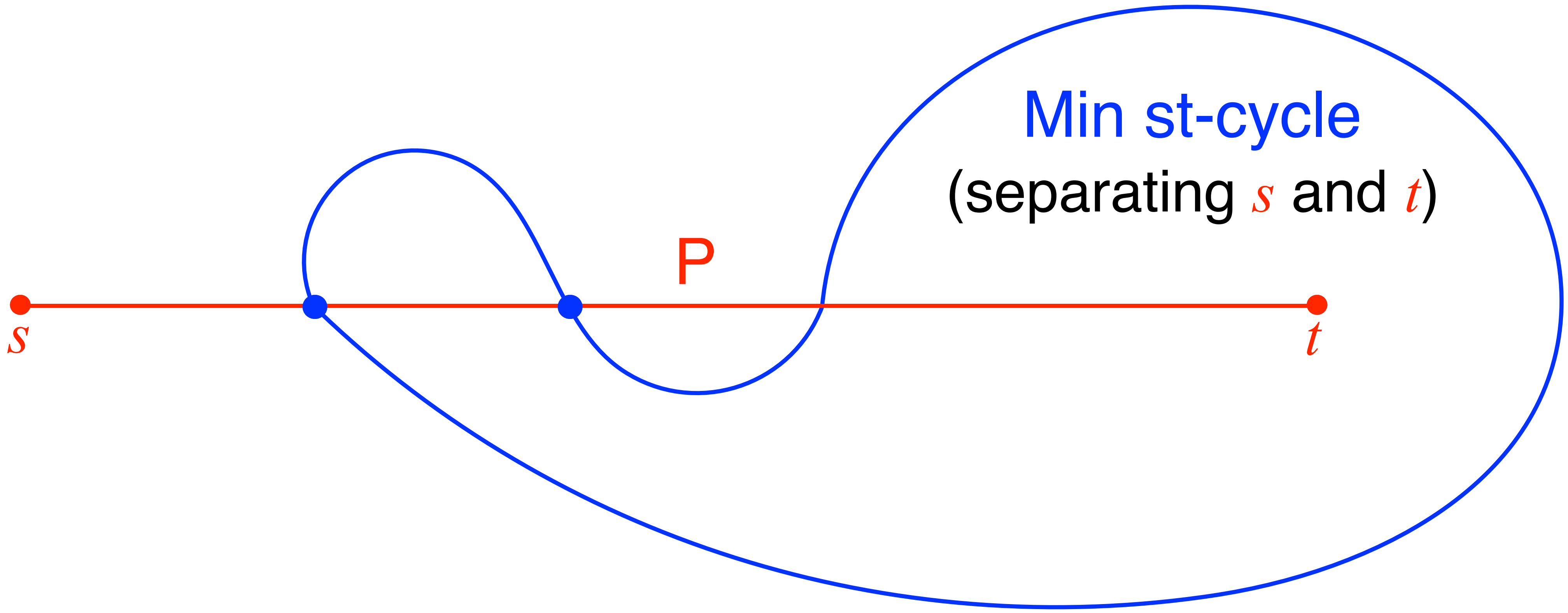

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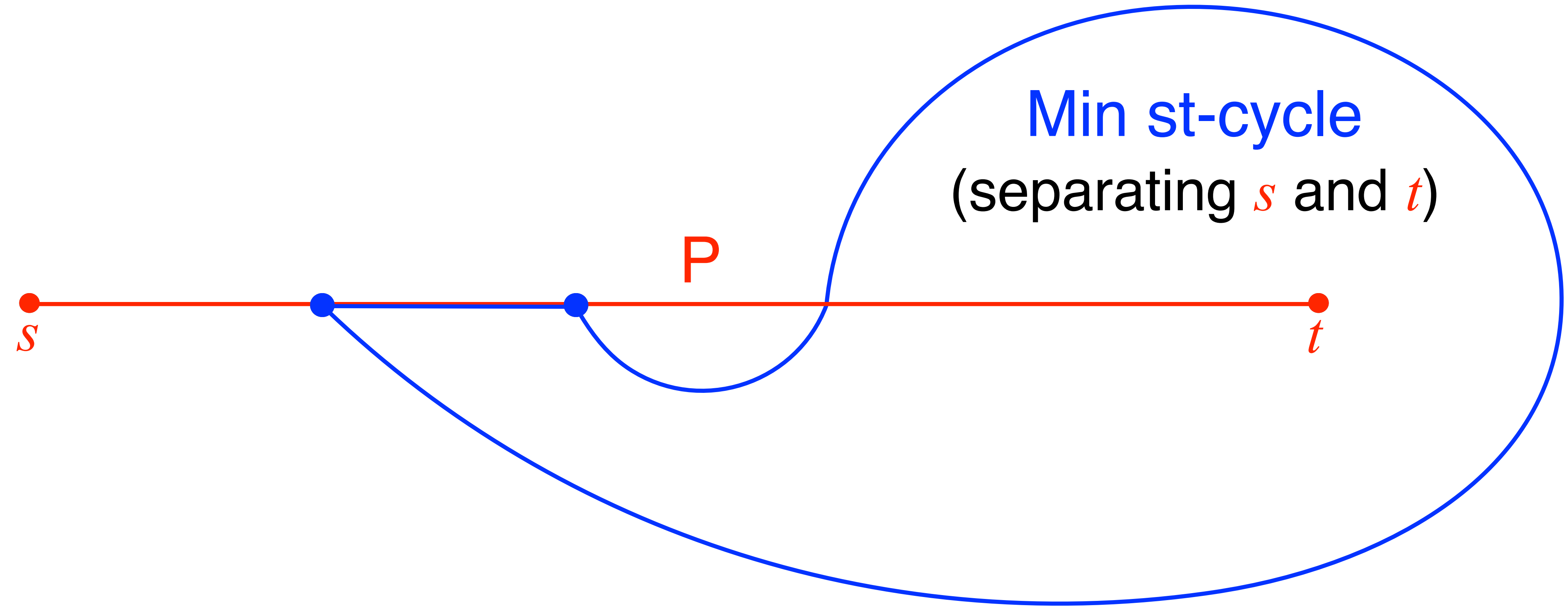

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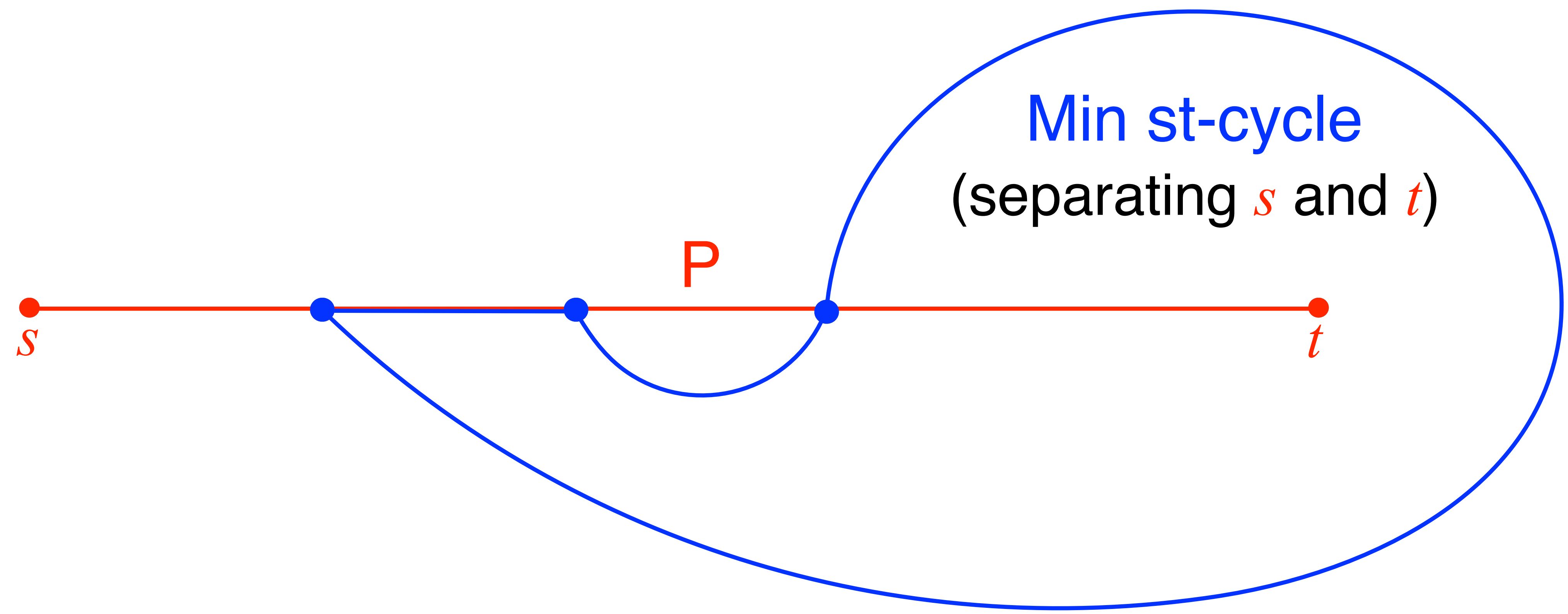

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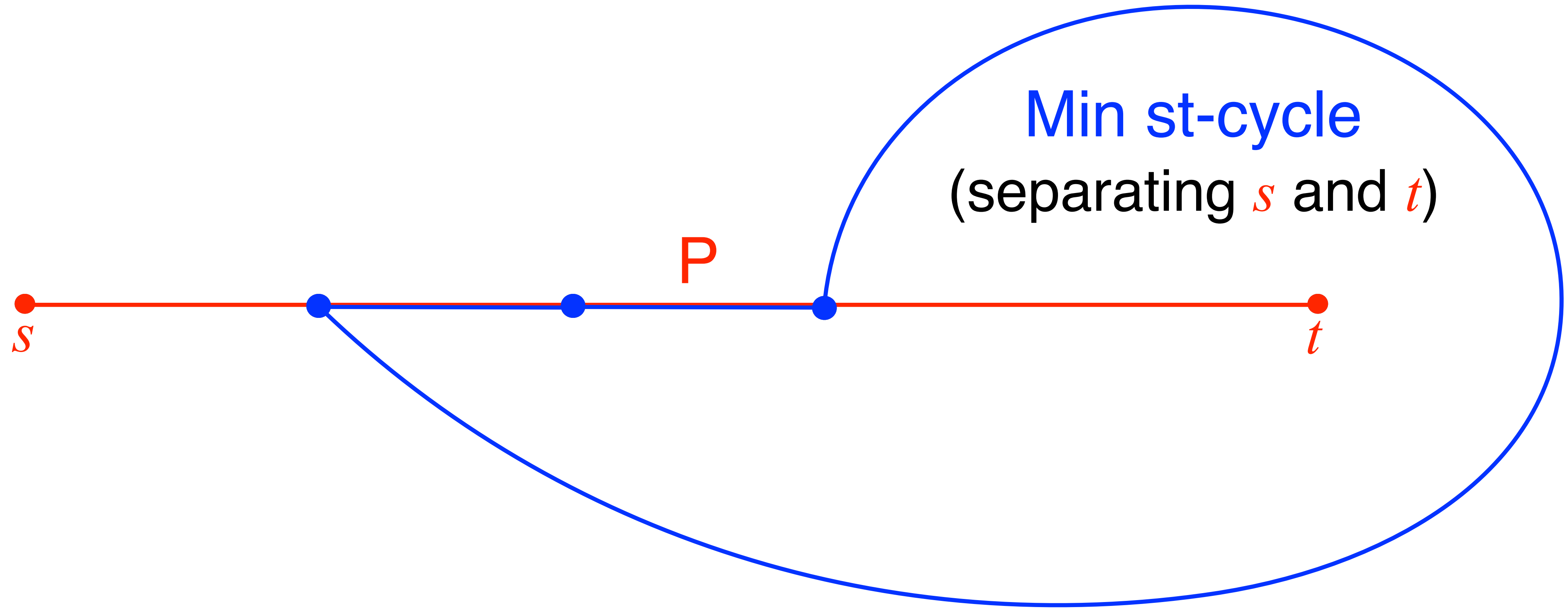
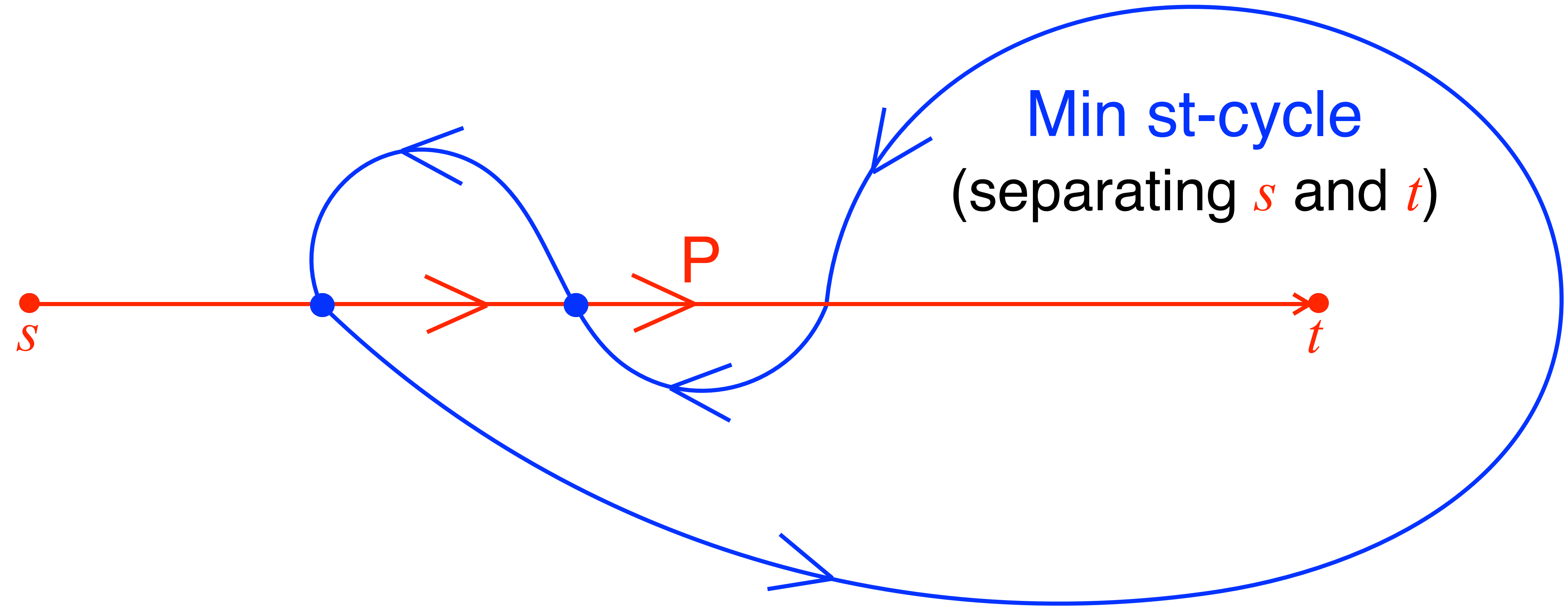
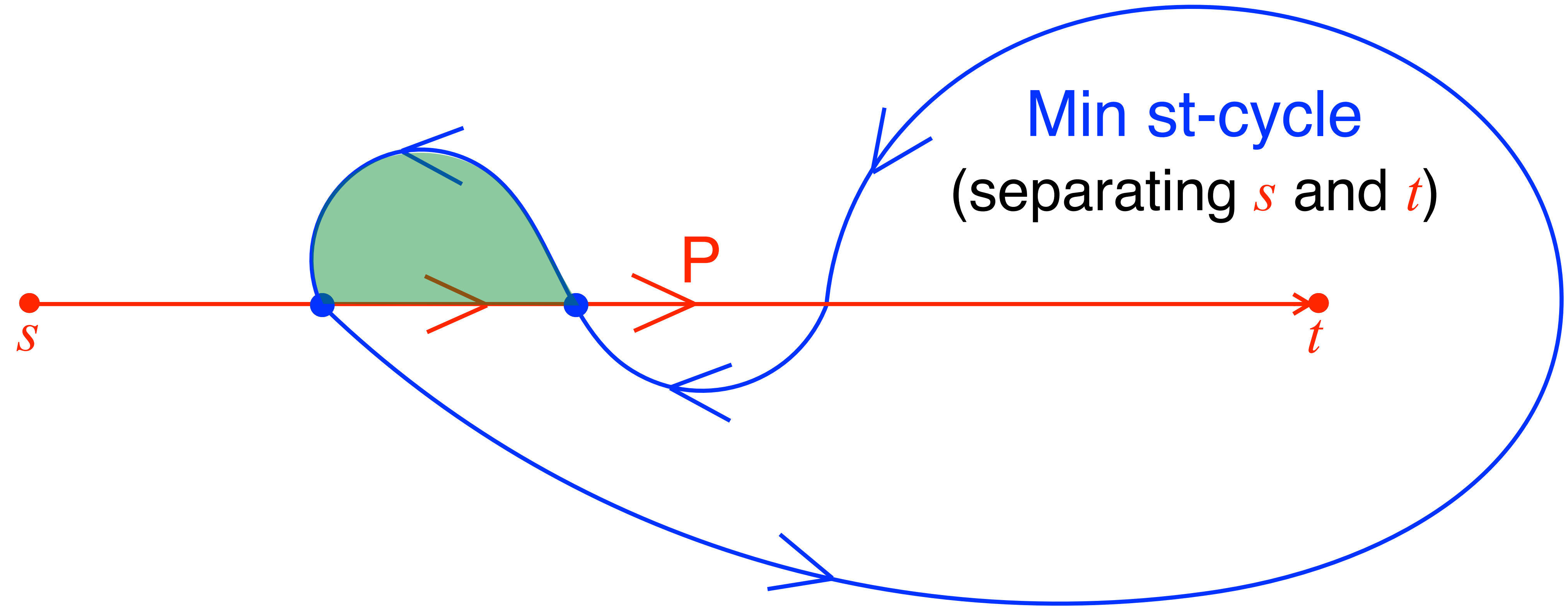


Directed Min st-cycle Crosses P Multiple Times



Directed Min st-cycle Crosses P Multiple Times

Directed Min cycle Crosses P at most Once!



Undirected: Reif's Algorithm [Reif 1983]

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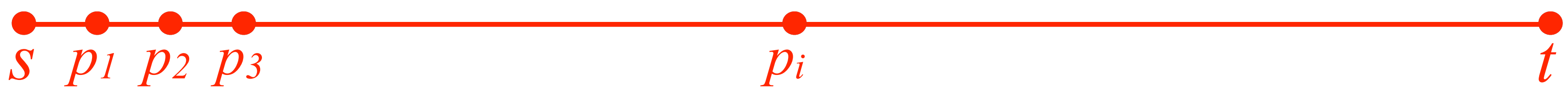

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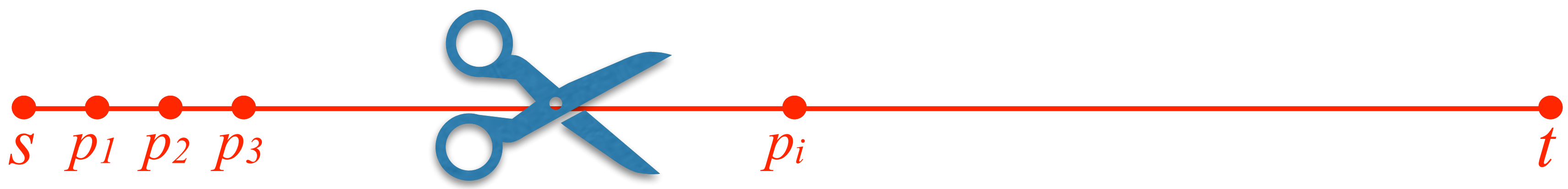

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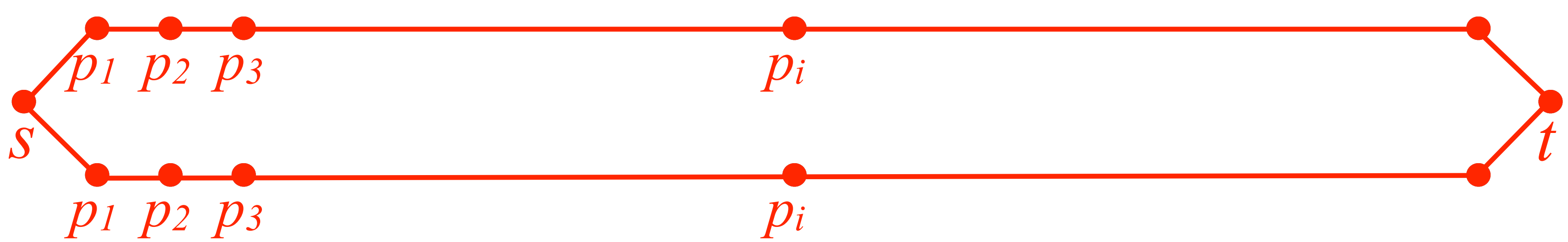

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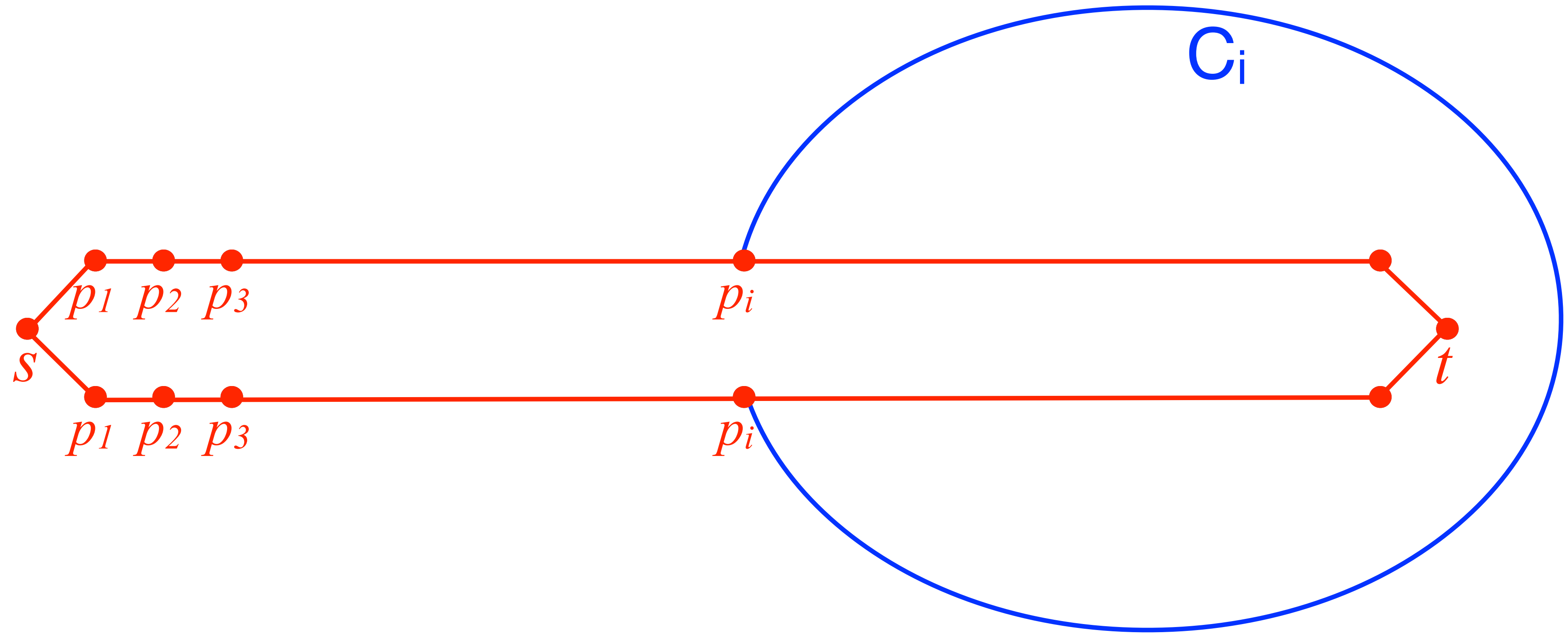

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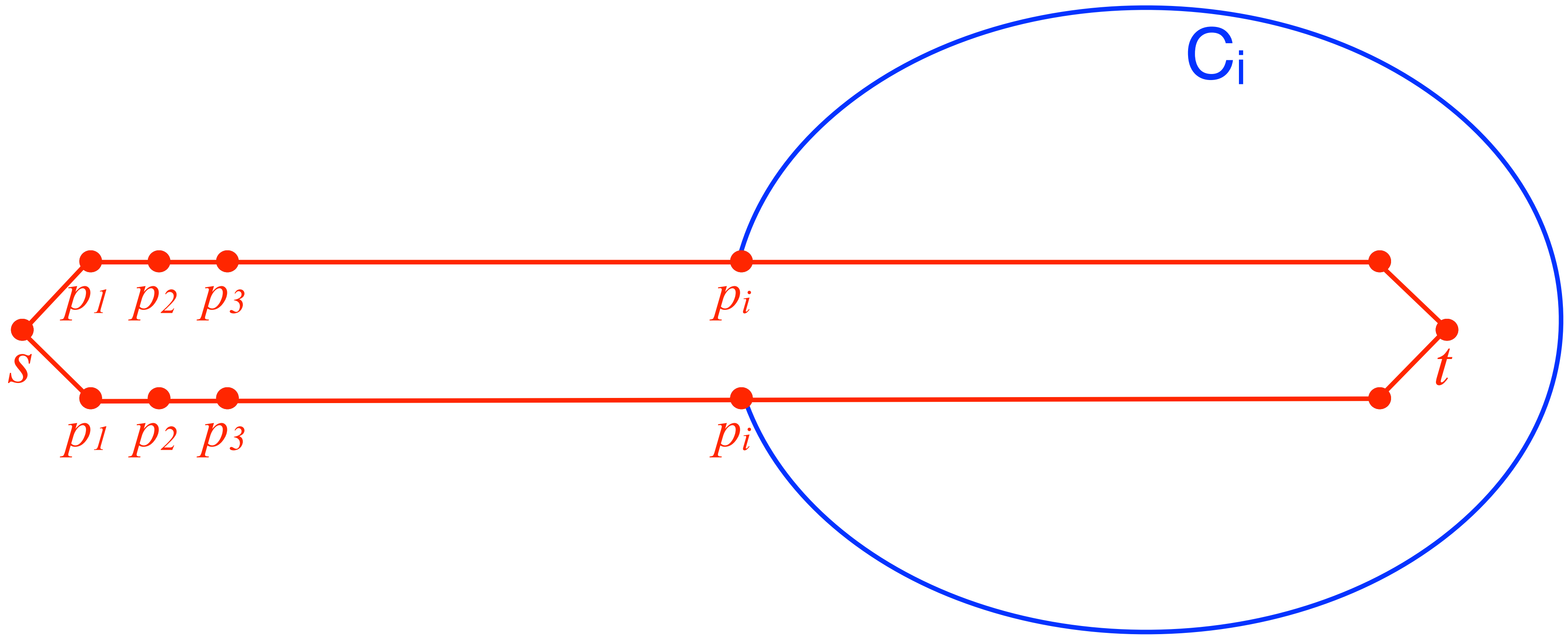

SSSP computation in O(n) time [Henzinger et al 1997]

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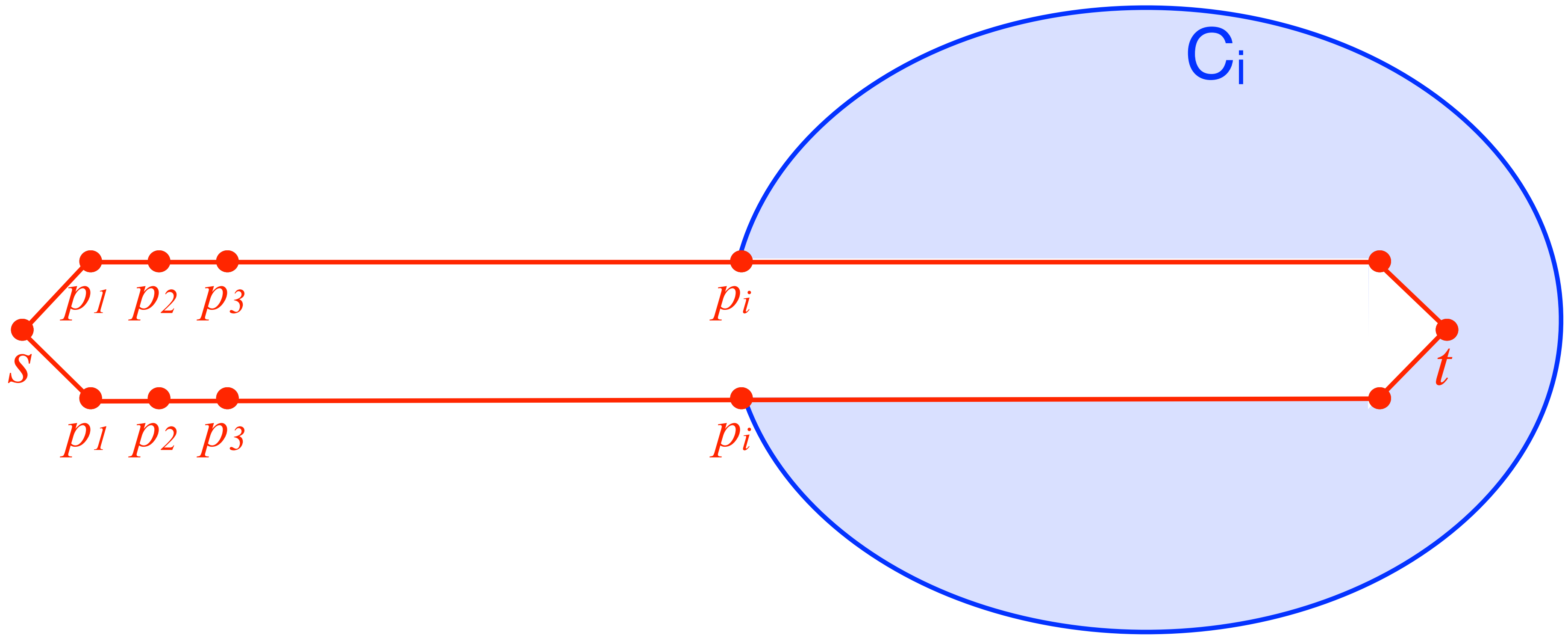

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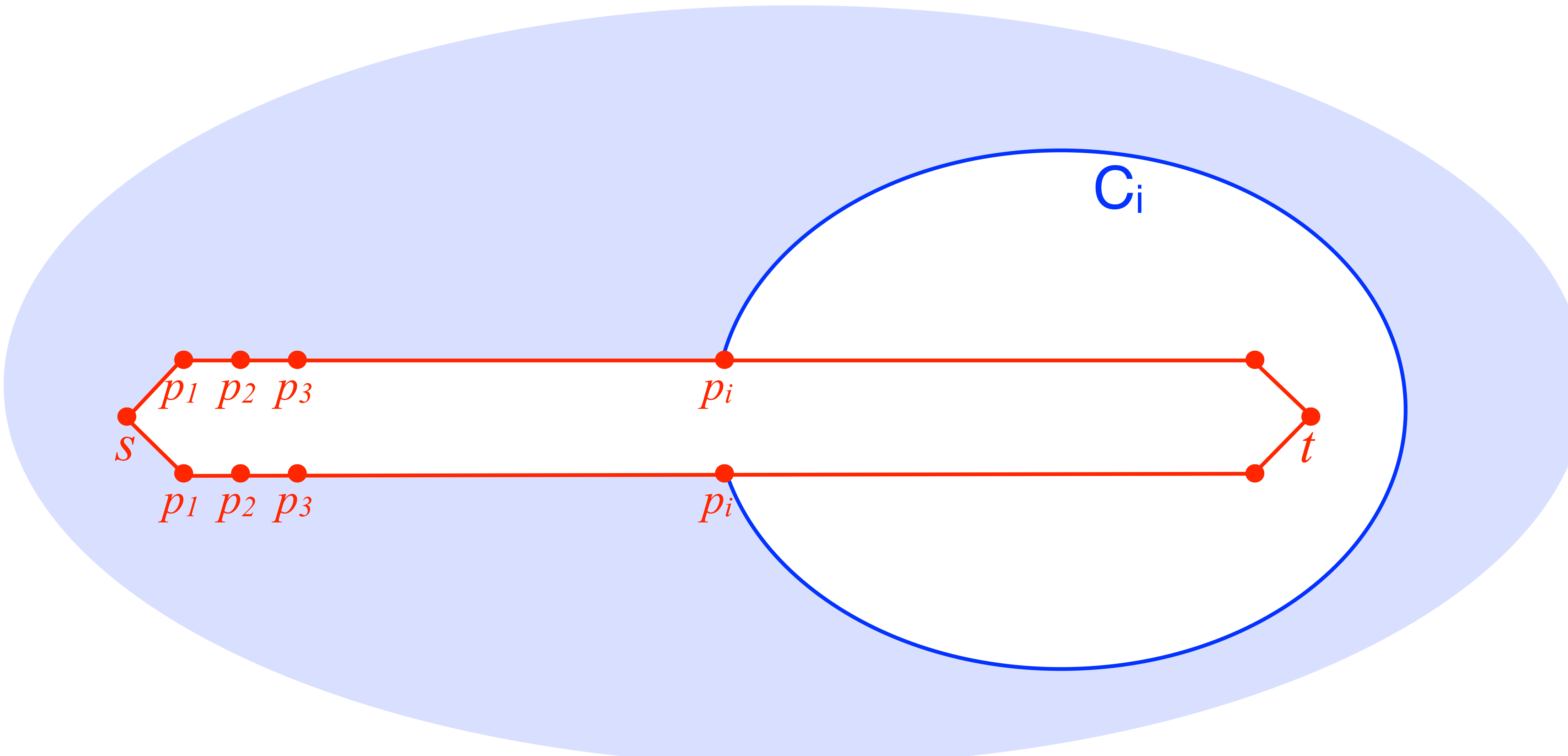

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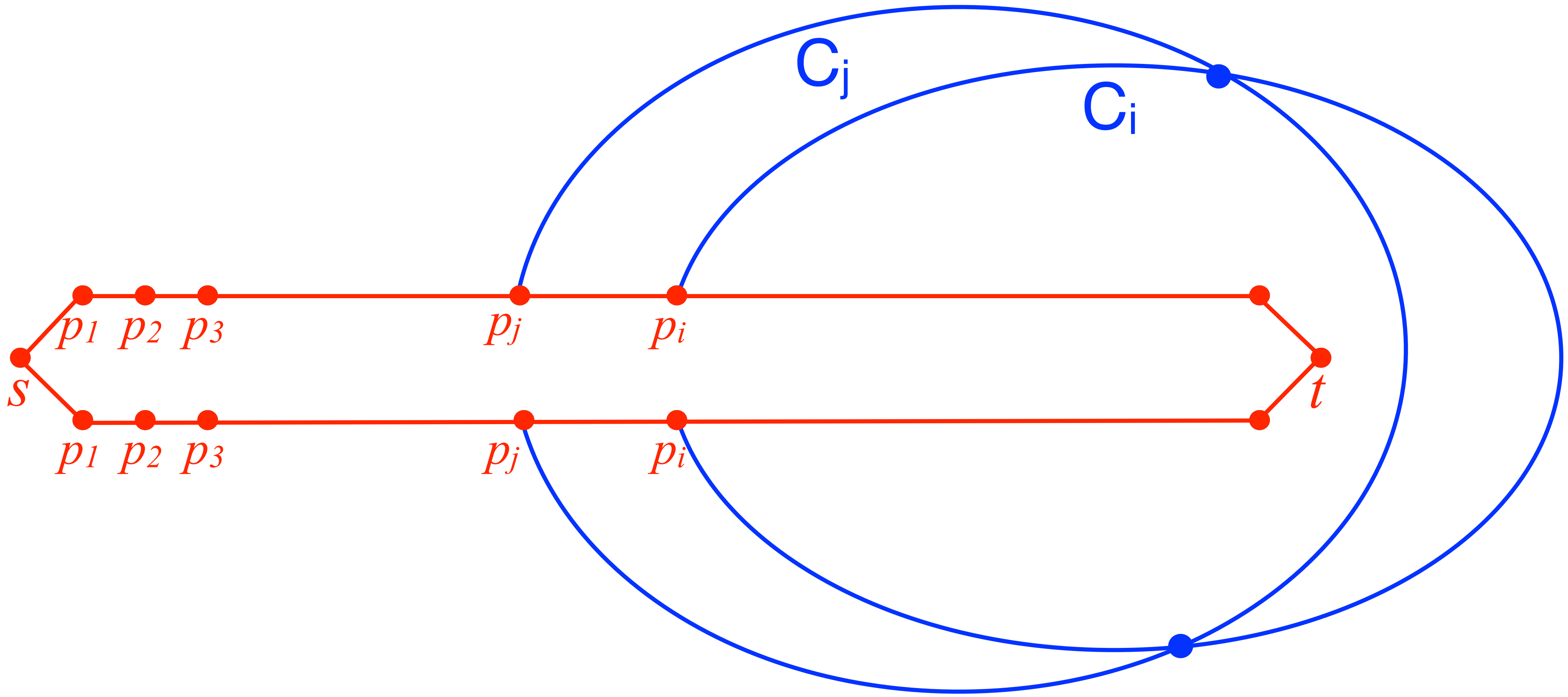

SSSP computation in O(n) time [Henzinger et al 1997]

Correctness: cycles do not cross

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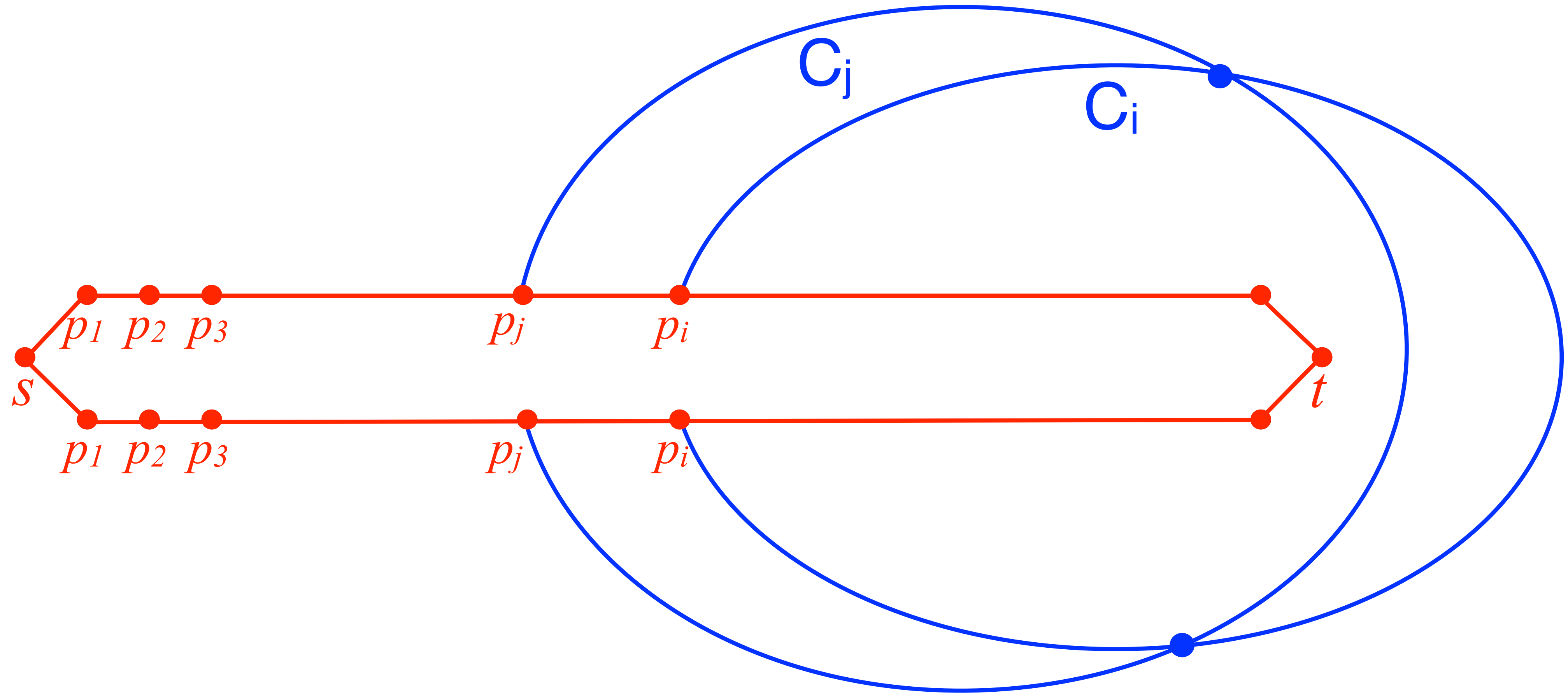

SSSP computation in O(n) time [Henzinger et al 1997]

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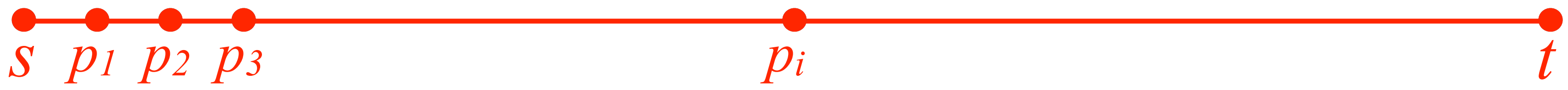
<u>Time</u>: O(nlogn)

Coarse Reif [Italiano,Nussbaum,Sankowski,Wulff-Nilsen STOC 2011]



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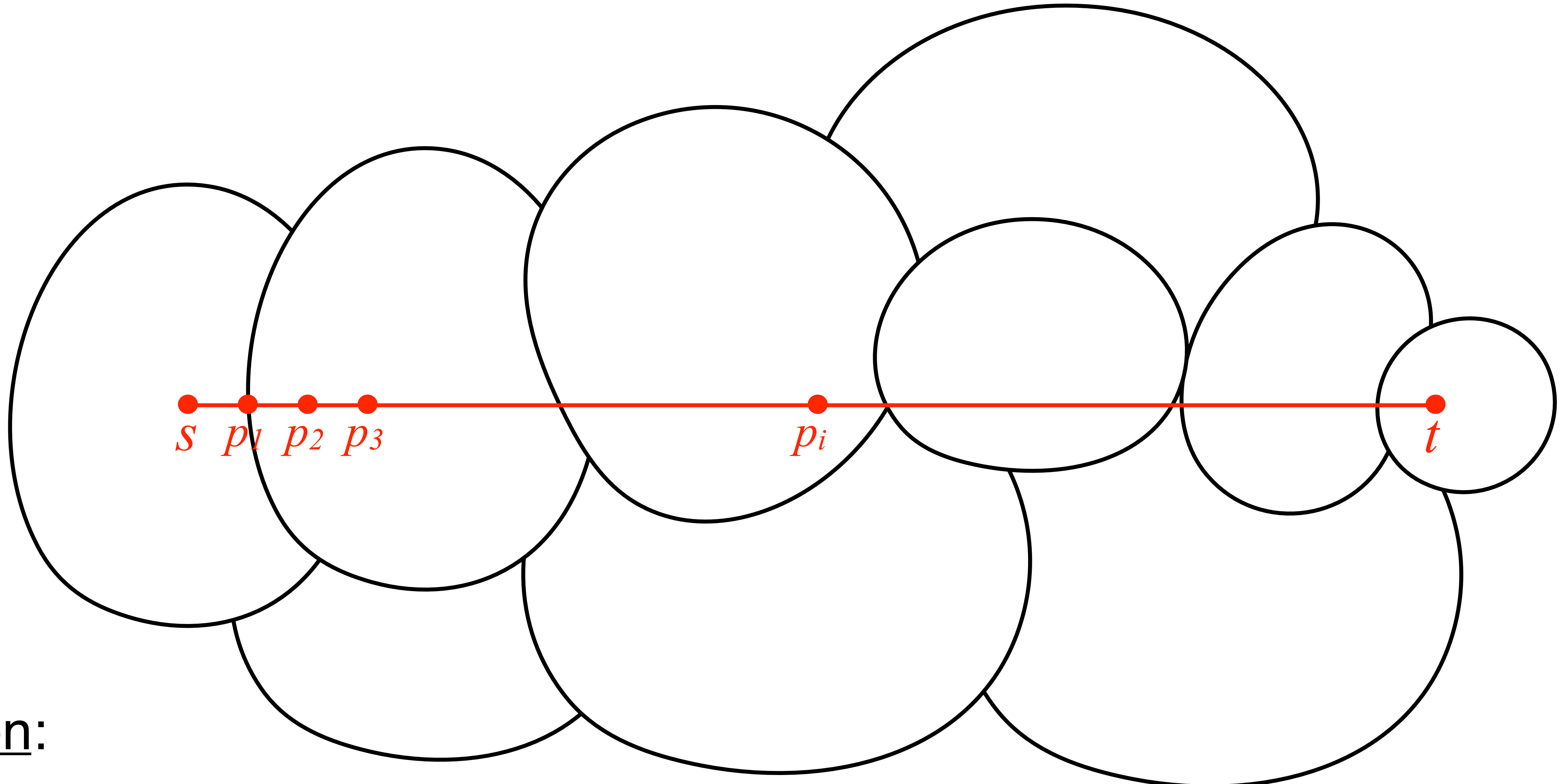

r-division:

O(n/r) pieces each with O(r) vertices and O(√r) boundary vertices

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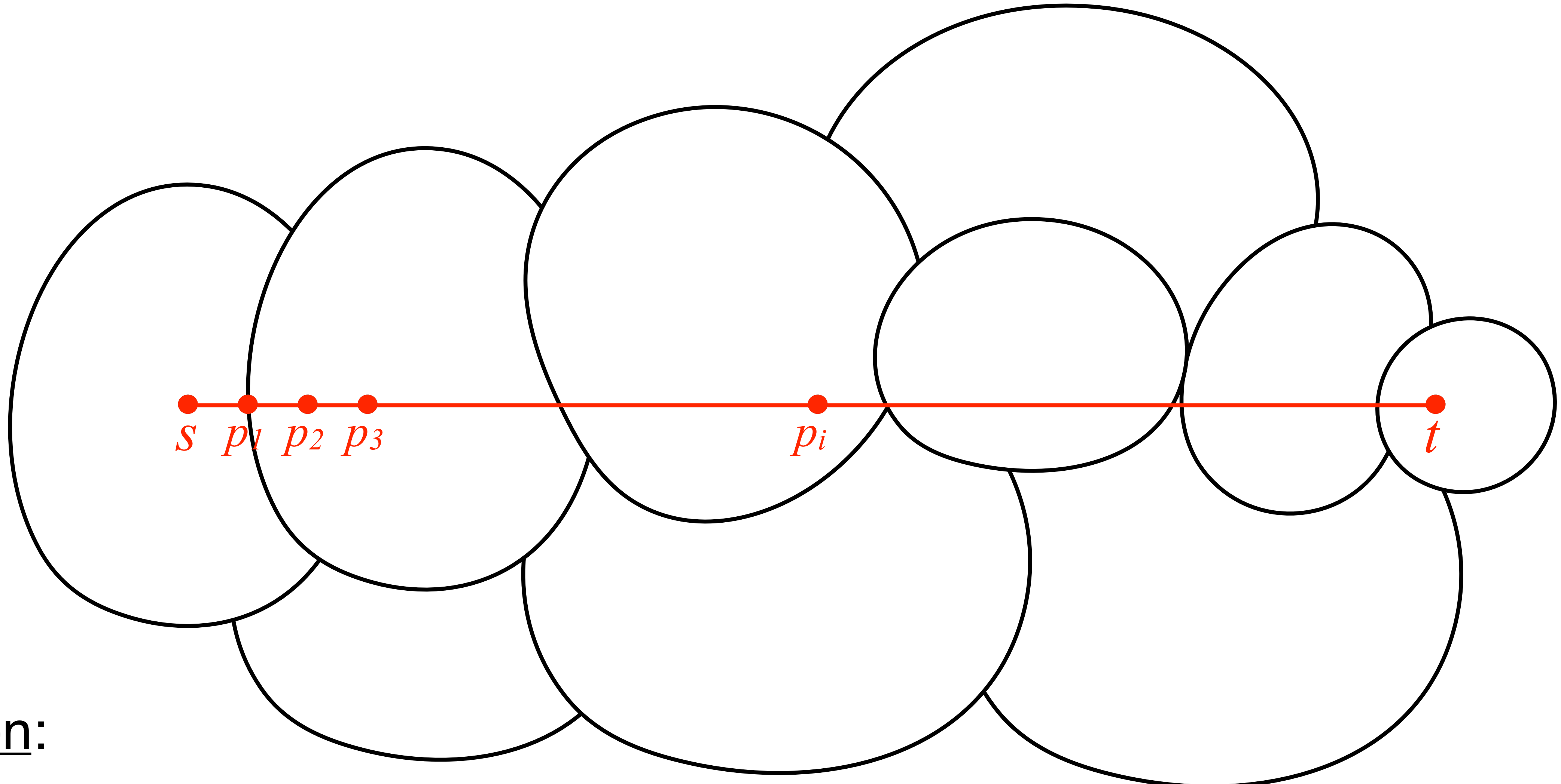

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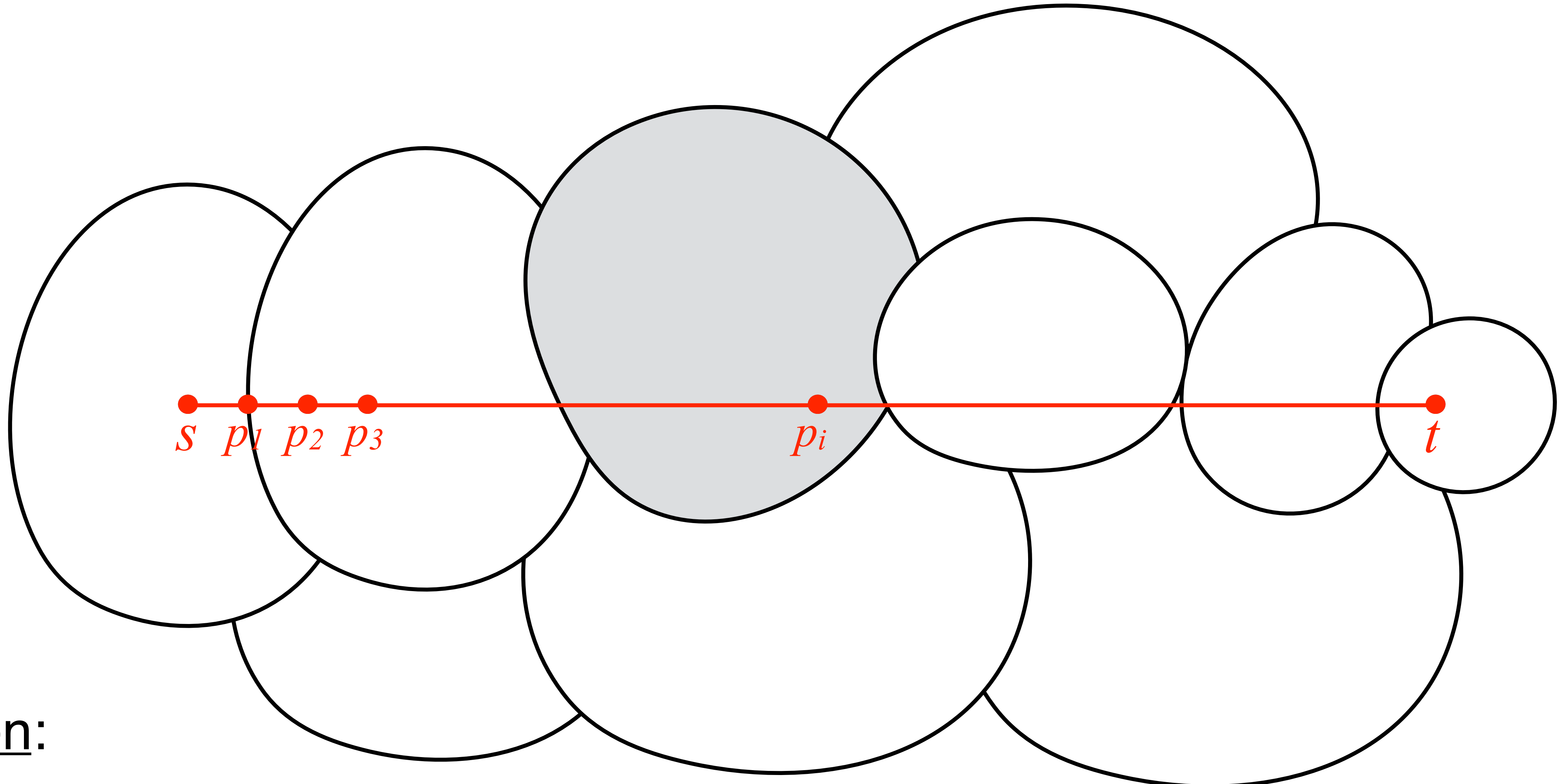

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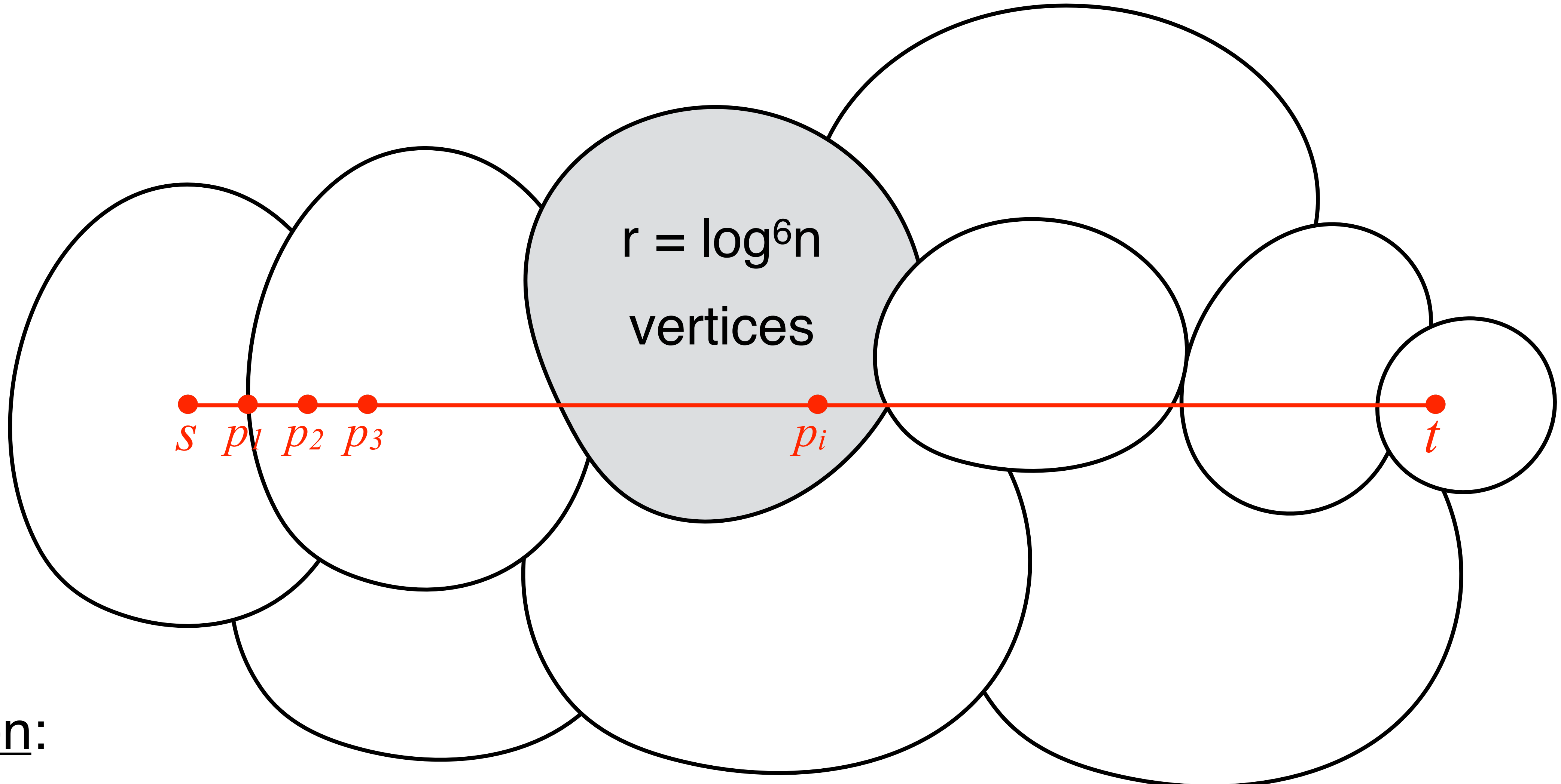

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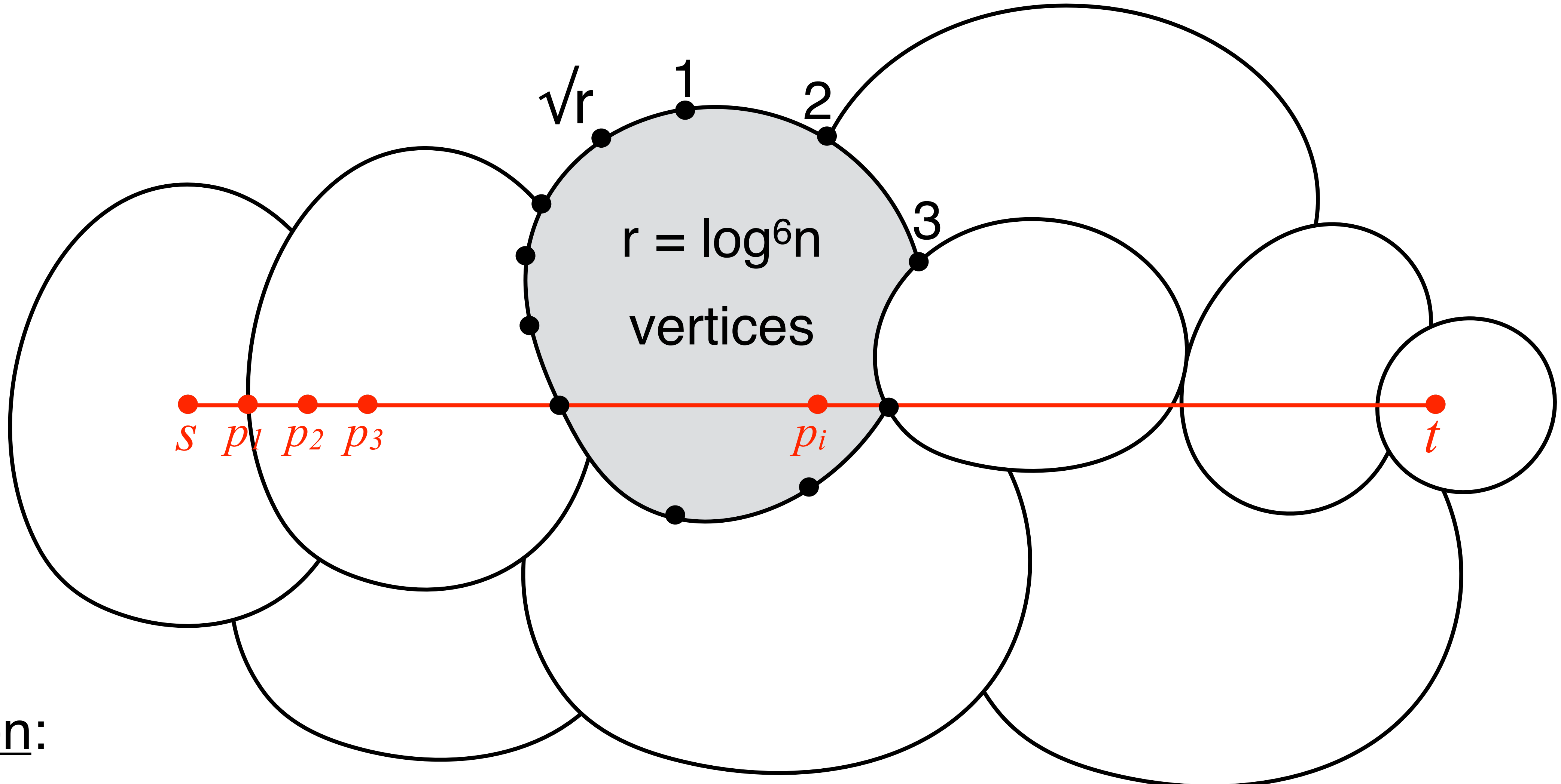

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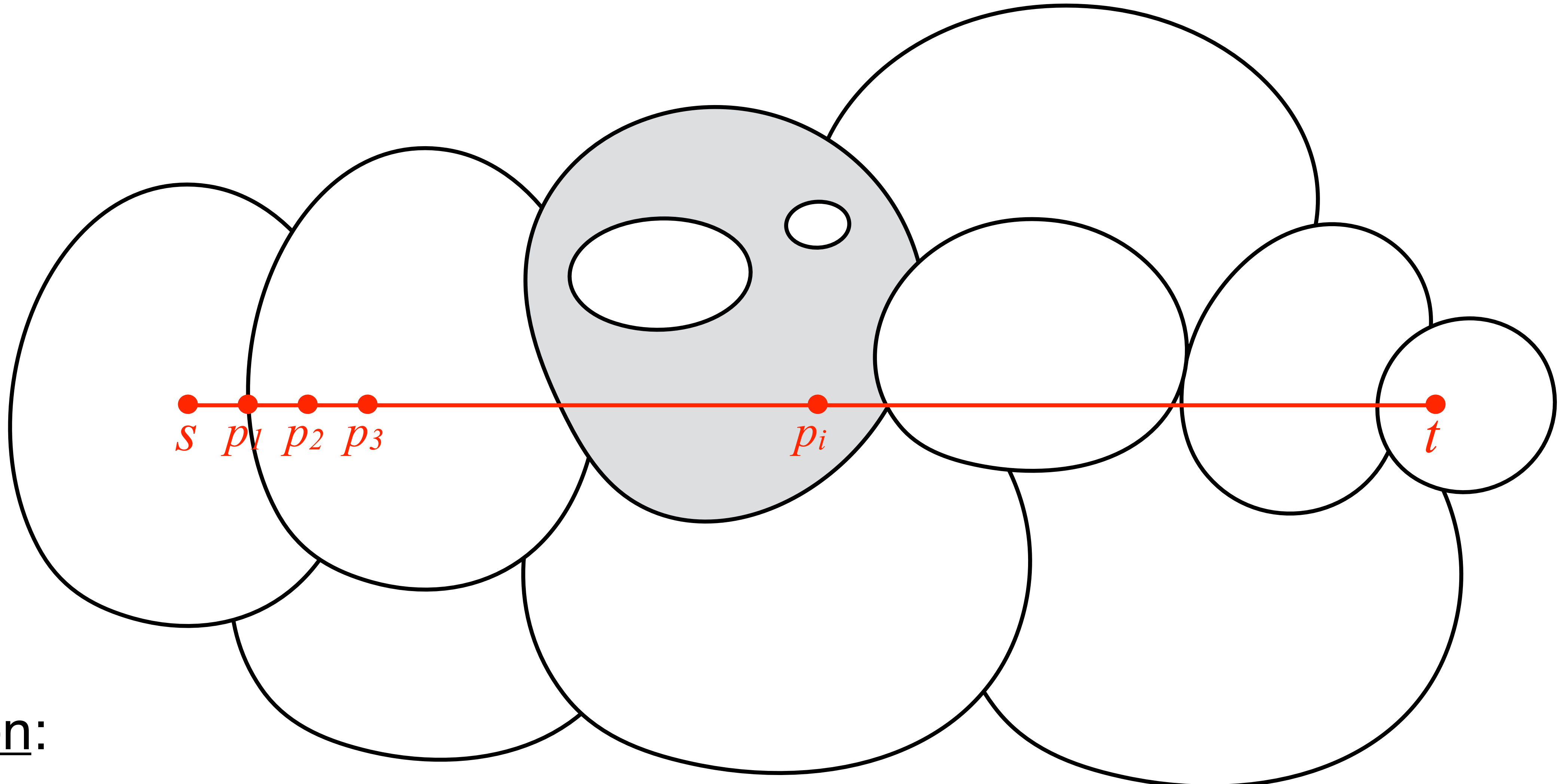

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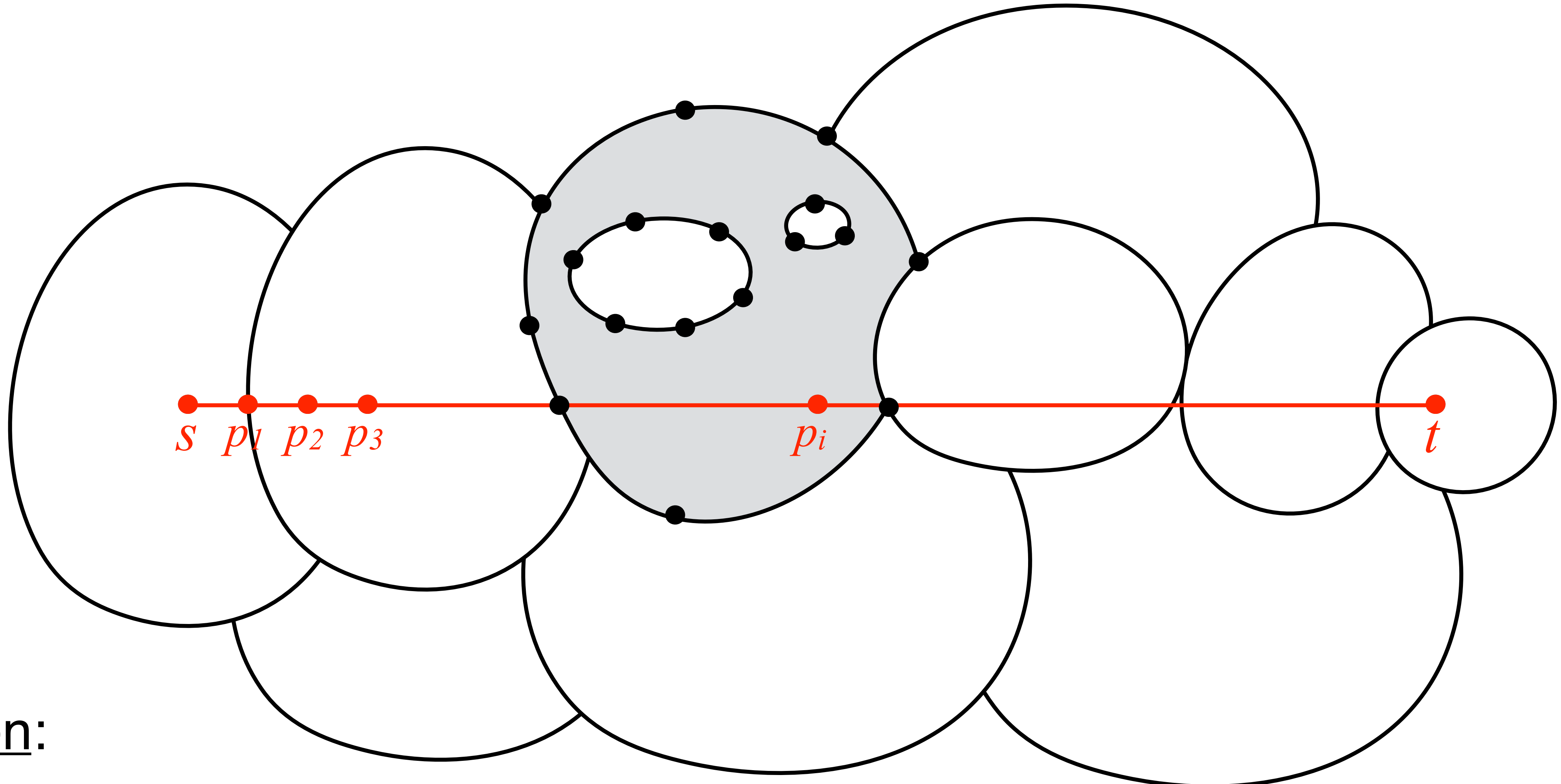

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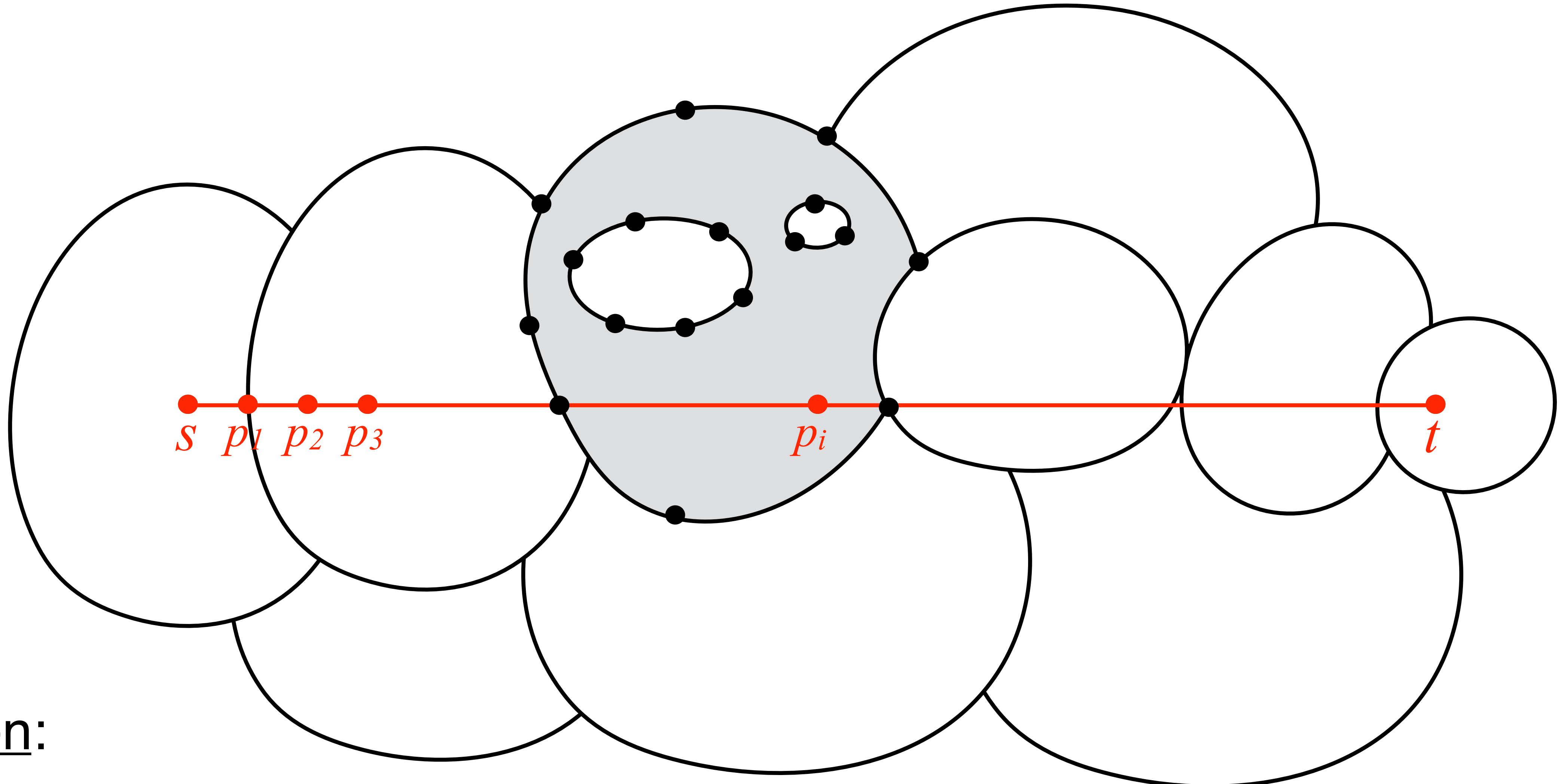

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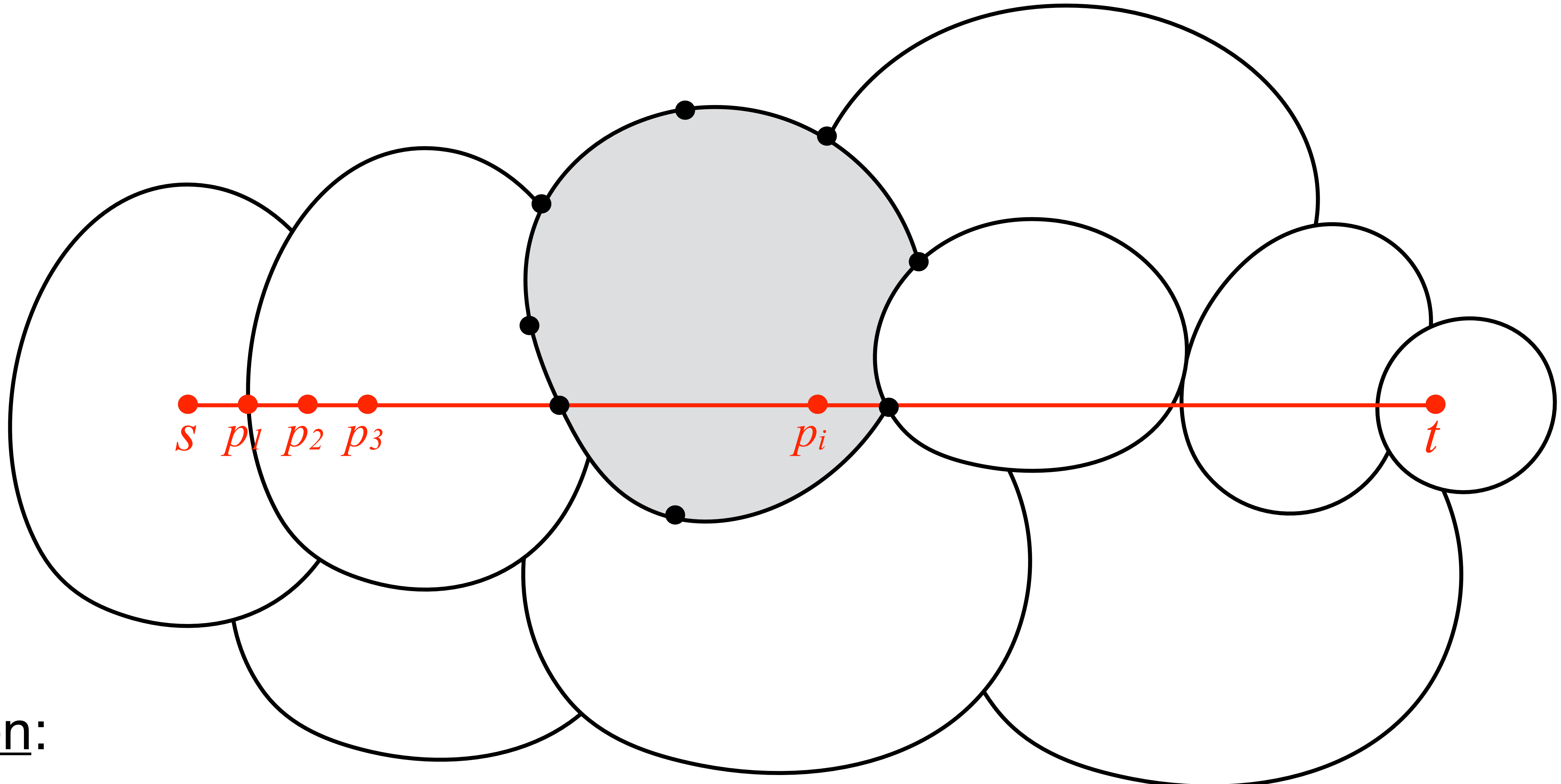

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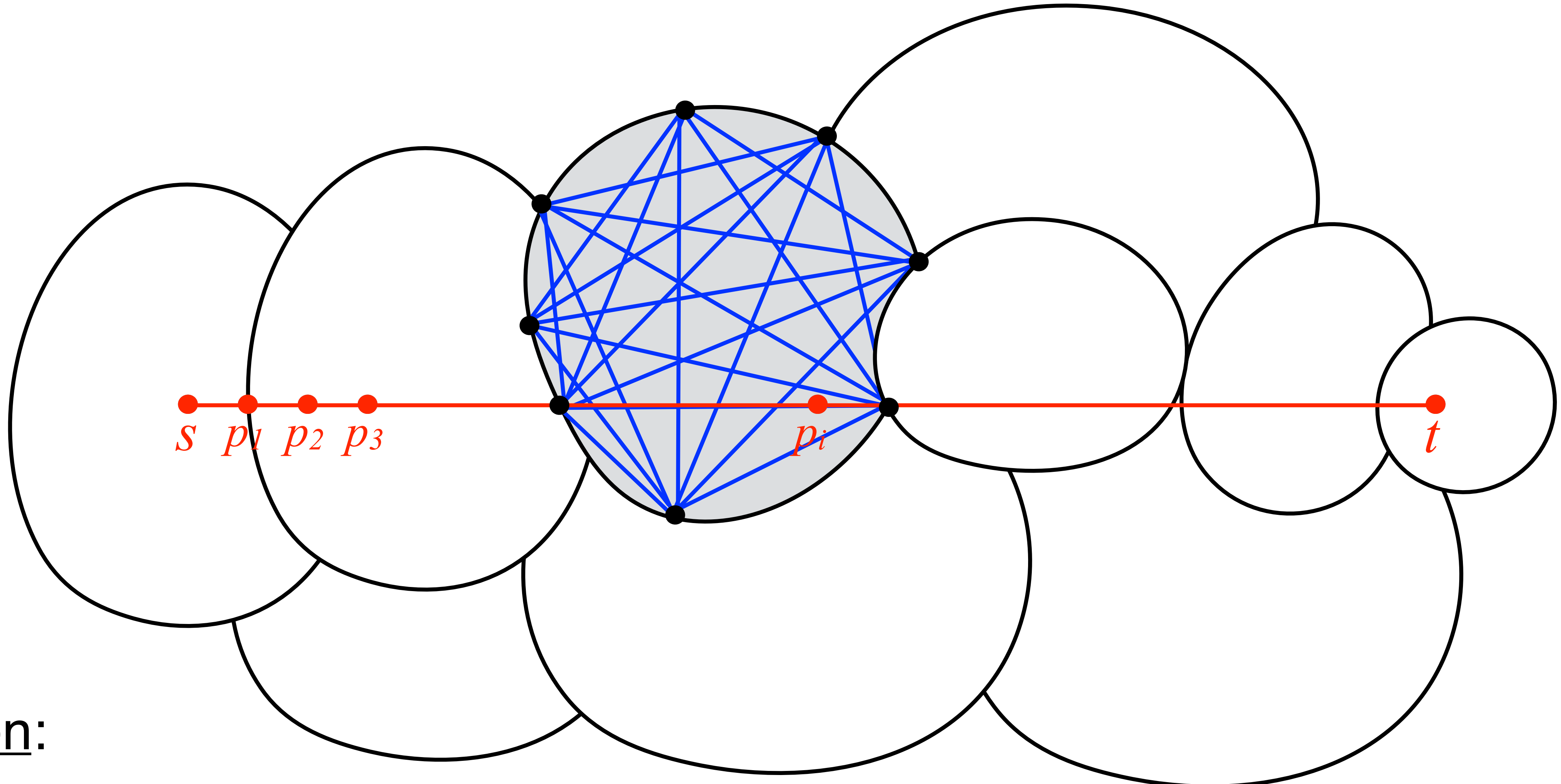

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Dense Distance Graph (DDG): all boundary-to-boundary distance matrices

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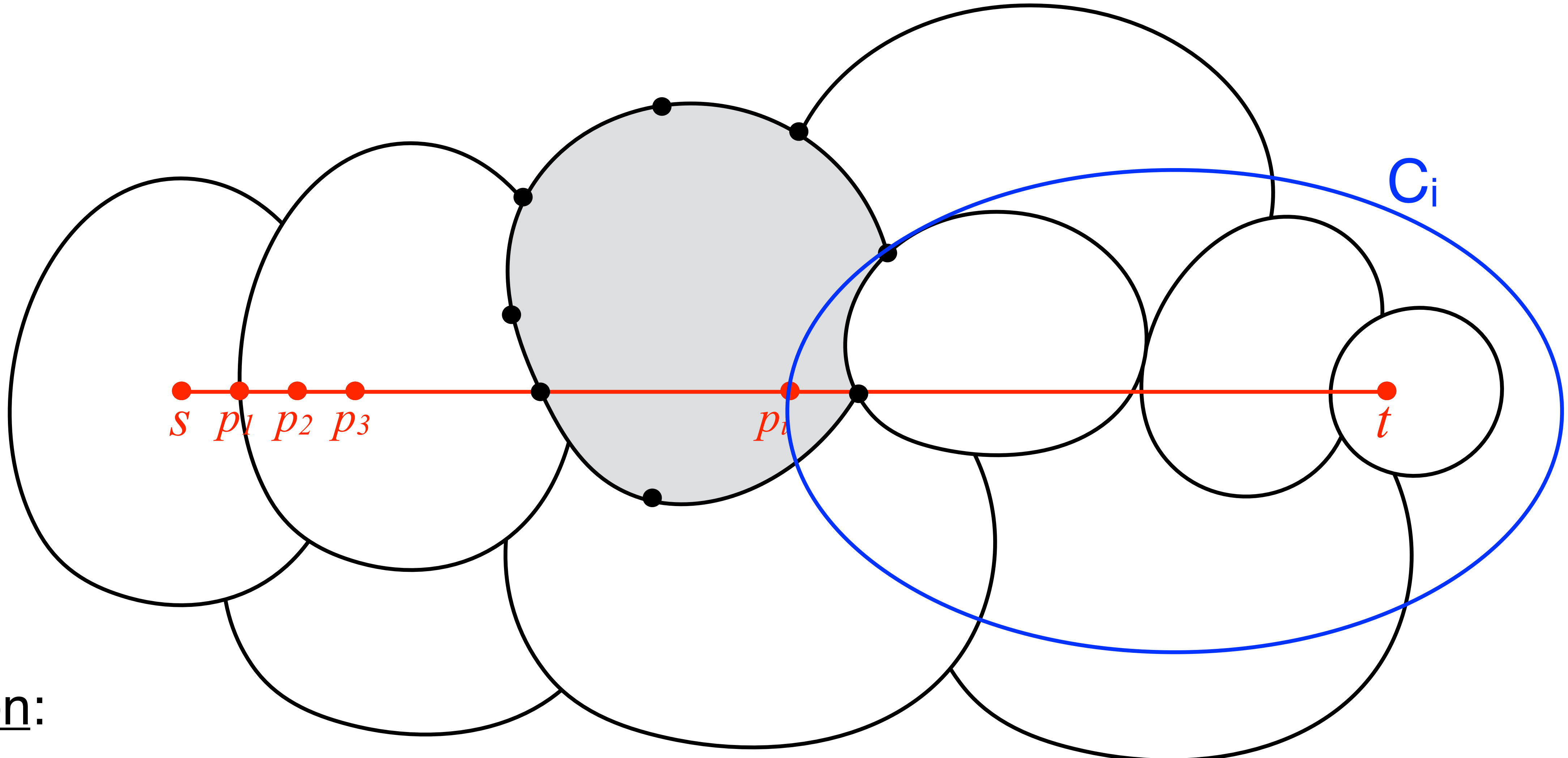

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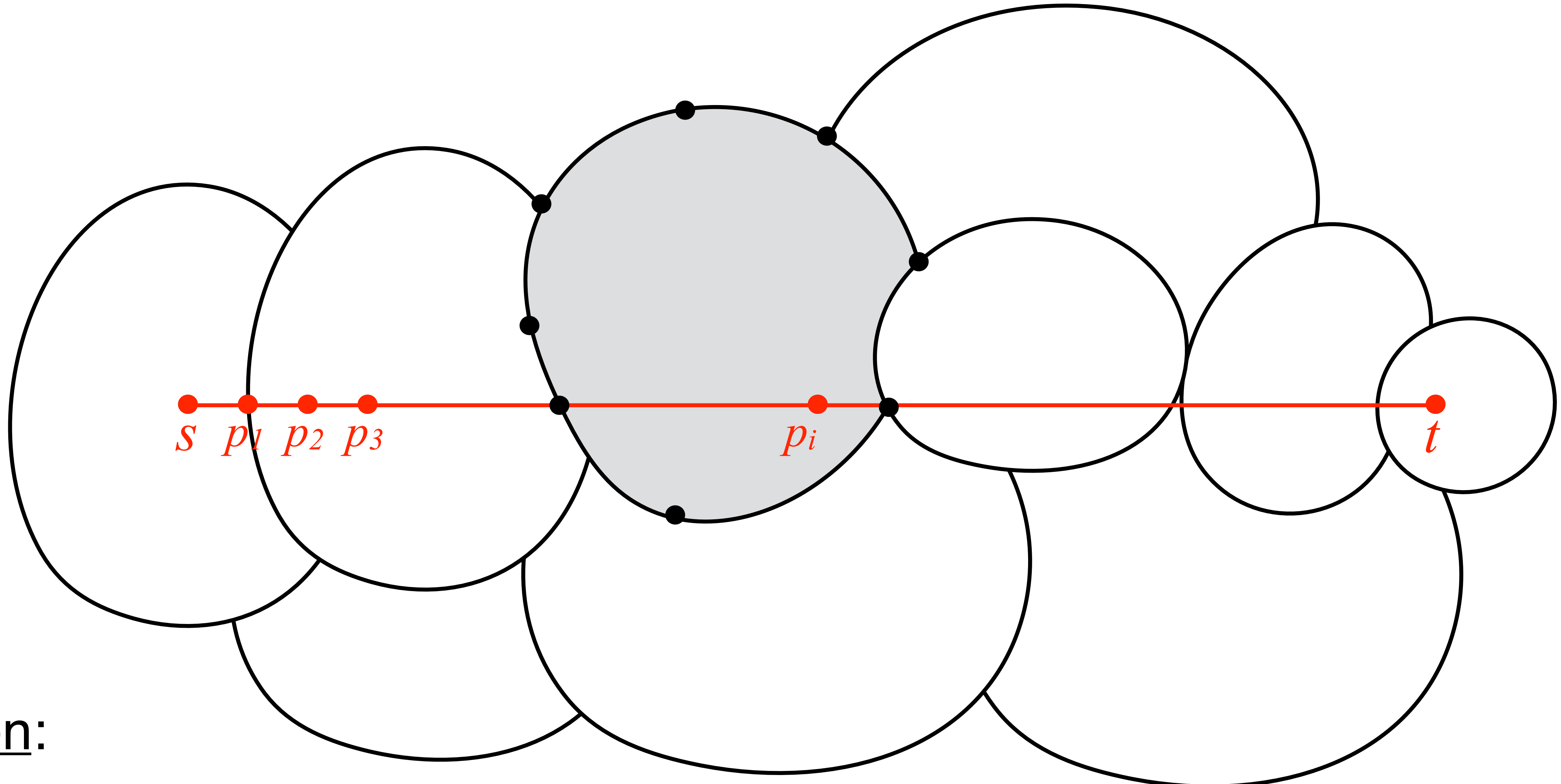

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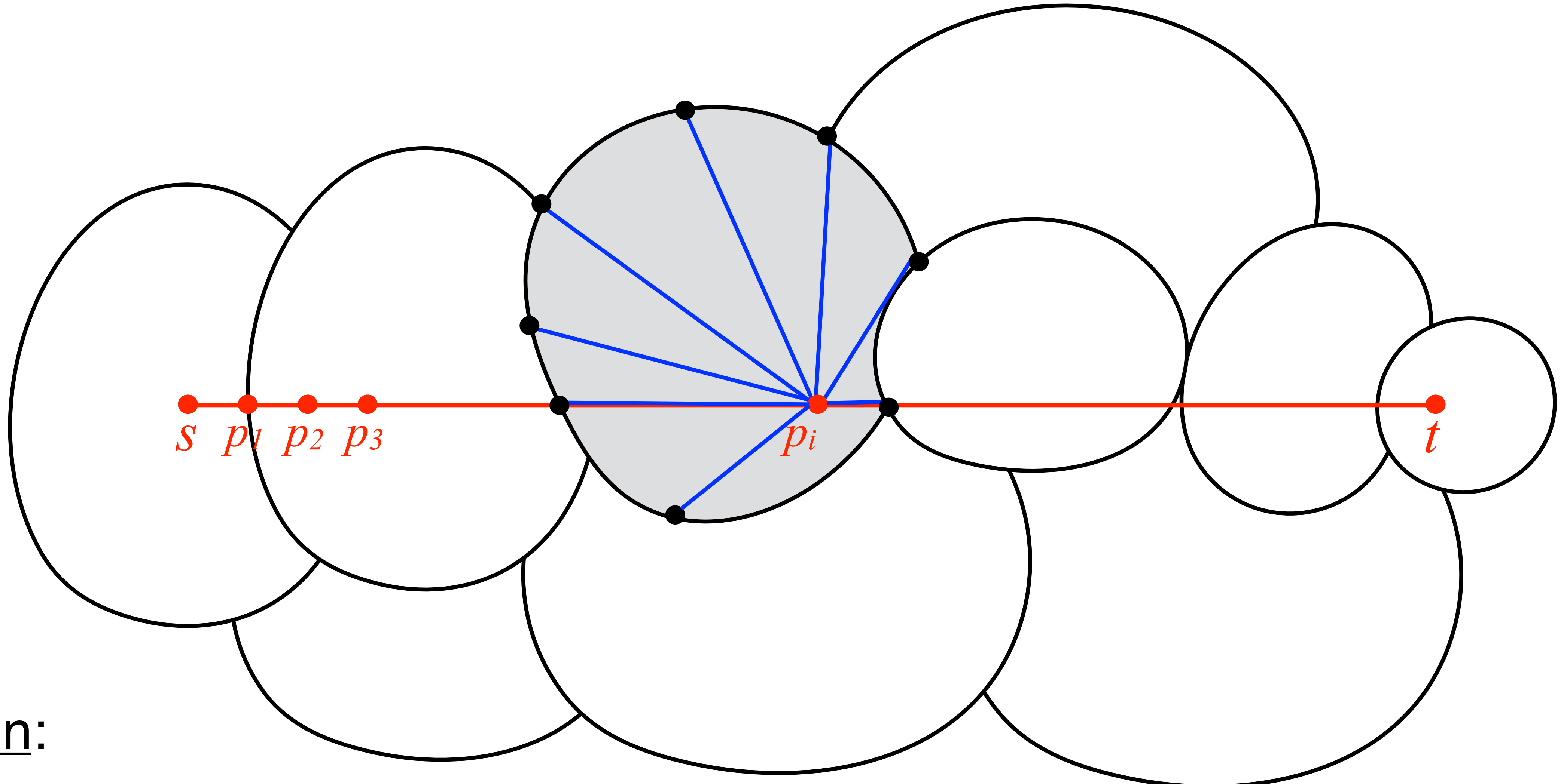

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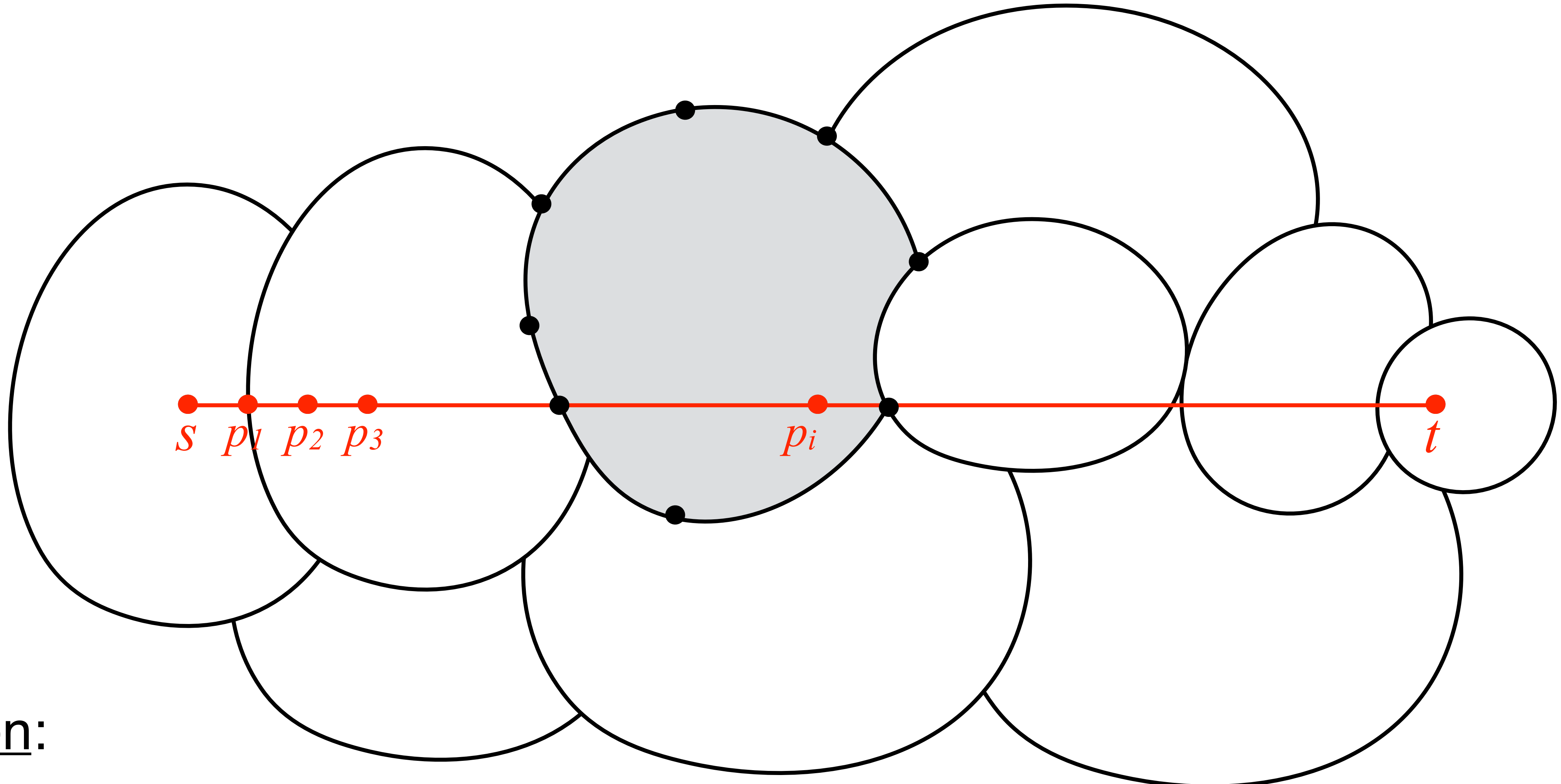

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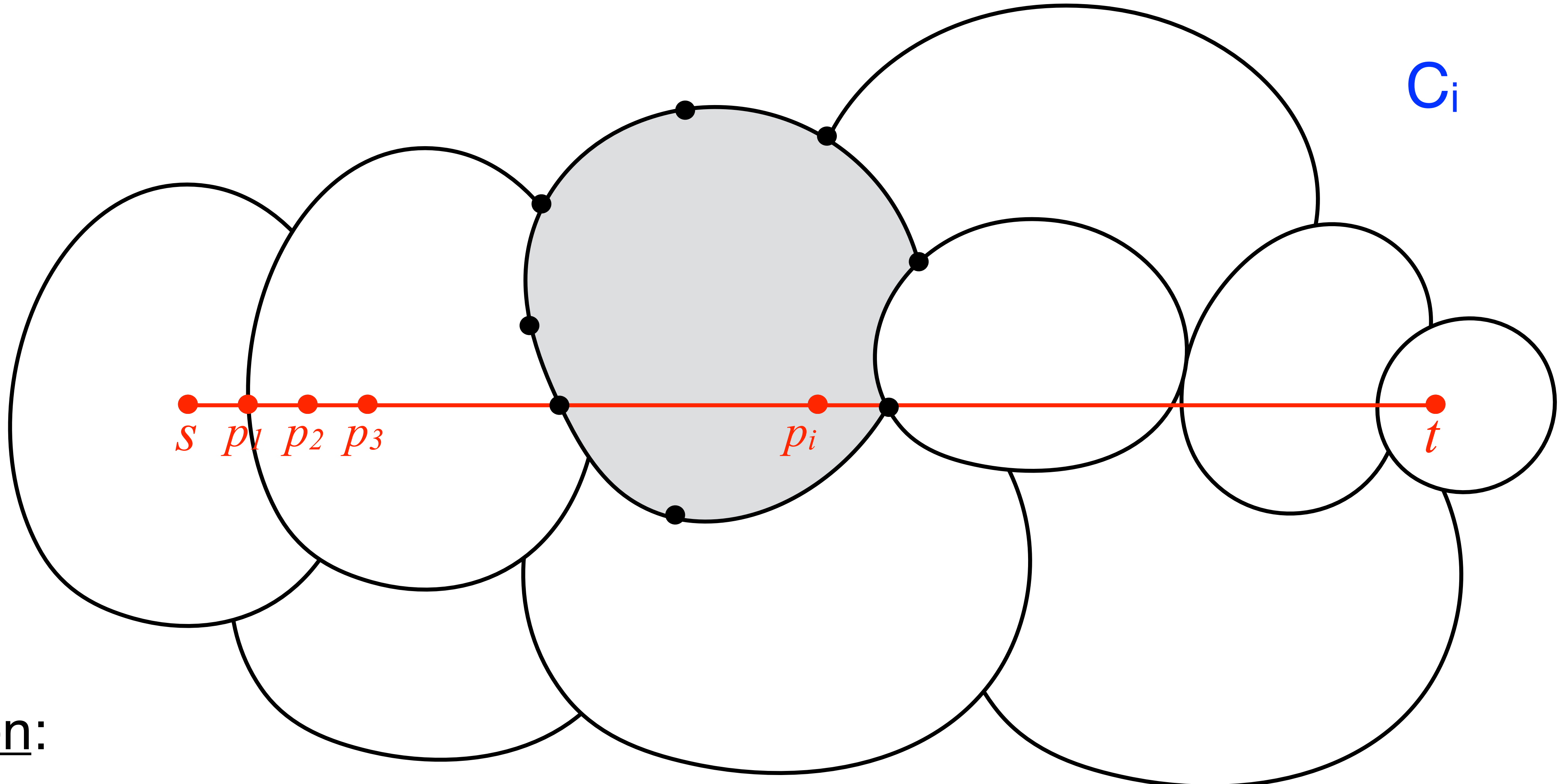

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Find C_i with SSSP on DDG:

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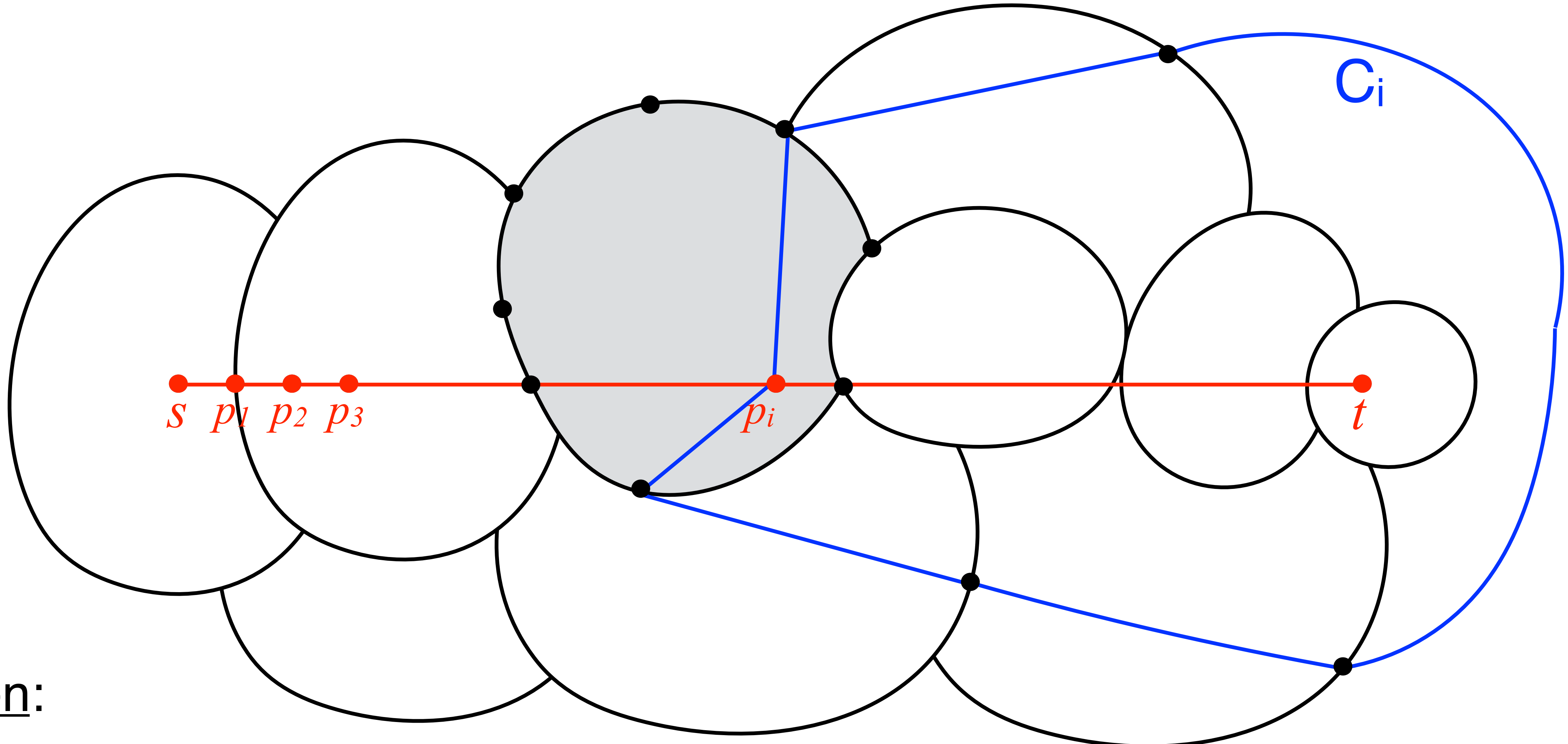

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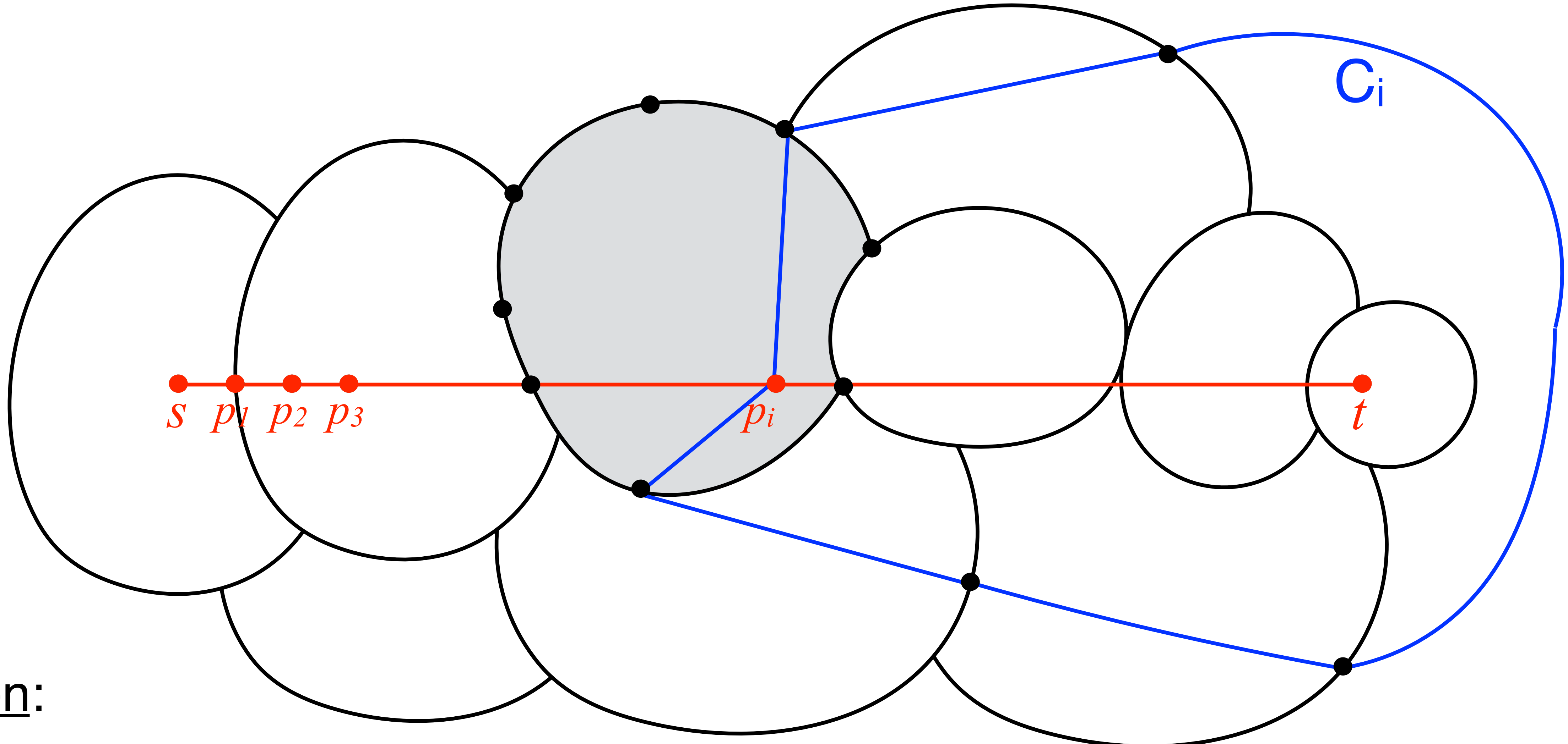

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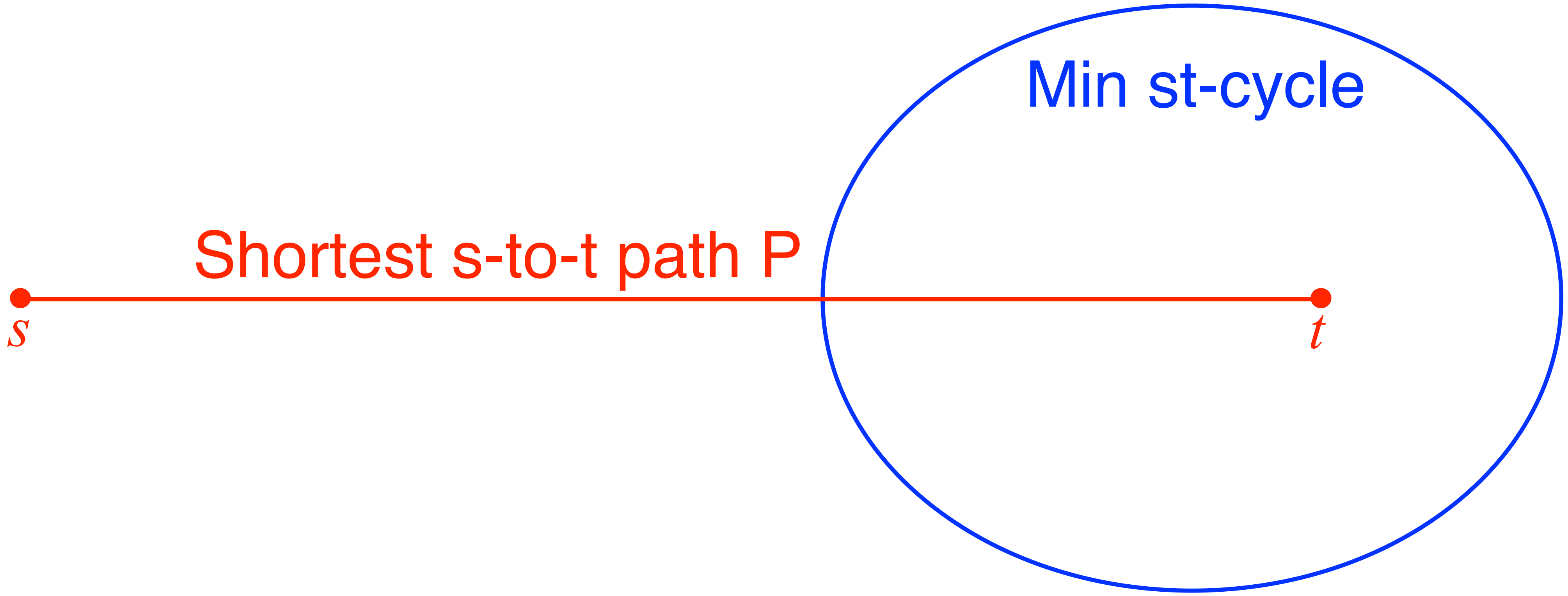
Find C_i with SSSP on DDG:

$O(n\log^2 n/\sqrt{r}) = O(n/\log n)$ sublinear time [Fakcharoenphol, Rao FOCS 2001]

Undirected Min st-cycle in O(nloglogn) time

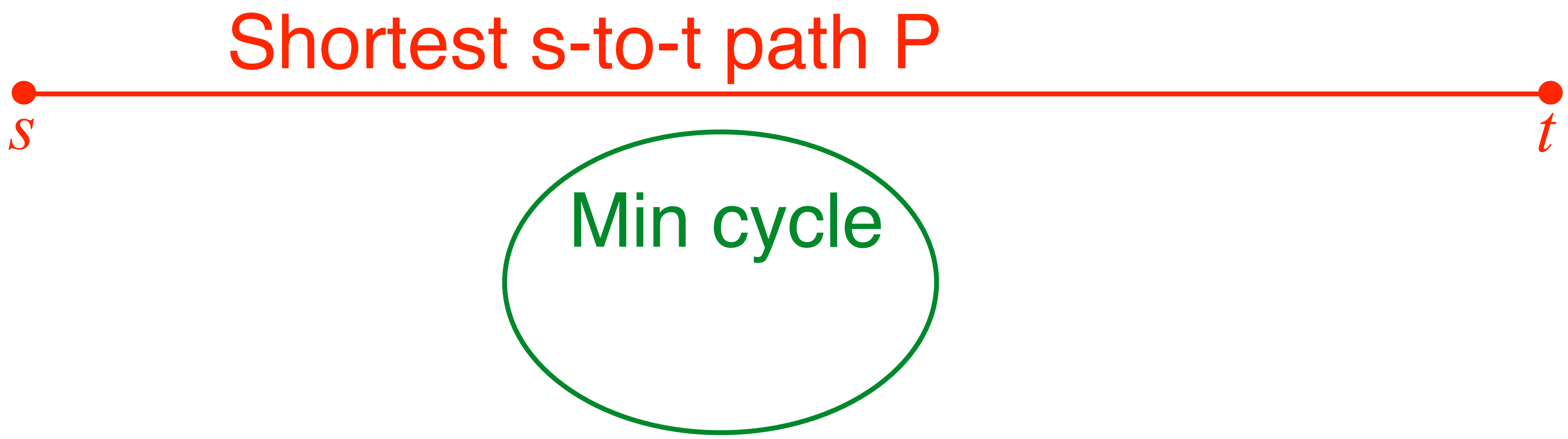
[Italiano,Nussbaum,Sankowski,Wulff-Nilsen STOC 2011]



Bottleneck: computing DDG (O(nloglogn) time)

Undirected Min cycle in O(nloglogn) time

[Łącki, Sankowski ESA 2011]



Undirected Min cycle in O(nloglogn) time

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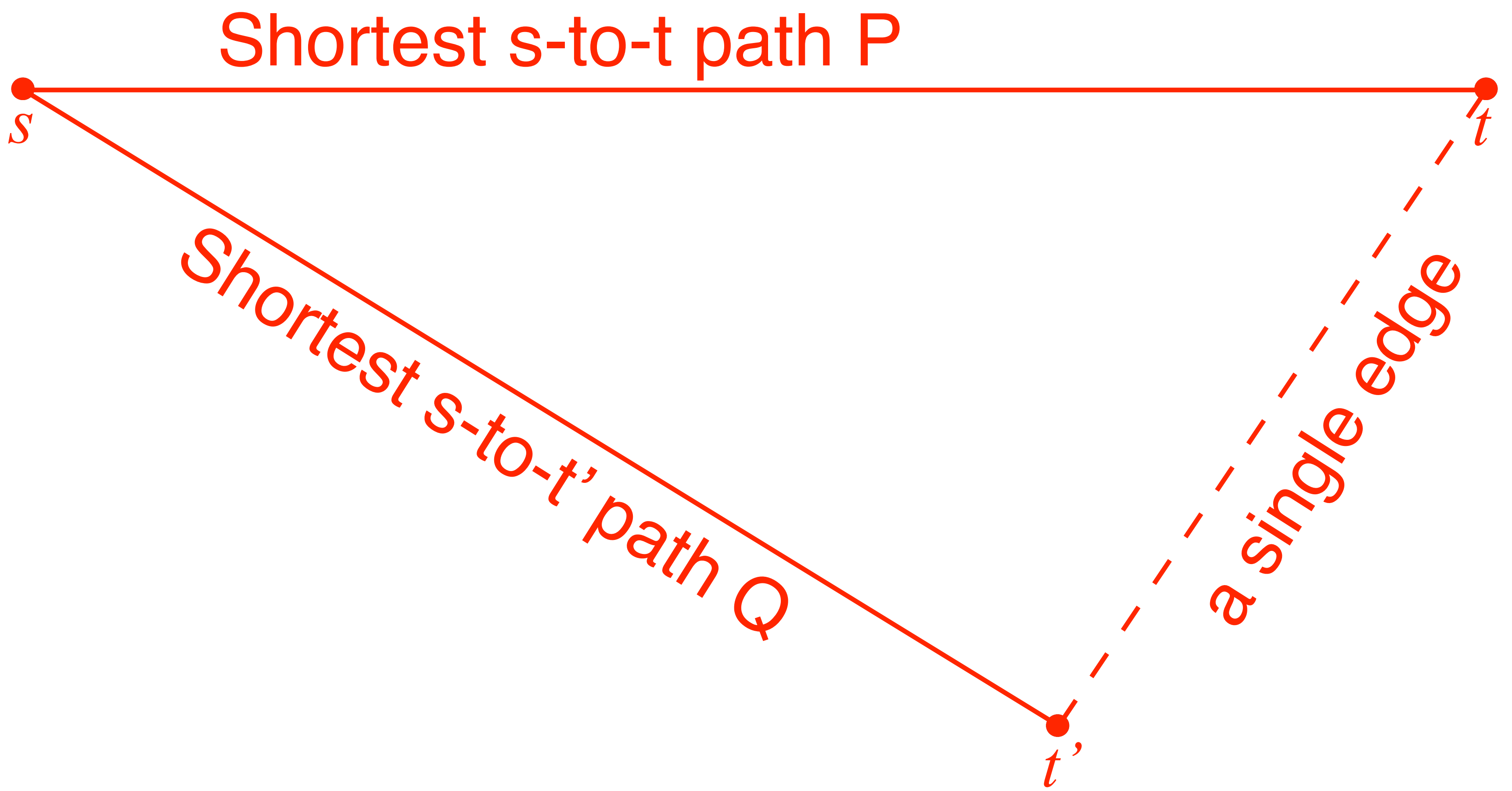
Shortest Path Separator



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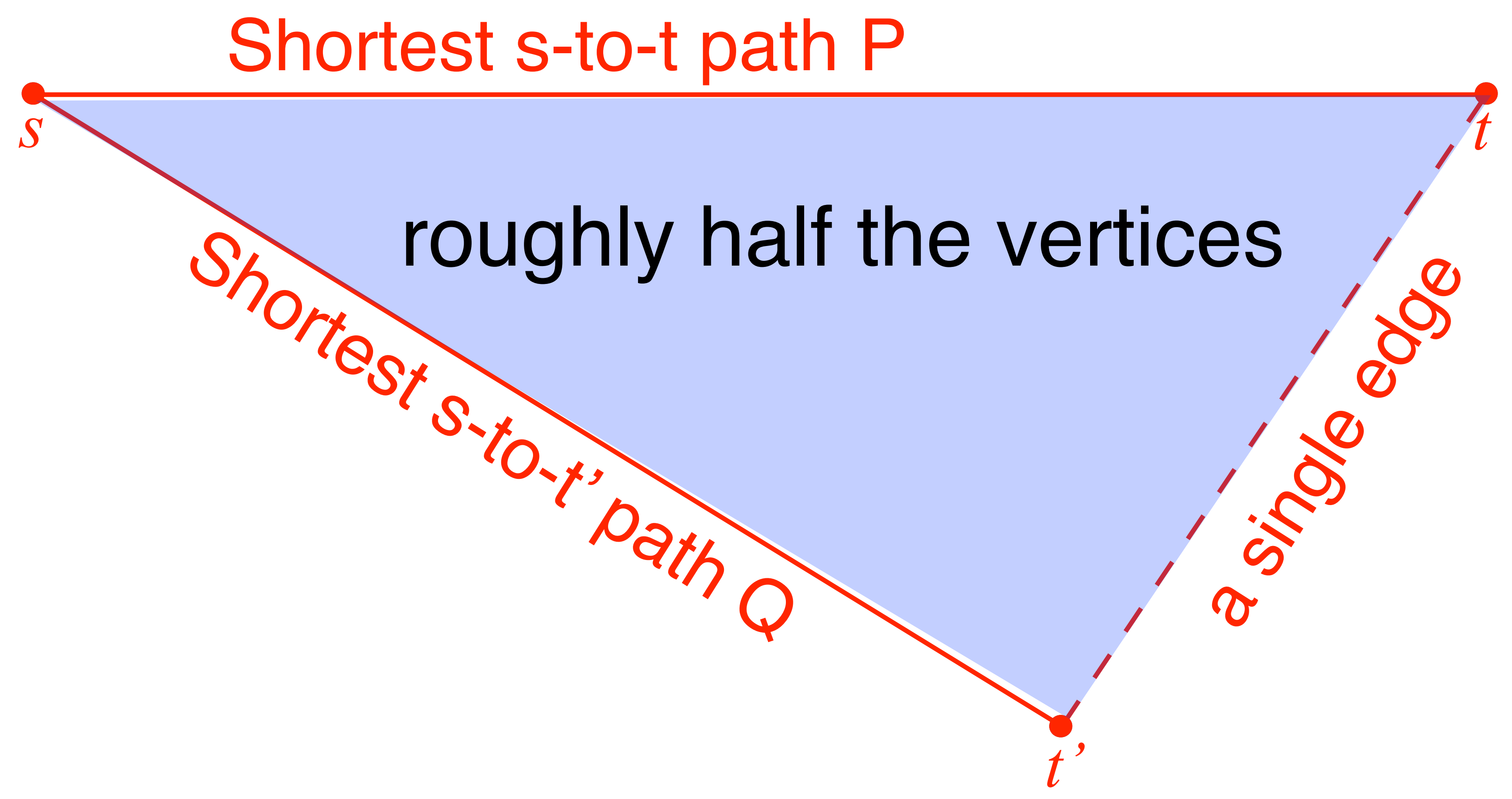

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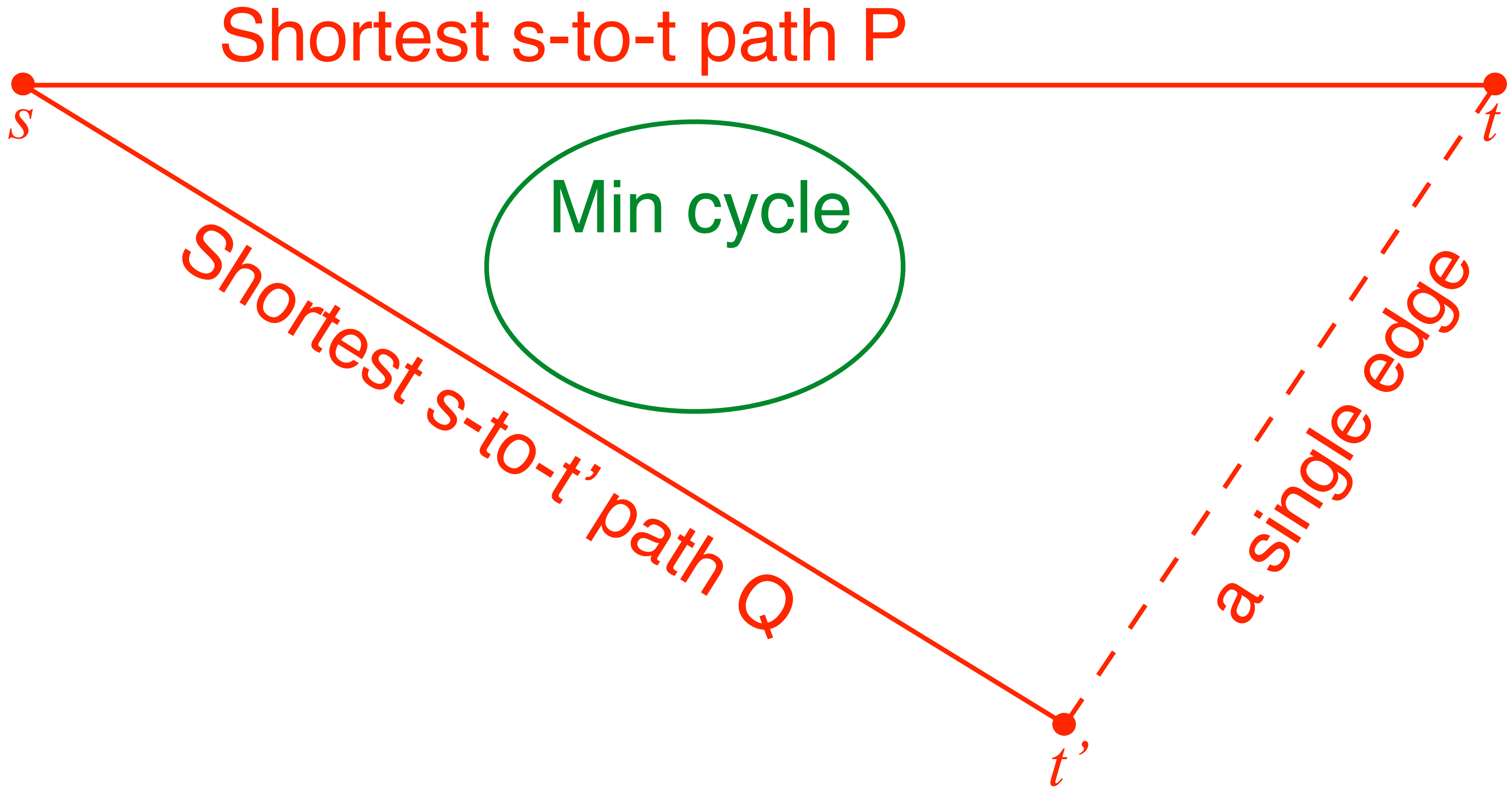

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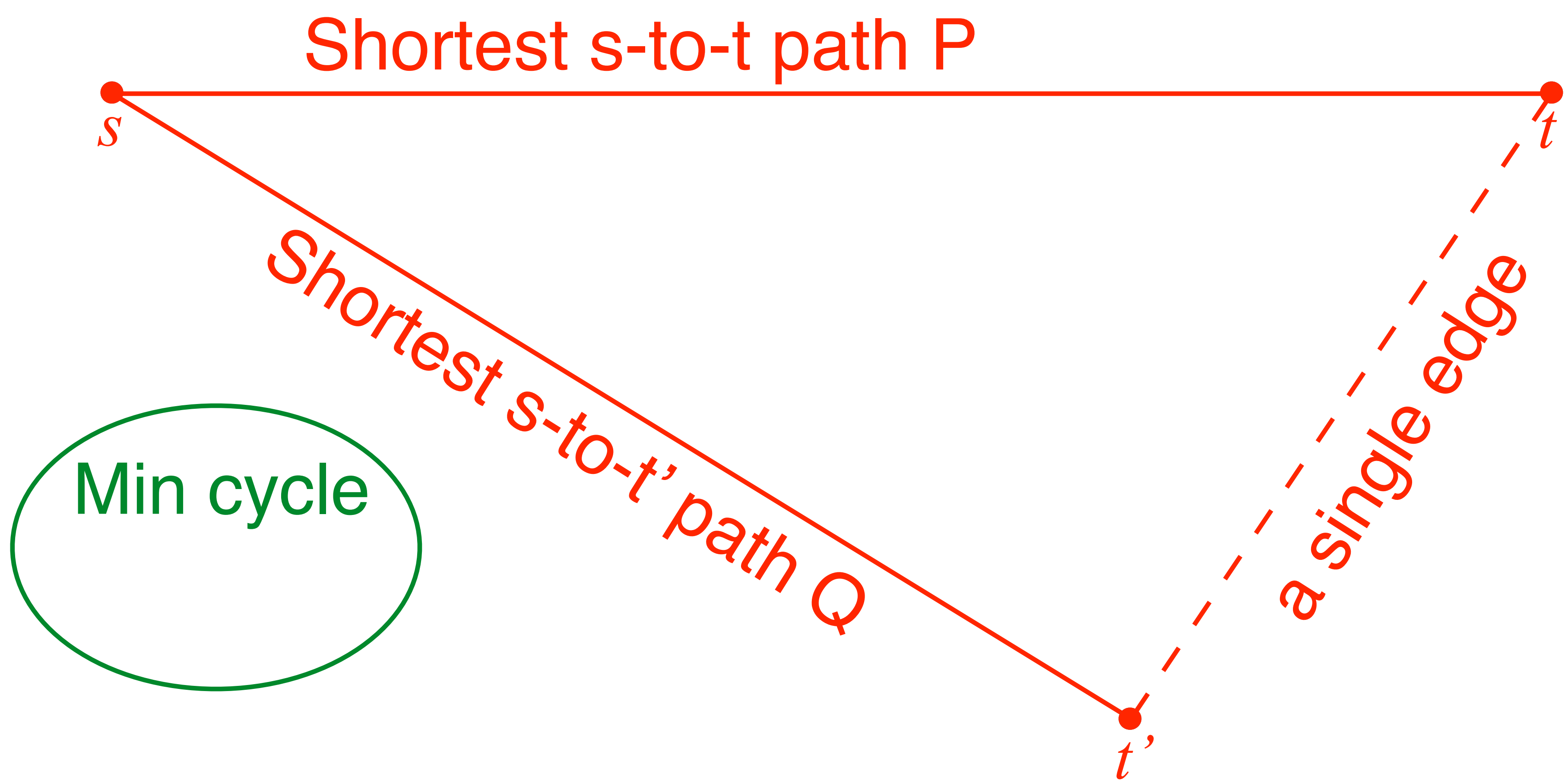

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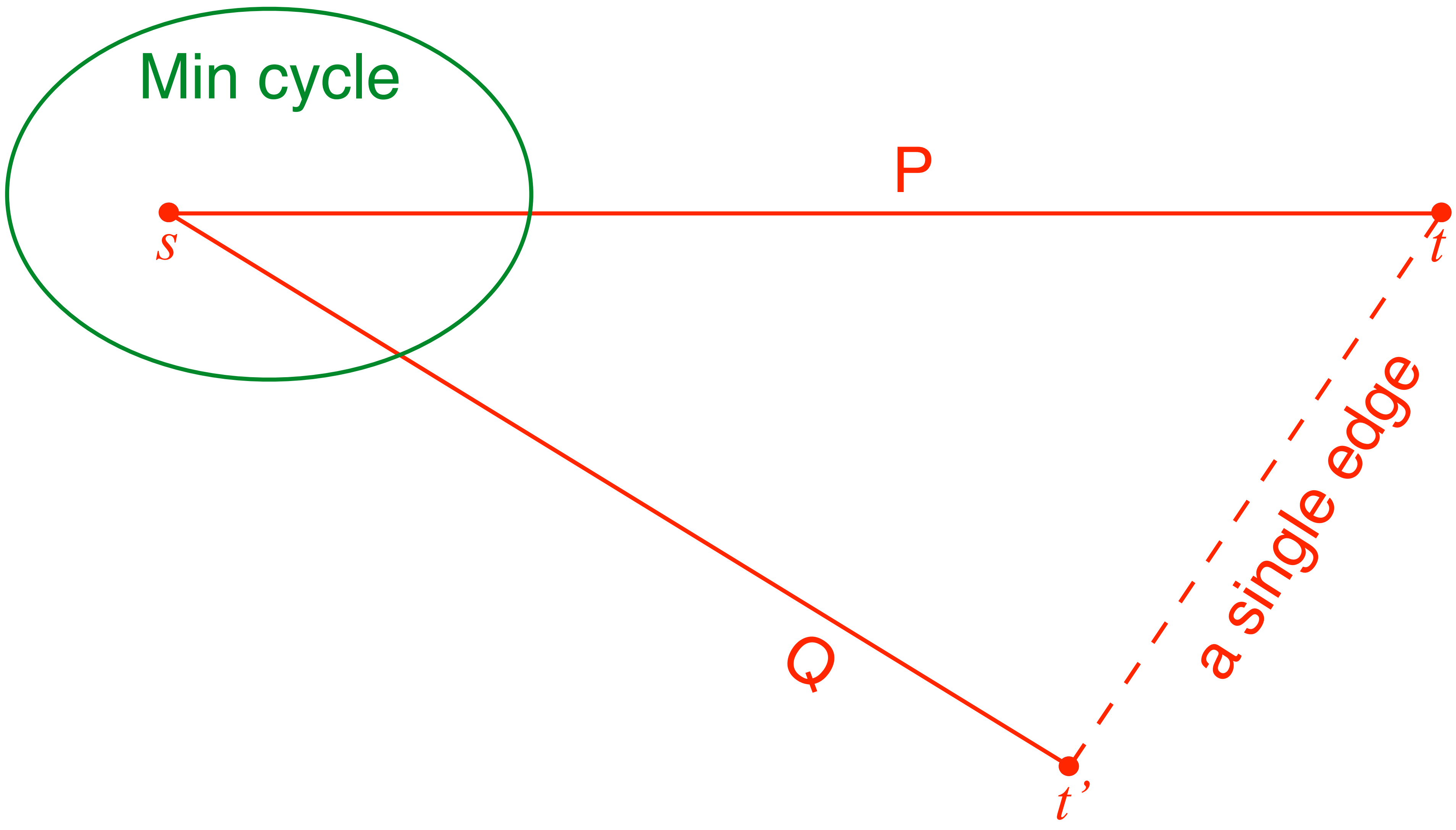

Undirected Min cycle in O(nloglogn) time

[Łącki, Sankowski ESA 2011]

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Find shortest cycle that crosses P once

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Bottleneck: recomputing DDG in each recursive level (O(nloglogn) time)

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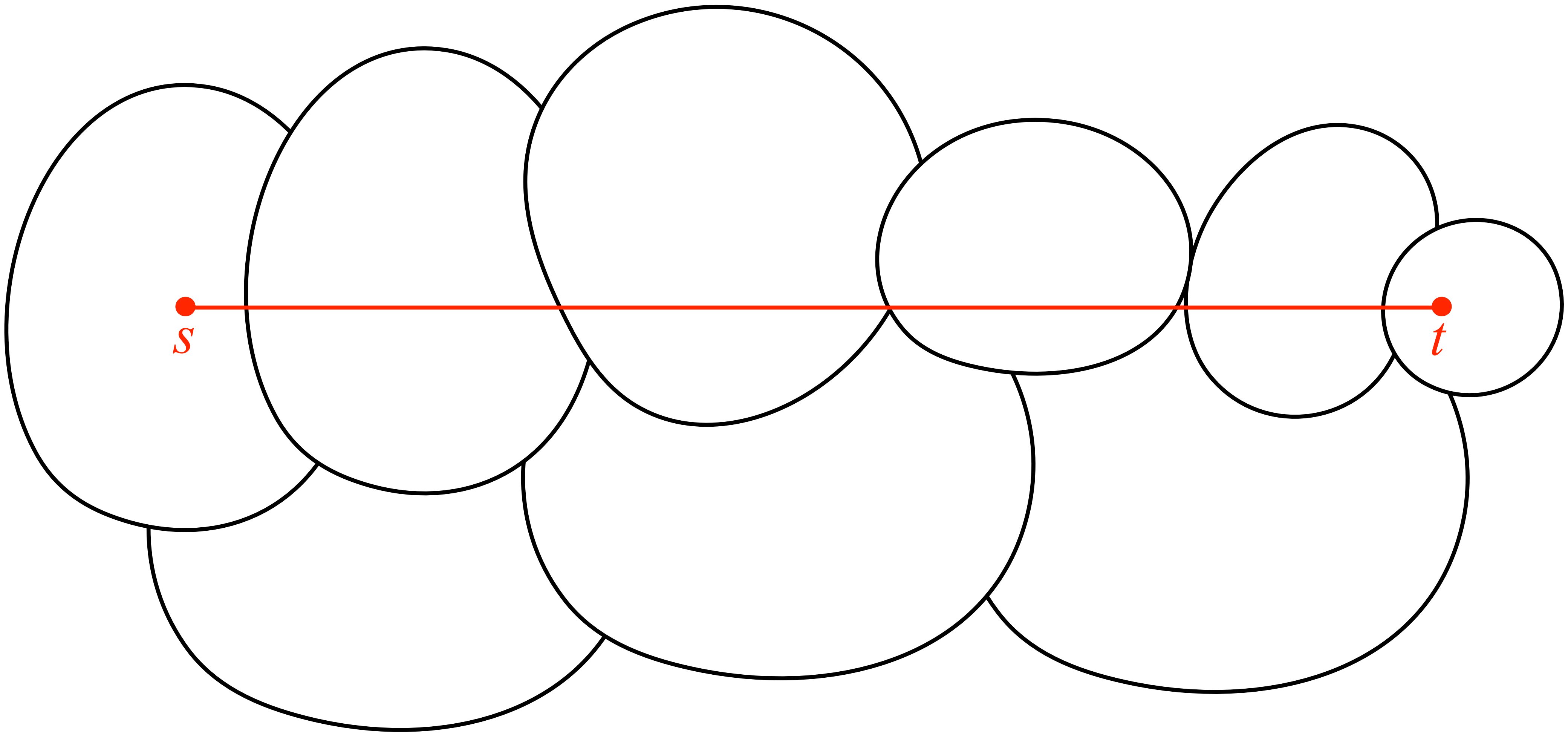

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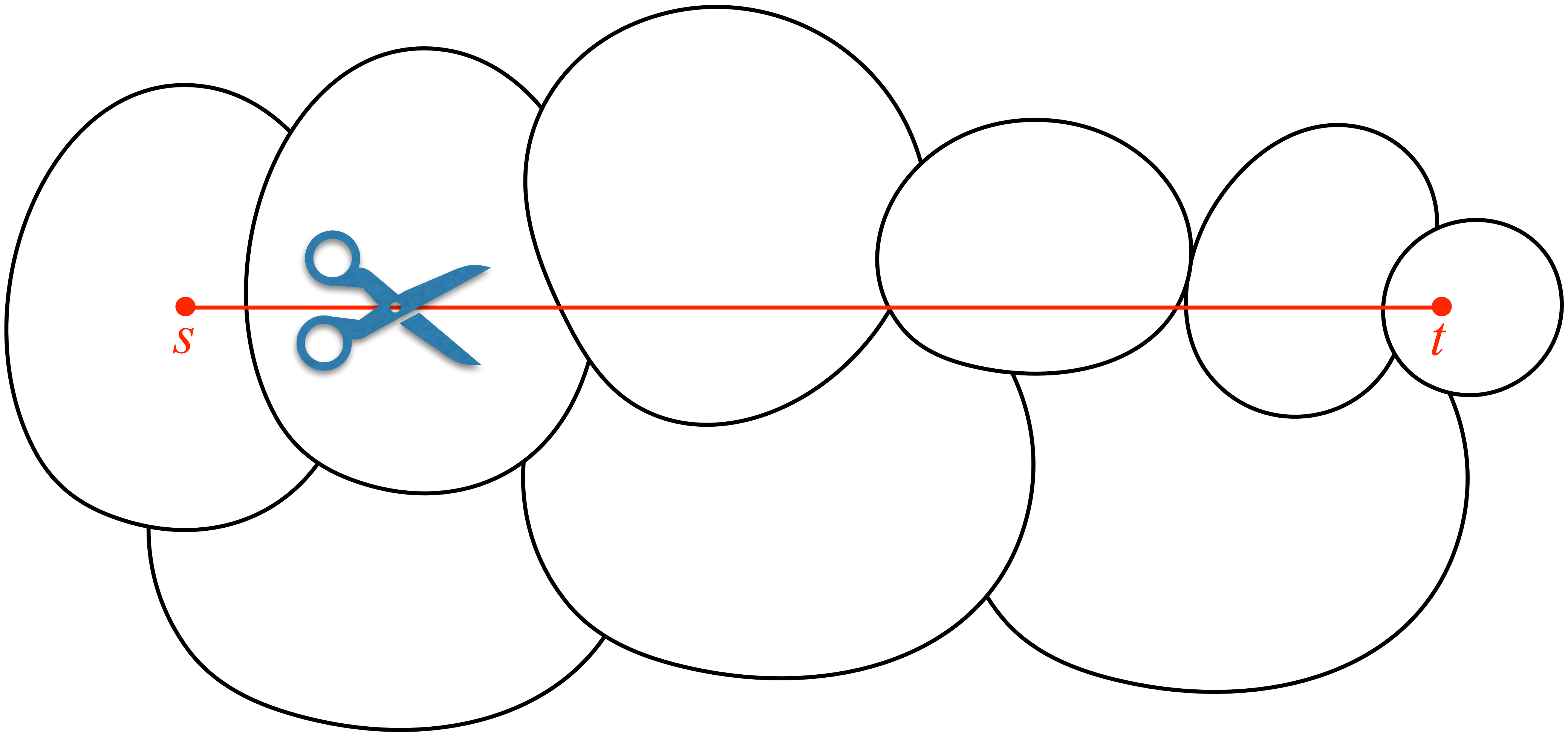

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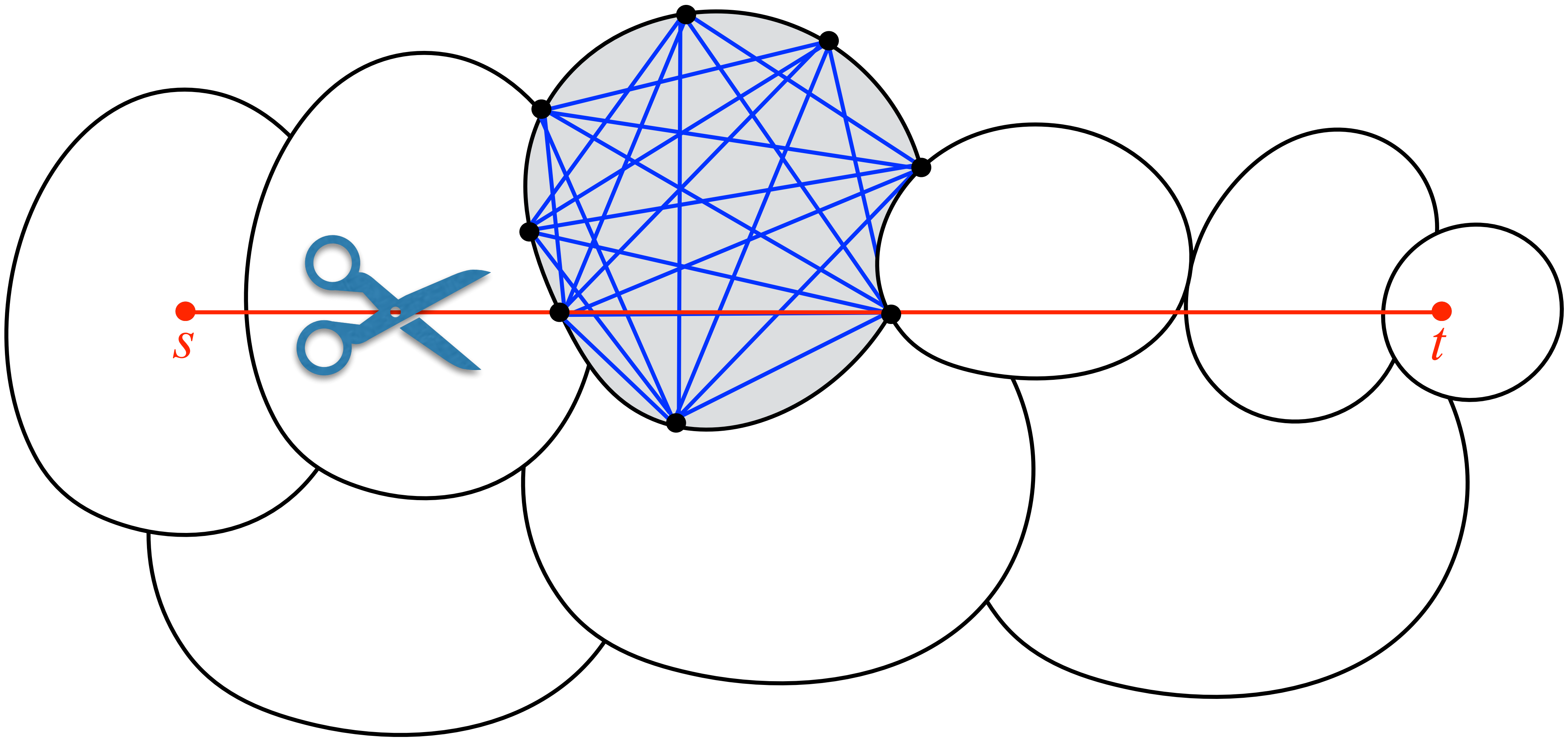

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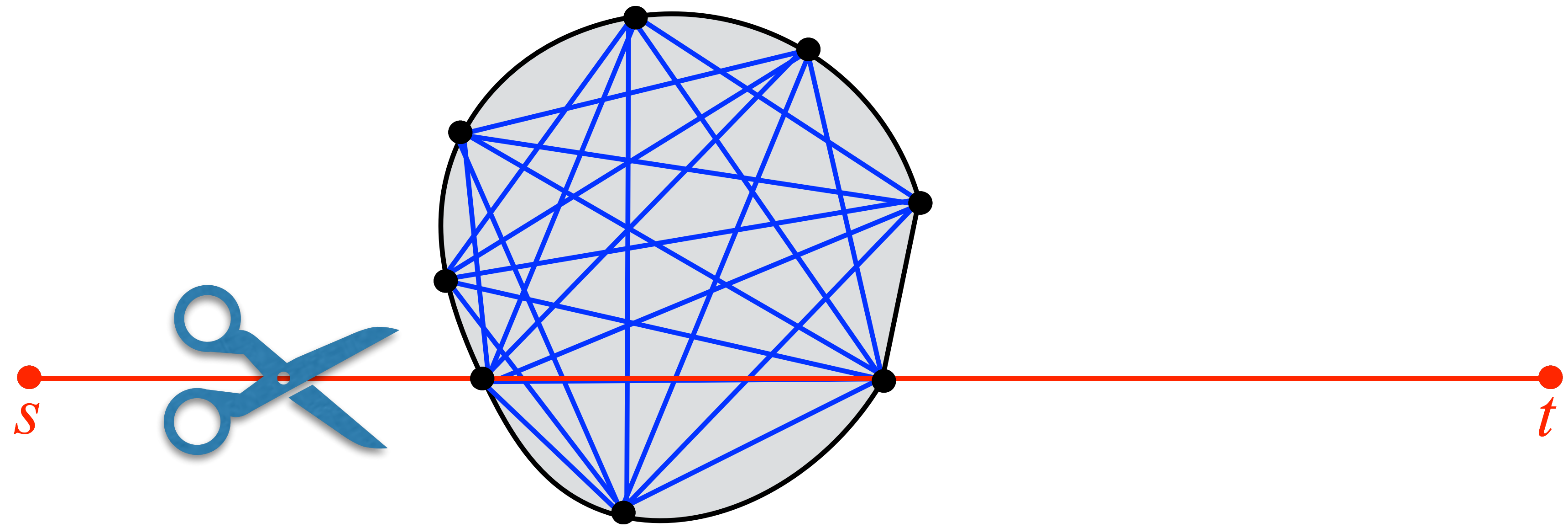

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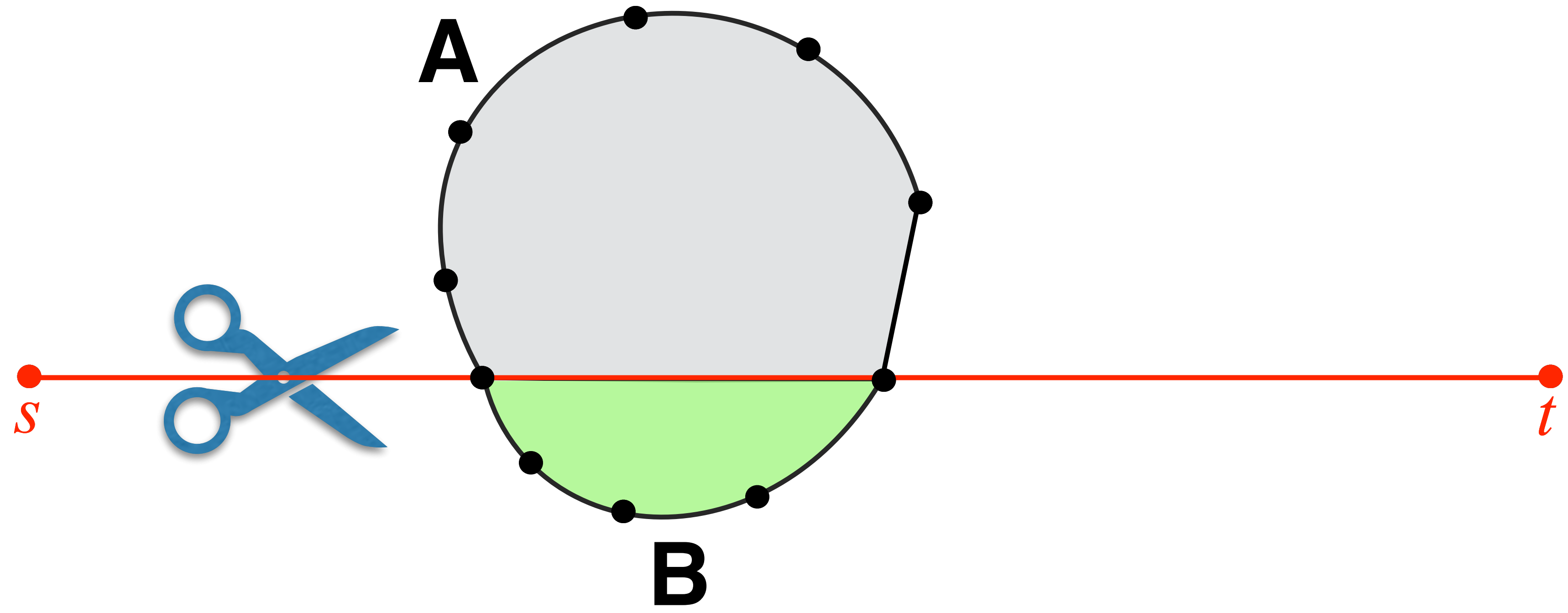

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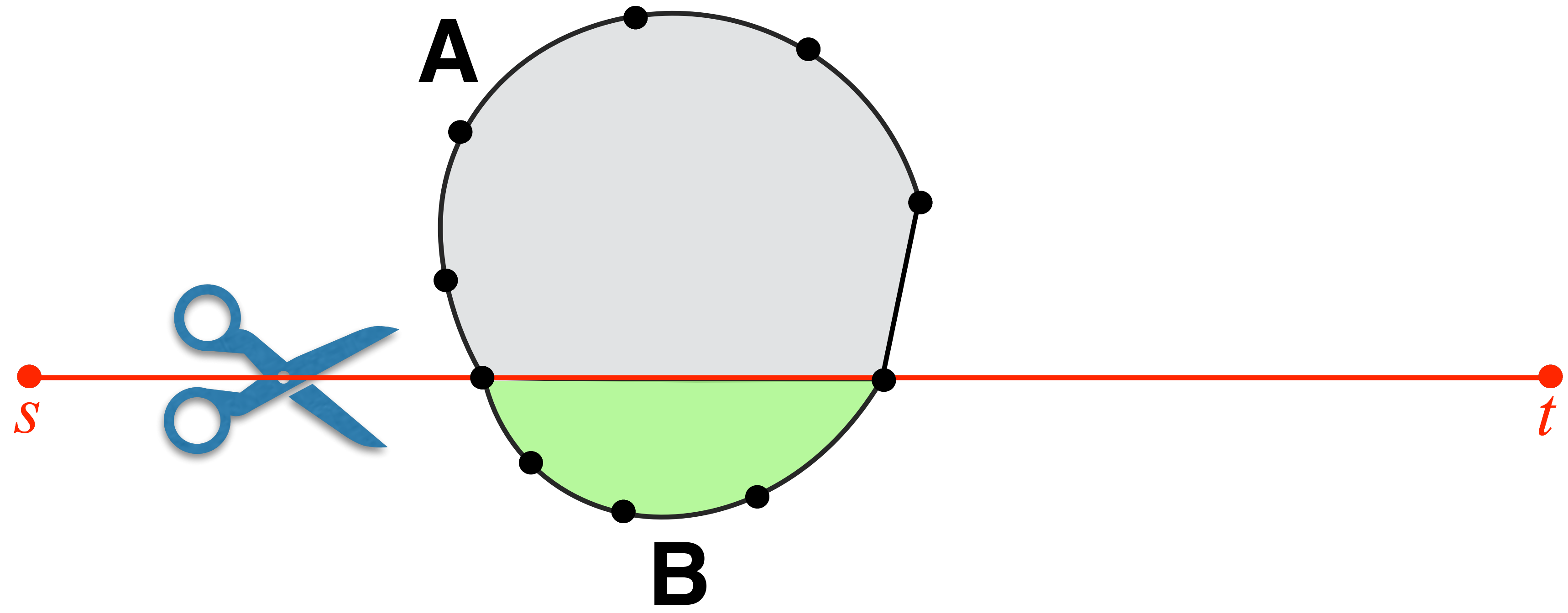
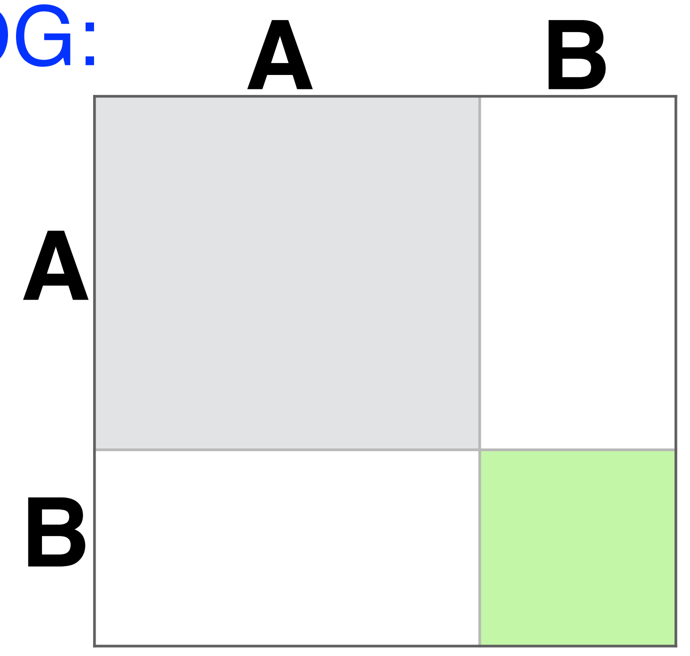


DDG:



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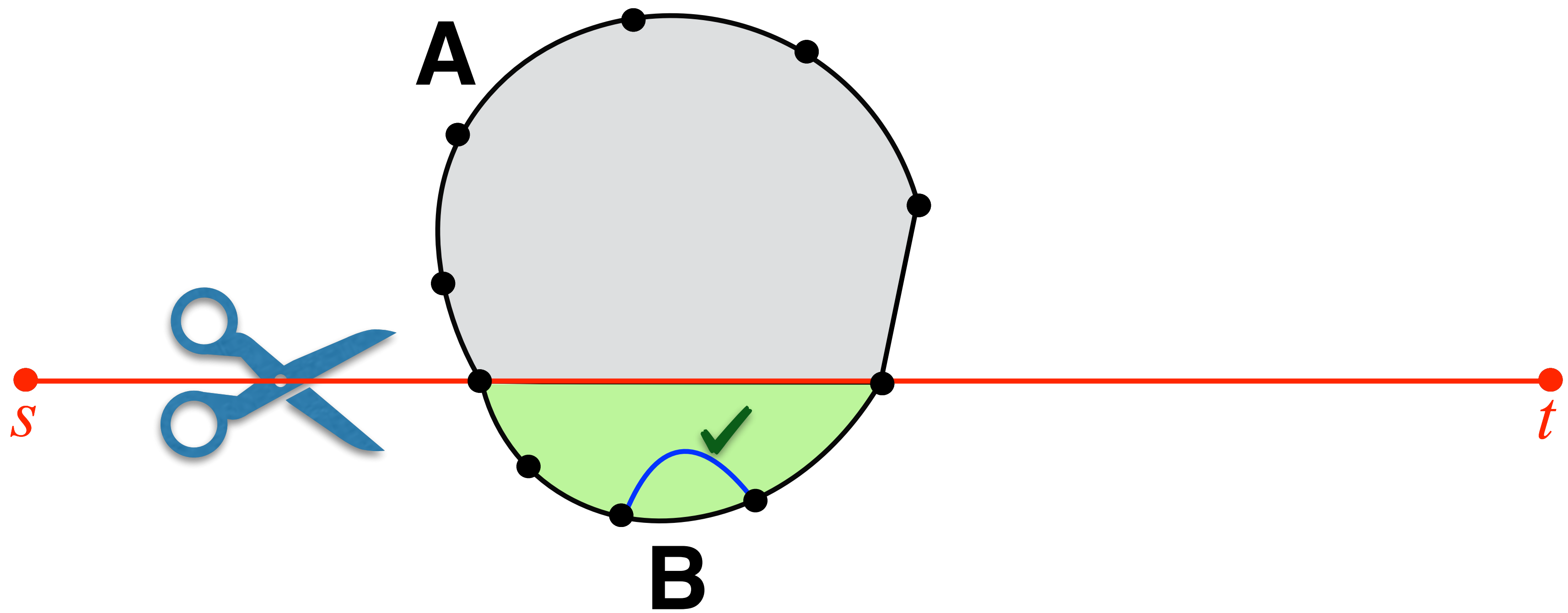

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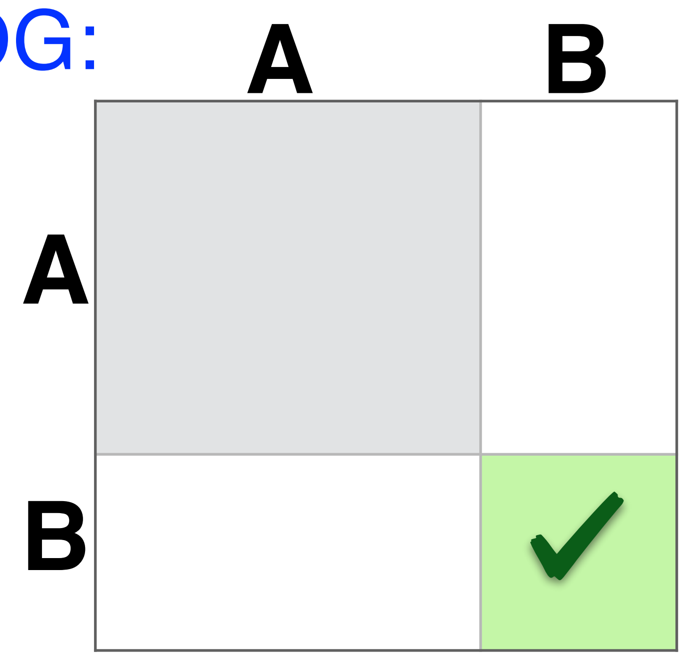

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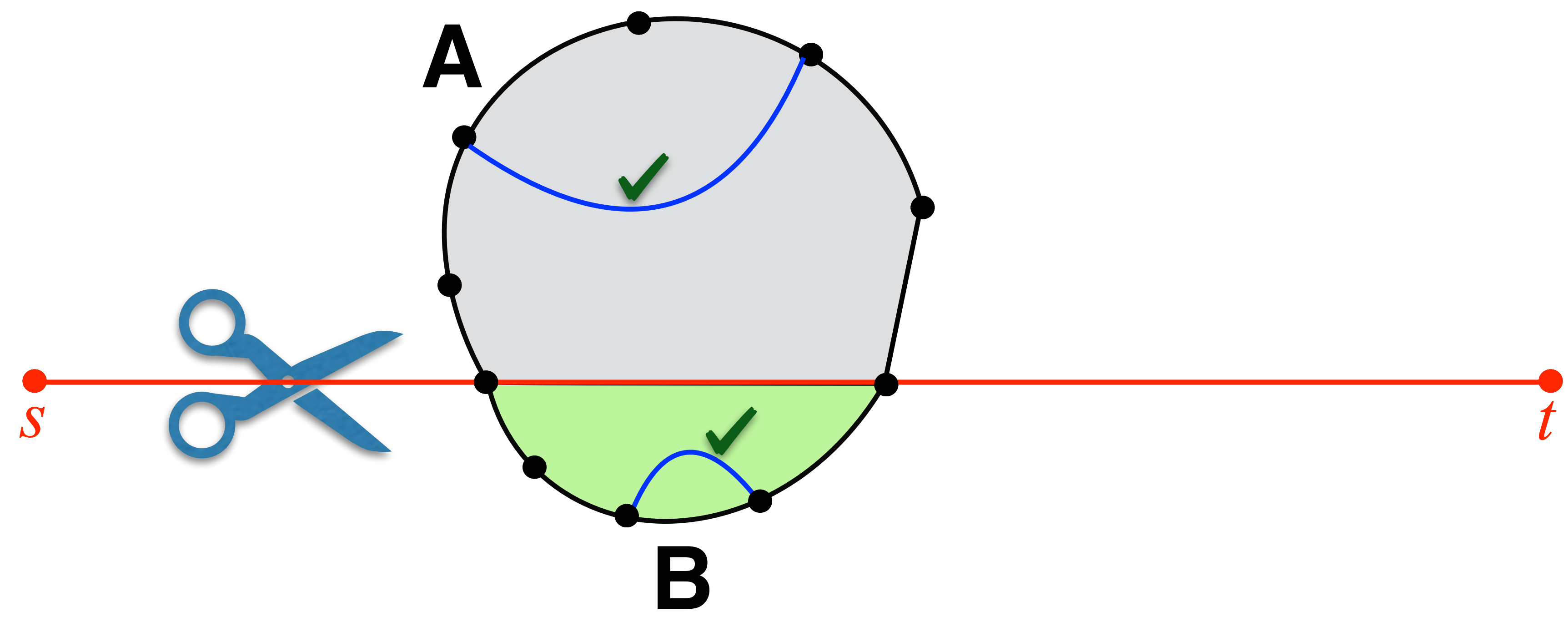

DDG:

	A	B
A	✓	
B		✓

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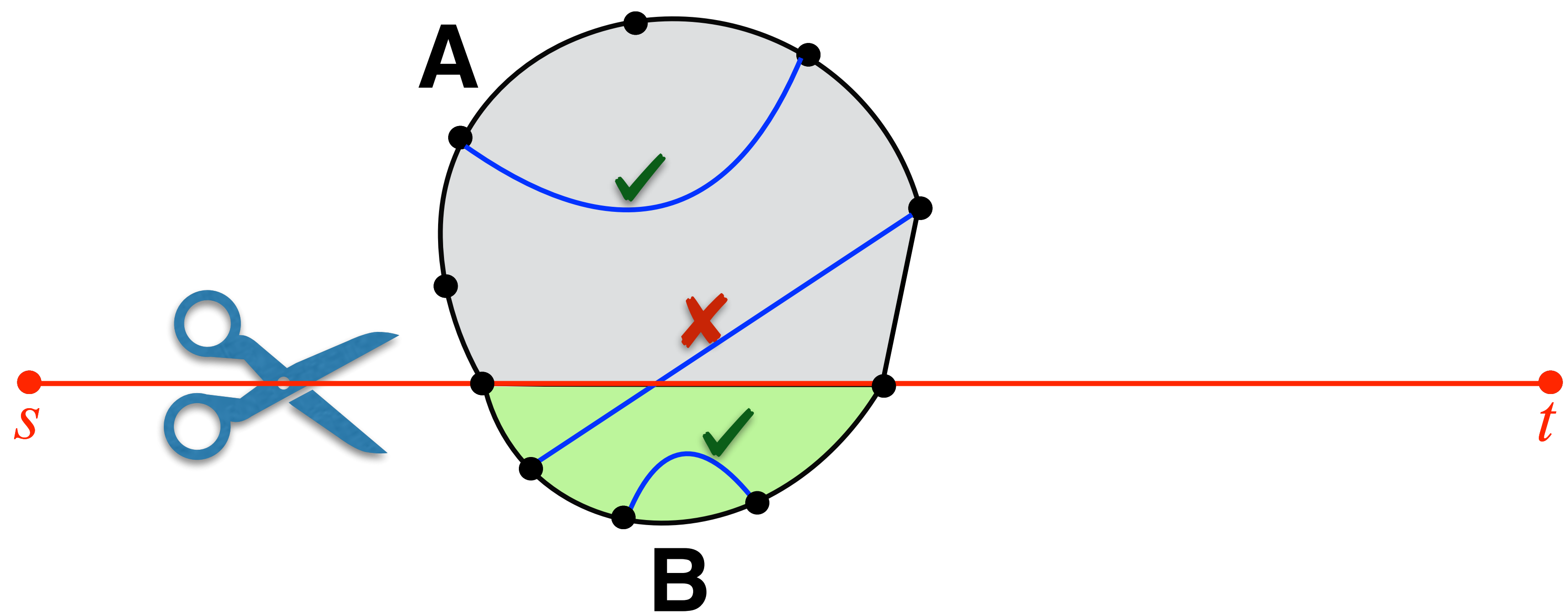

DDG:

	A	B
A	✓	✗
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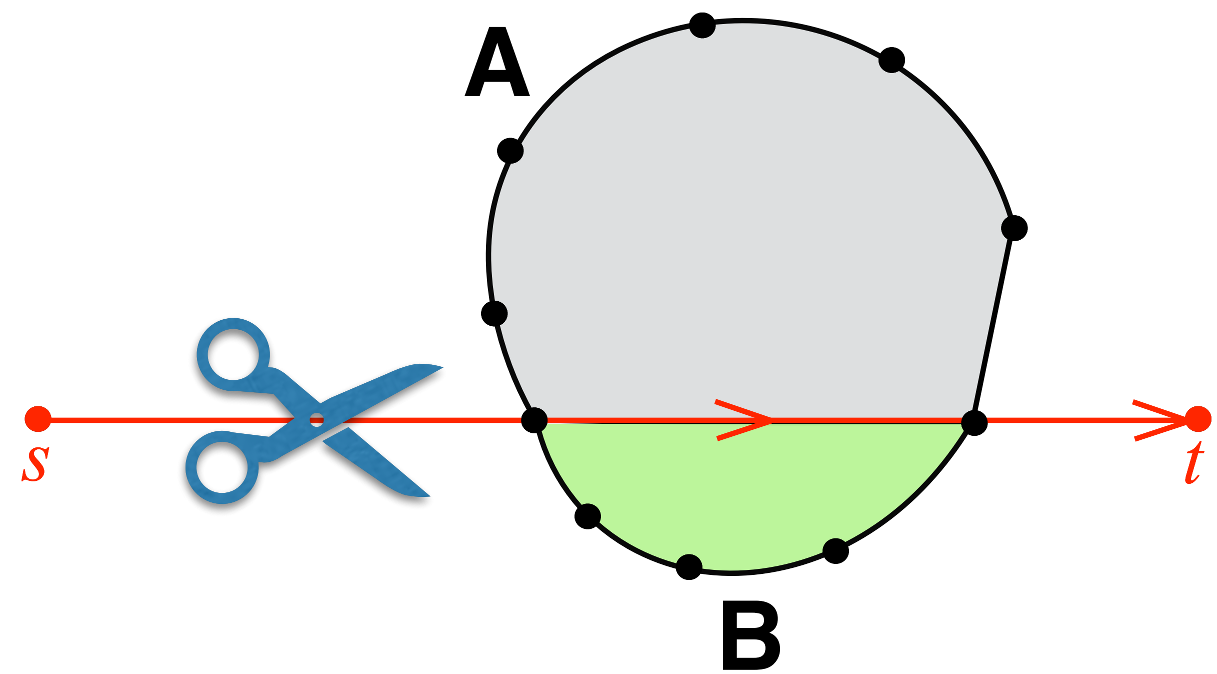
Directed Min cycle in O(nloglogn) time



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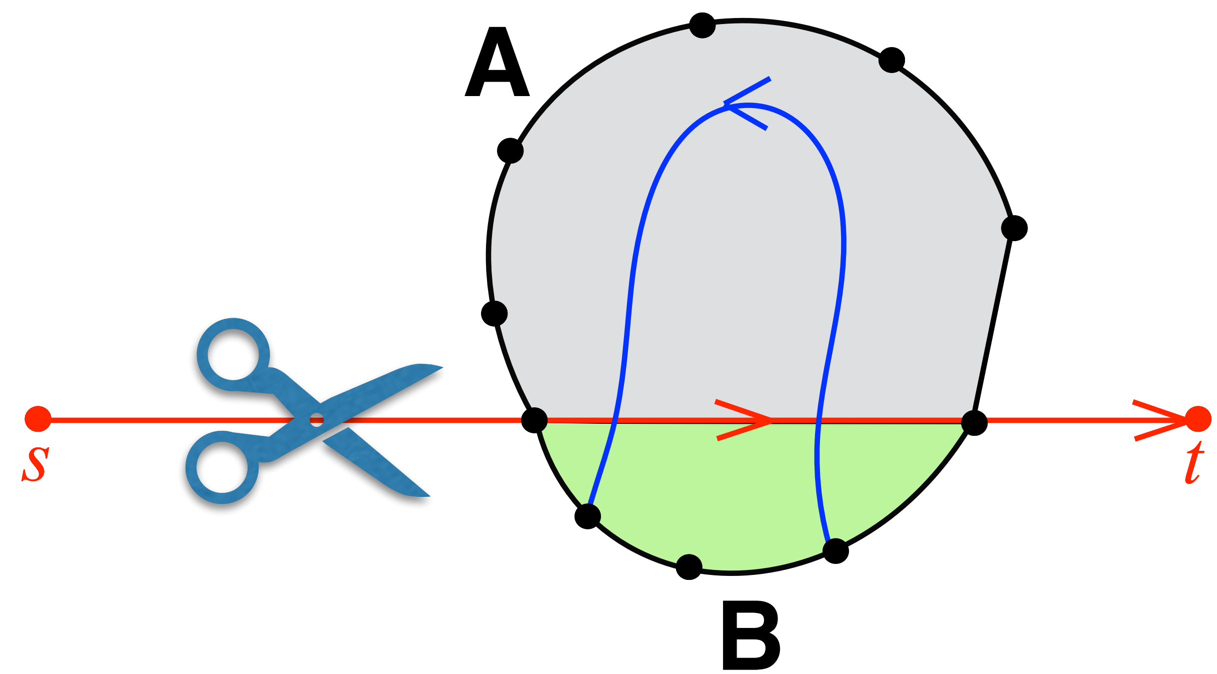

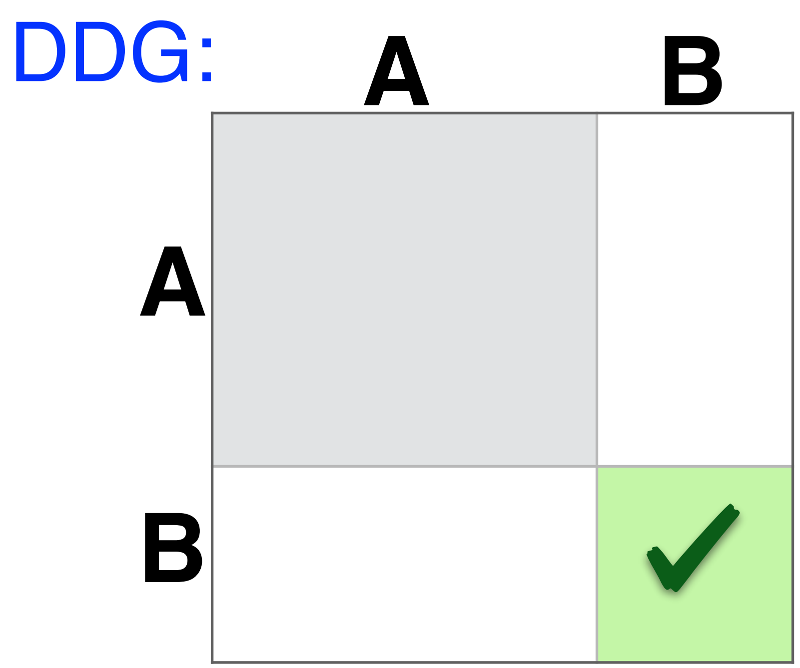

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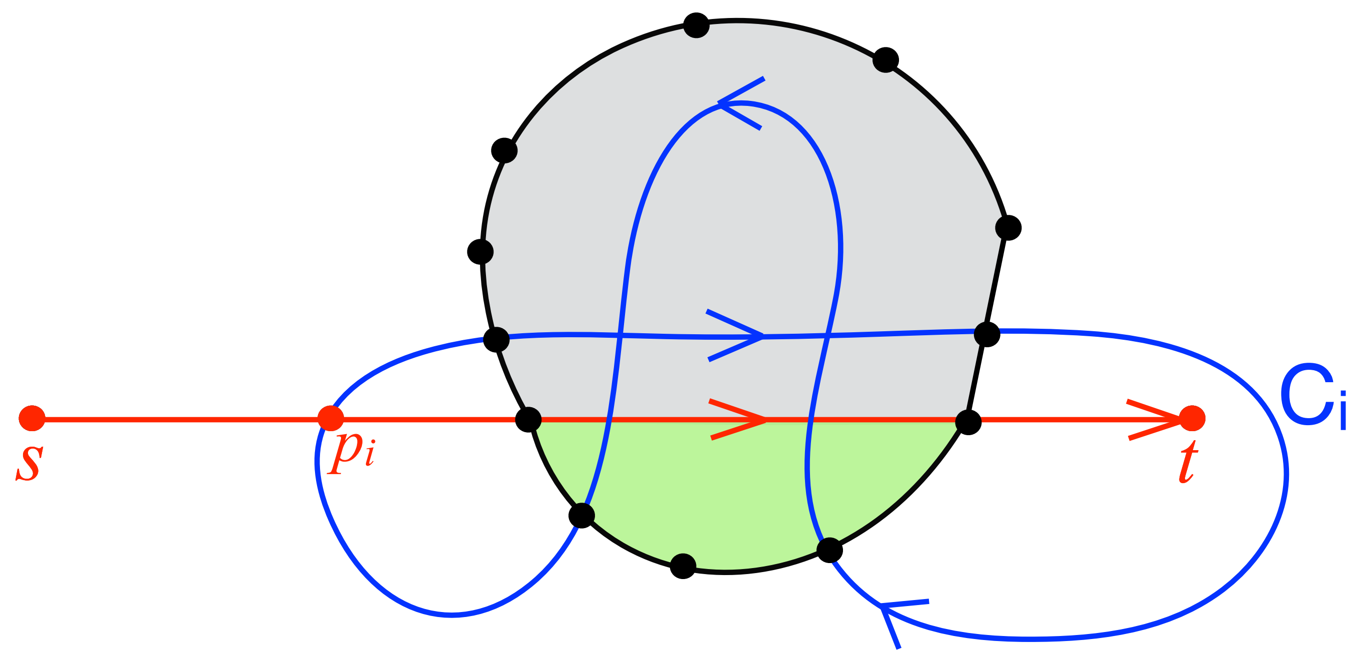

The underlying C_i might be non-simple!

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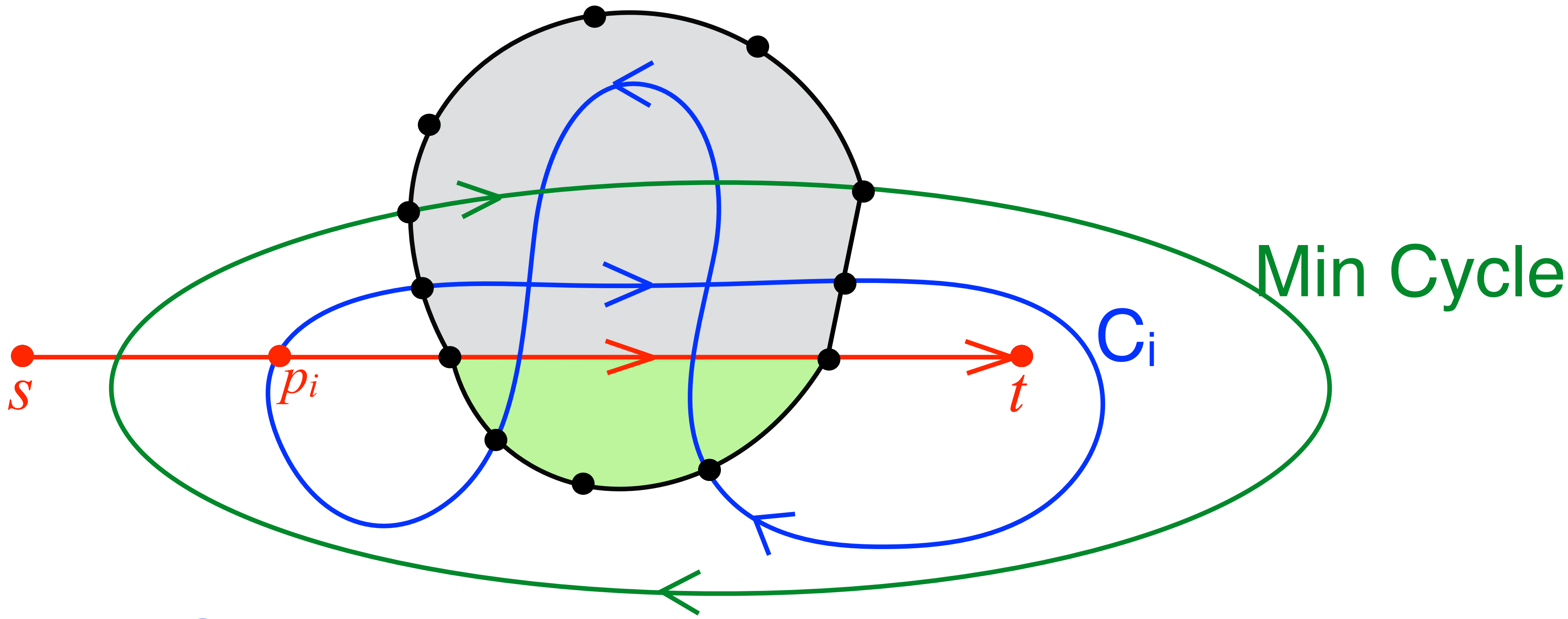

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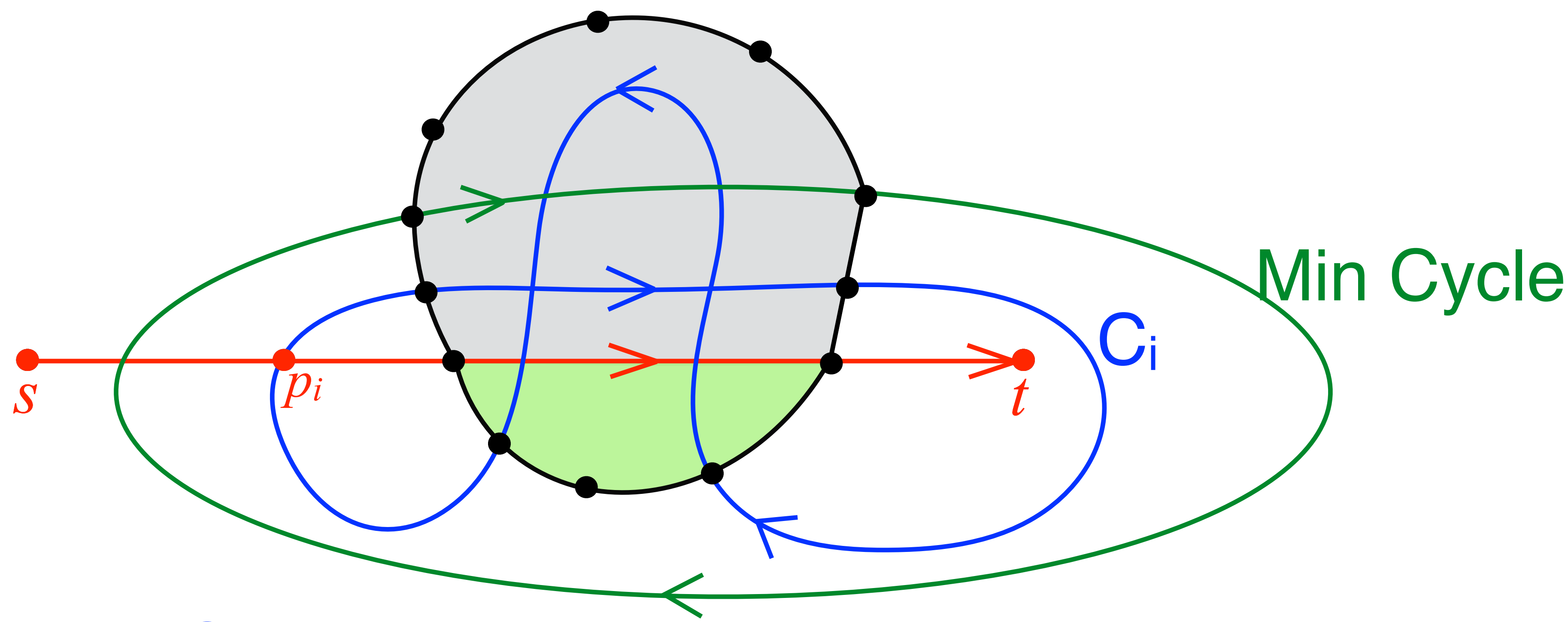

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Theorem:
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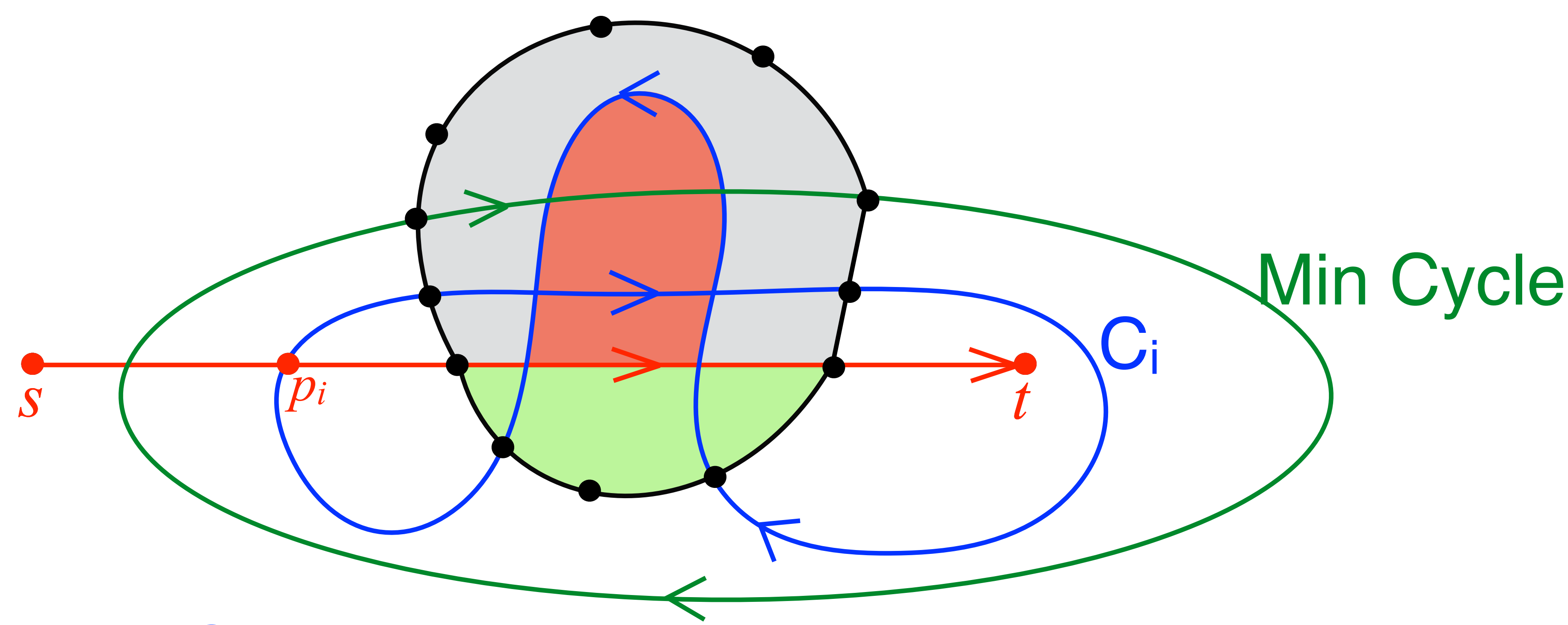

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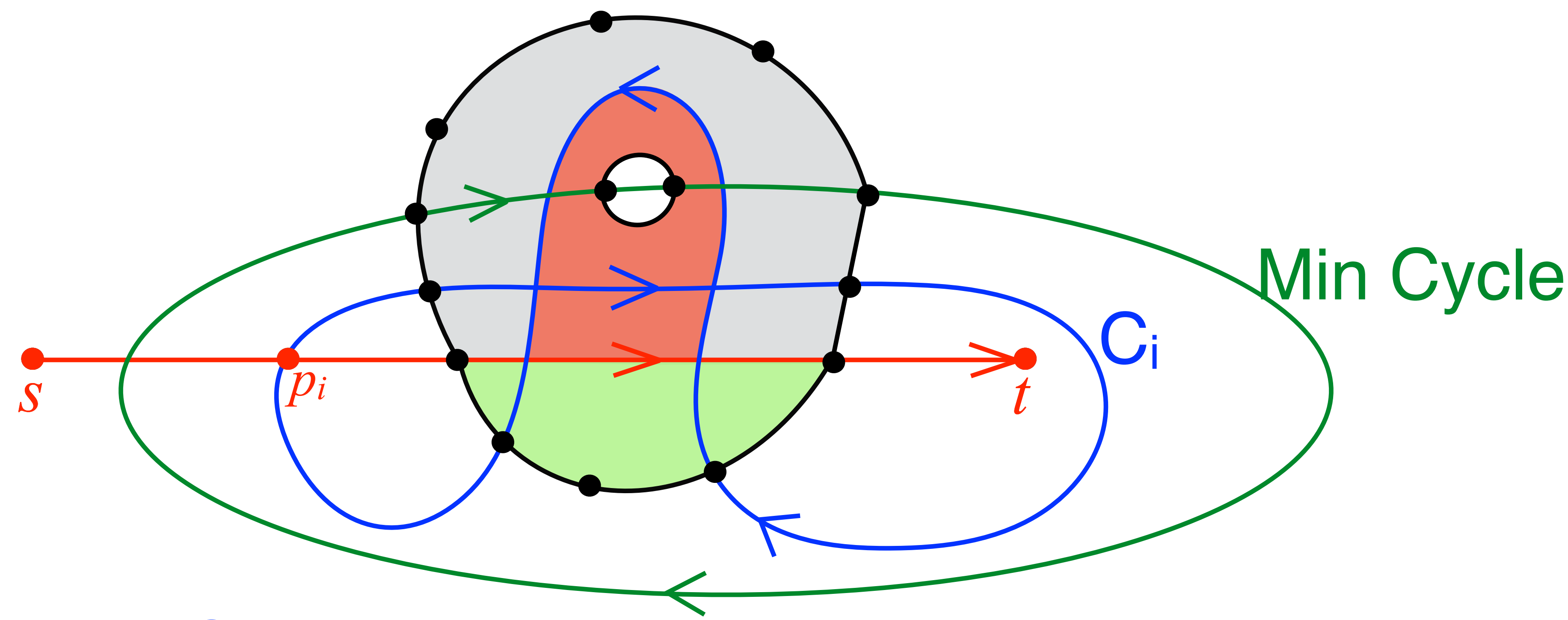

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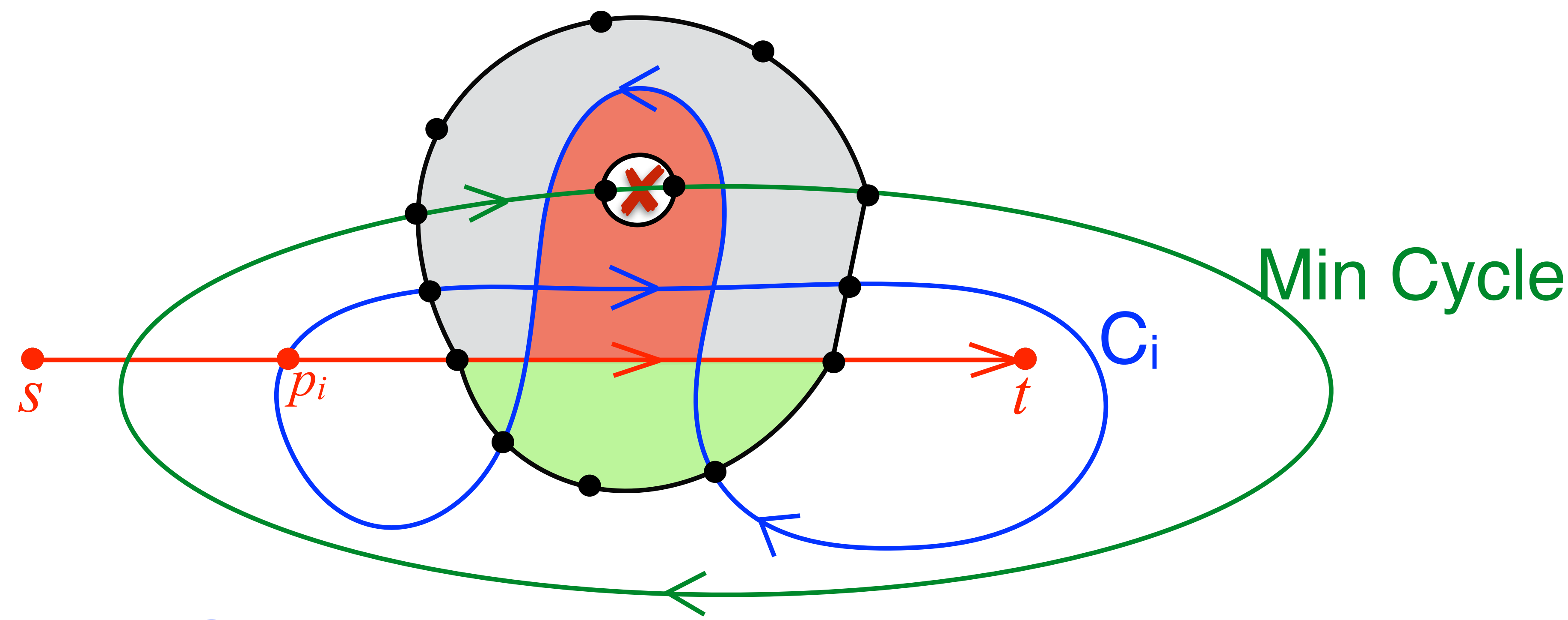

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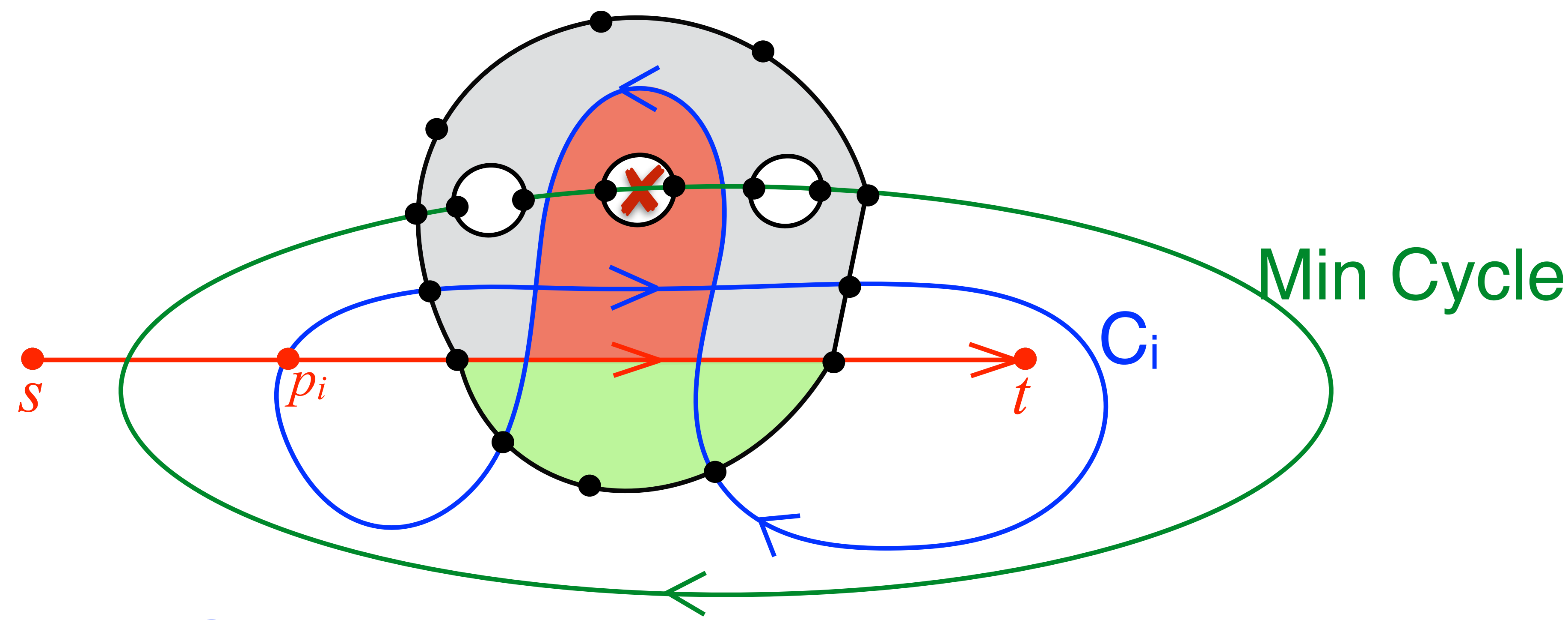

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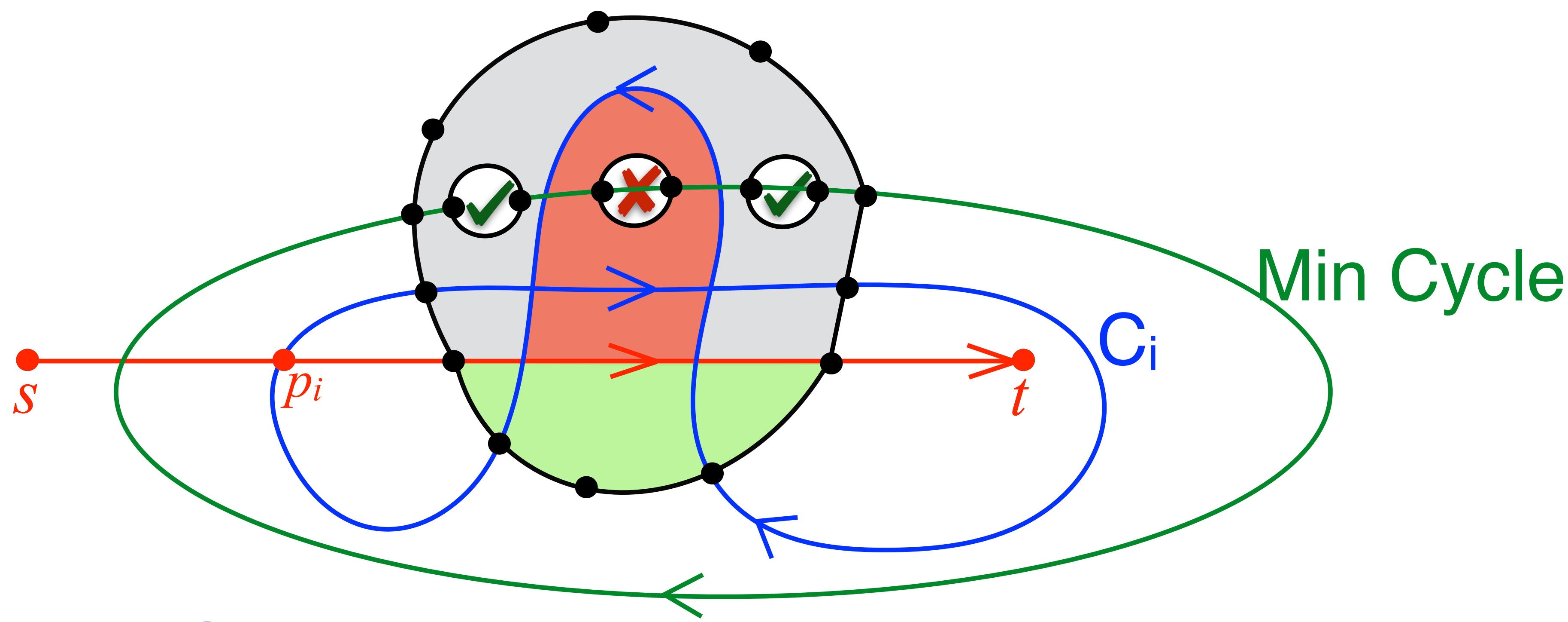

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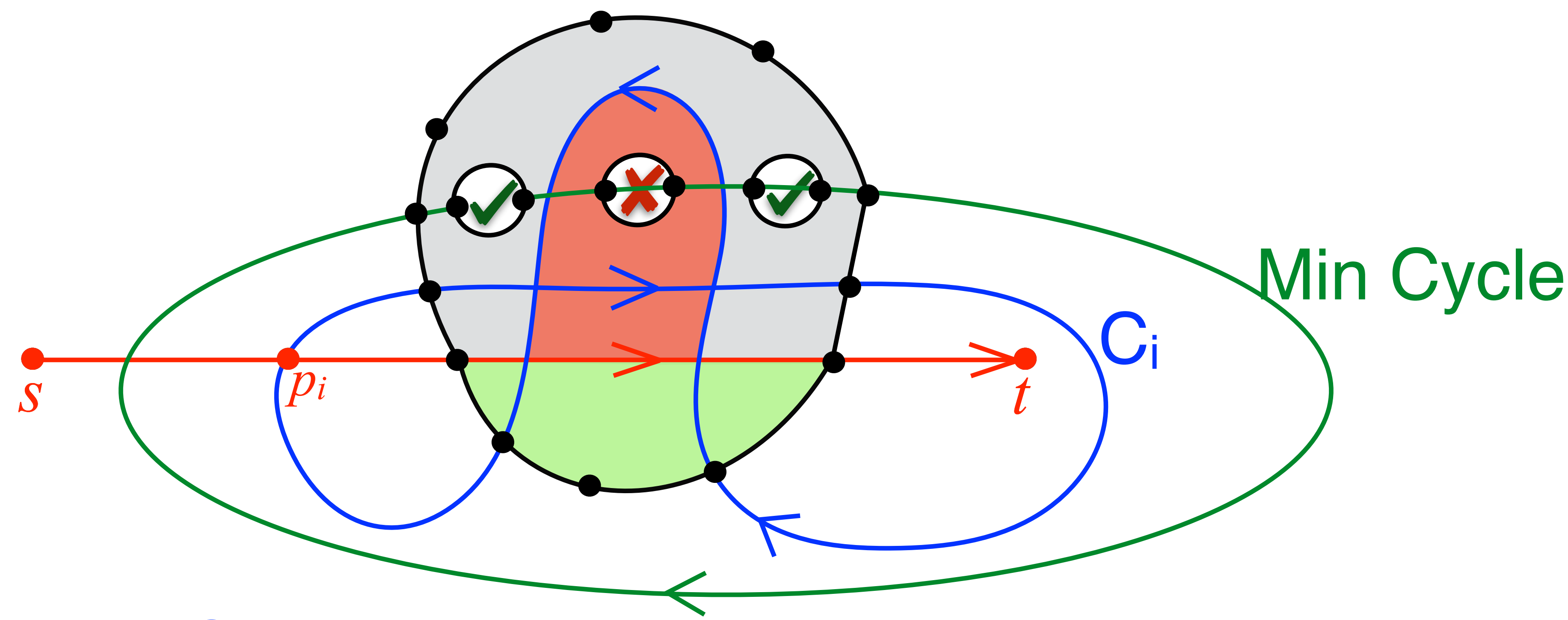

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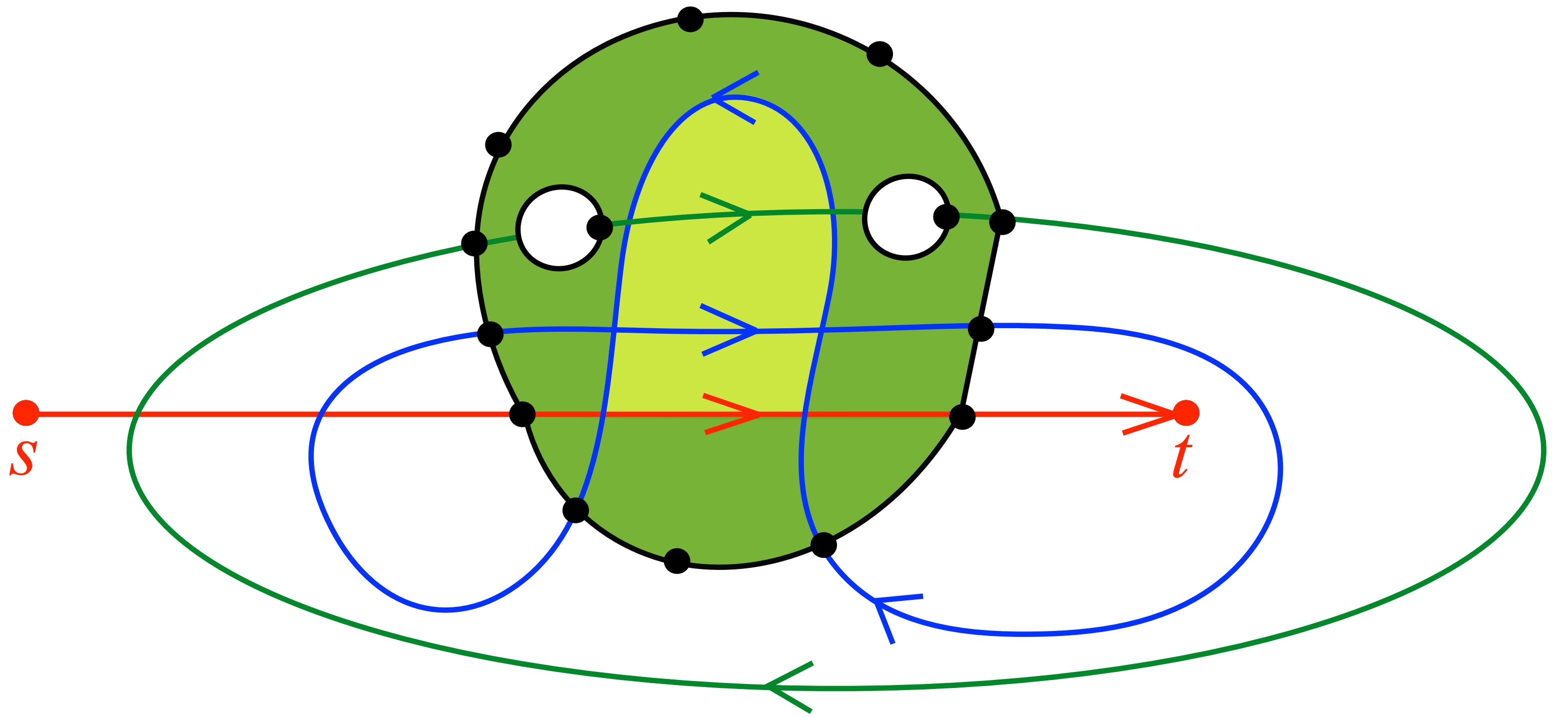
Challenges:

1. prove this theorem.
2. prevent holes in crossing area (by building one DDG for every hole configuration, using a bounded genus graph)

Conclusions



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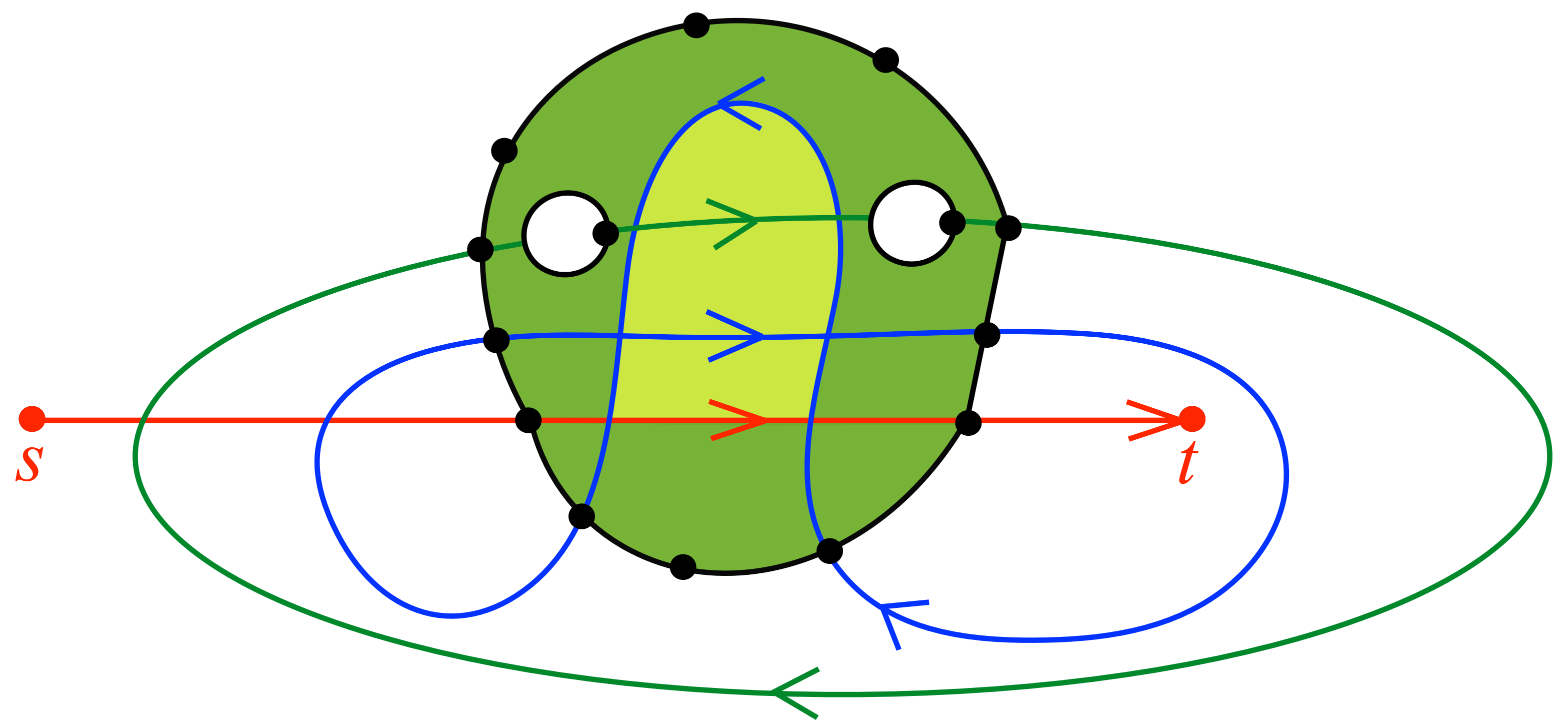

Undirected min st-cut: O(n loglogn)
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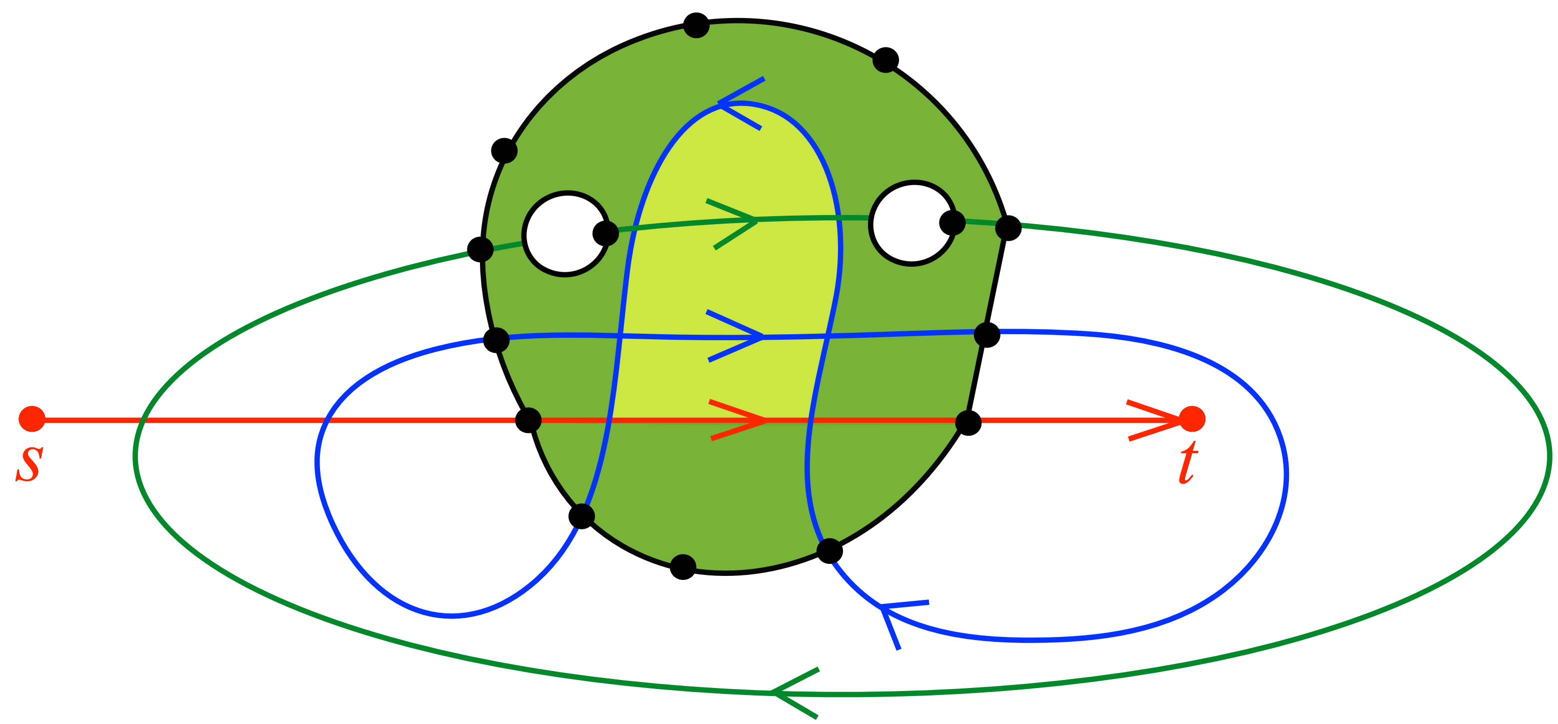

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Thank Yu