Tree Edit Distance Cannot be Computed in Strongly Subcubic Time (unless APSP can)

Karl Bringmann,
Paweł Gawrychowski,
Shay Mozes,
Oren Weimann

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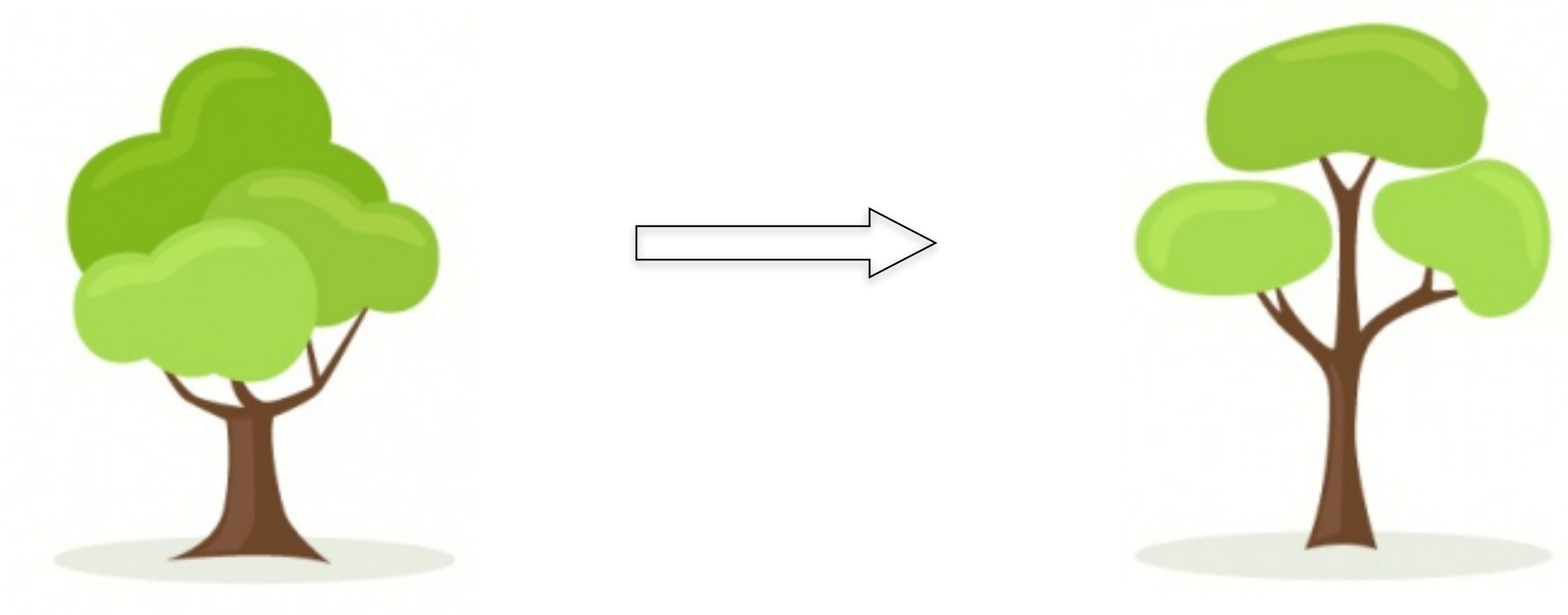
Minimum edits to transform one tree into the other



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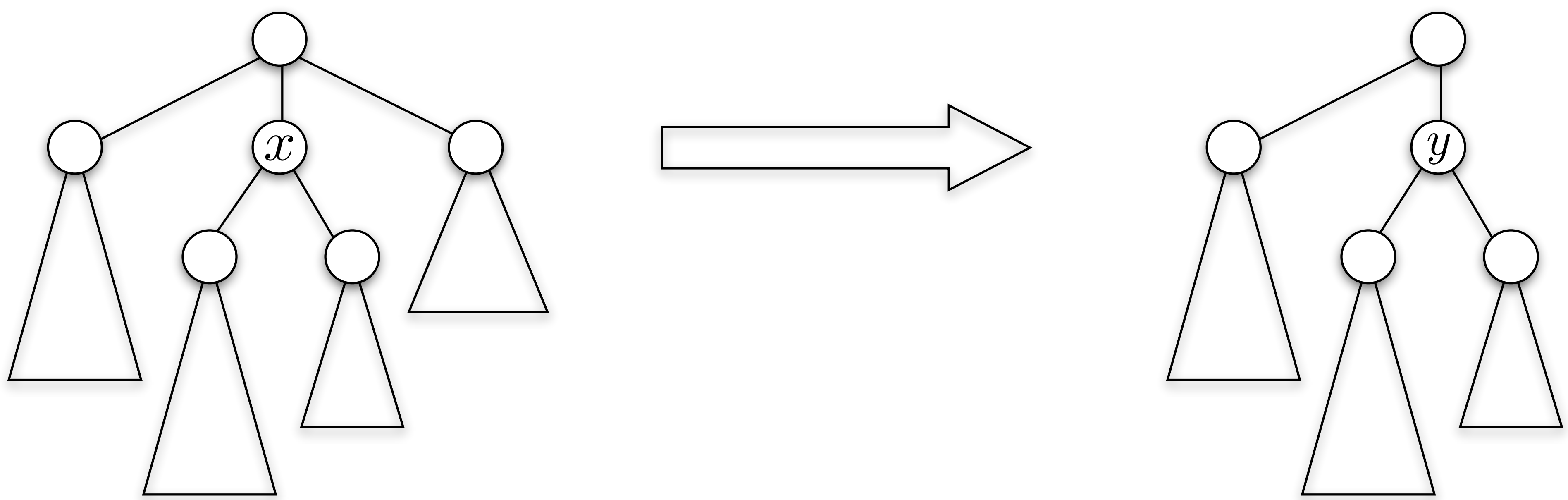
Minimum edits to transform one tree into the other

rooted, ordered trees with node labels



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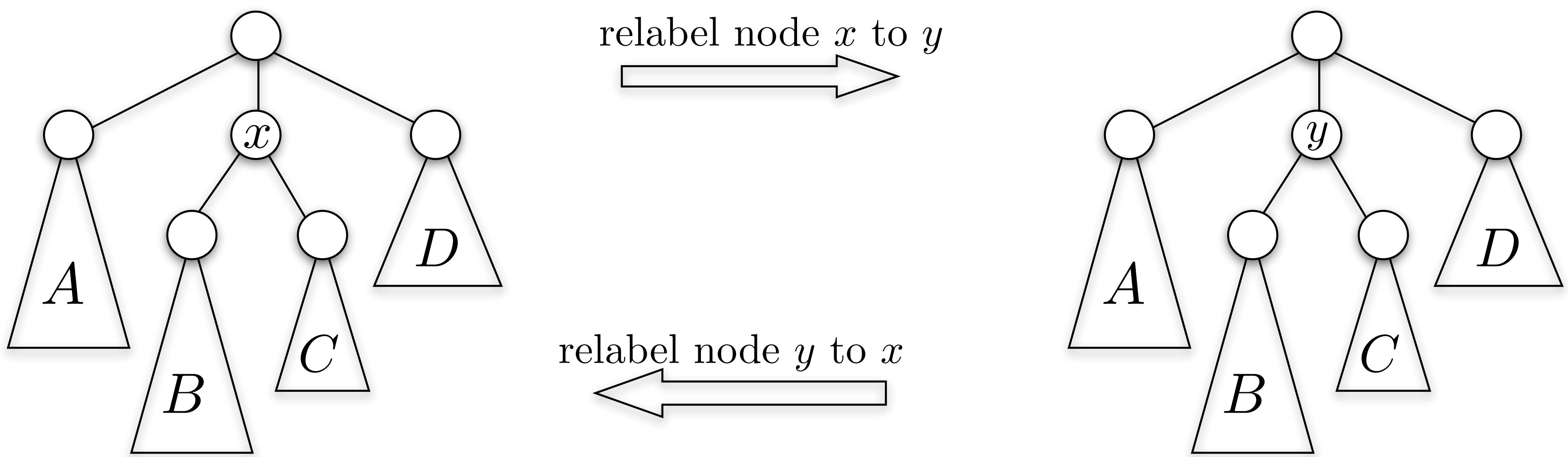

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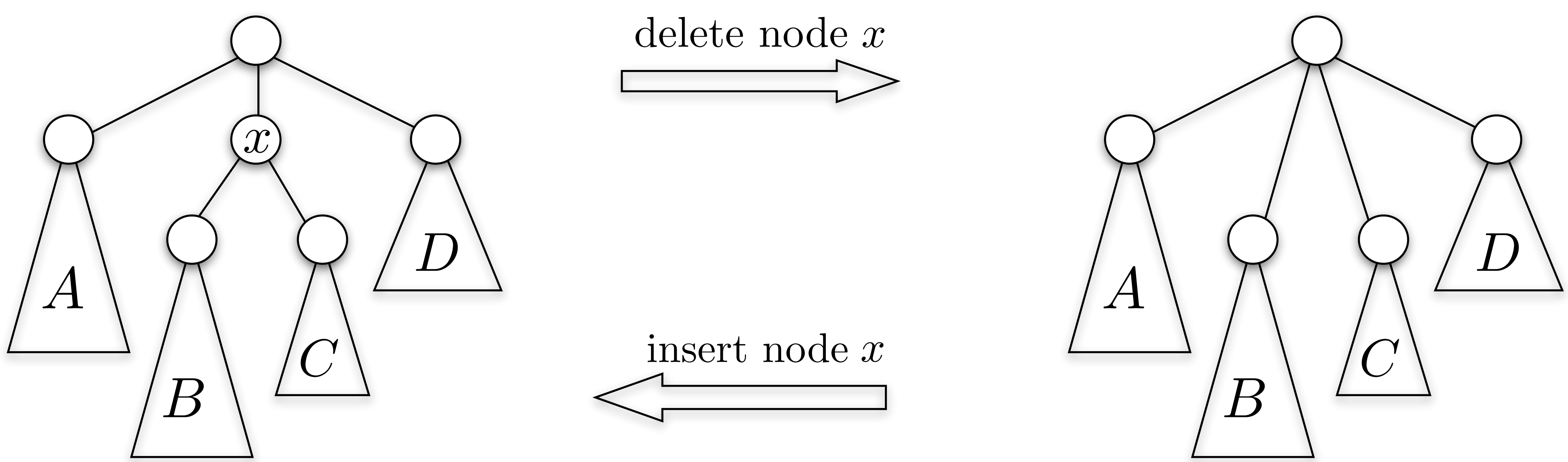

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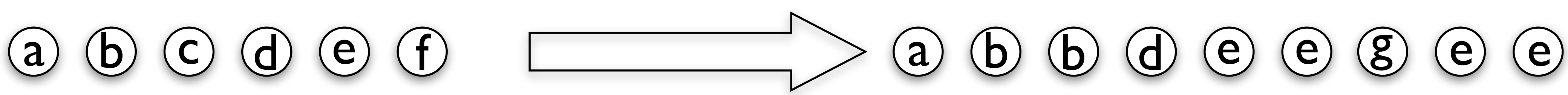

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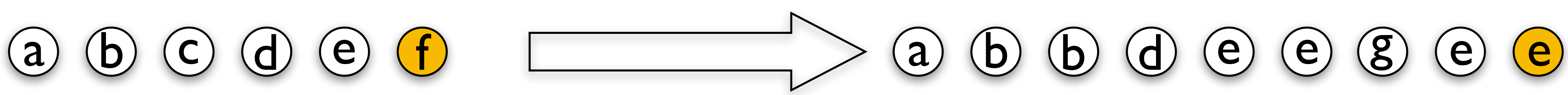

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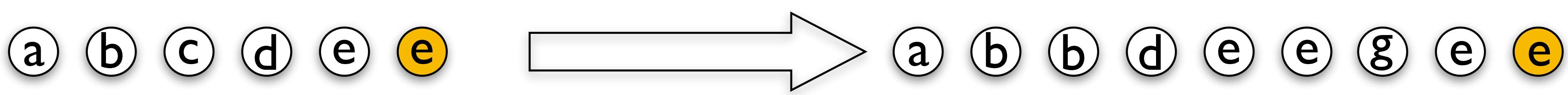

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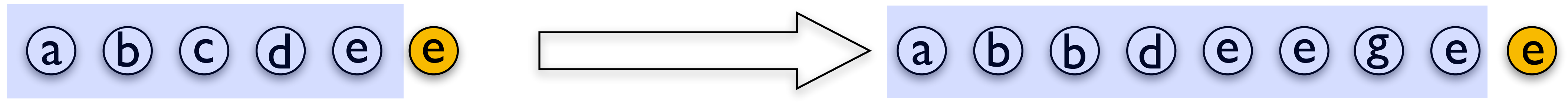

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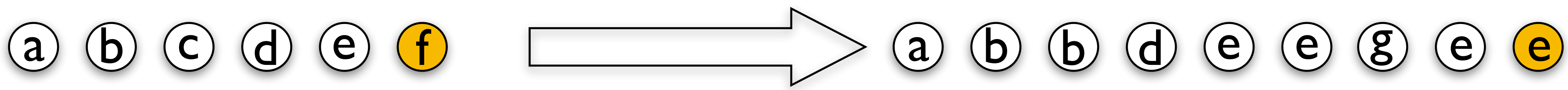

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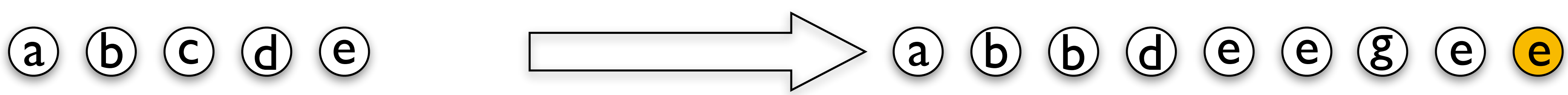

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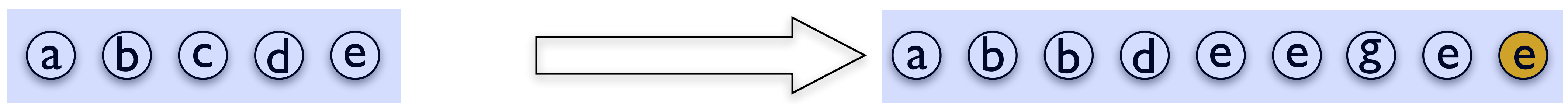

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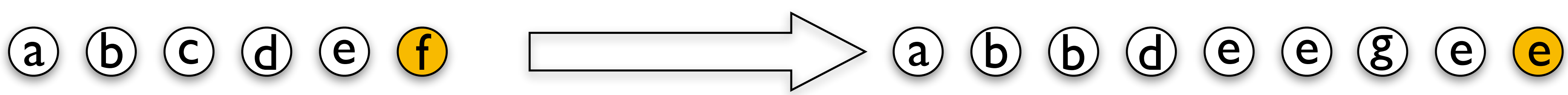

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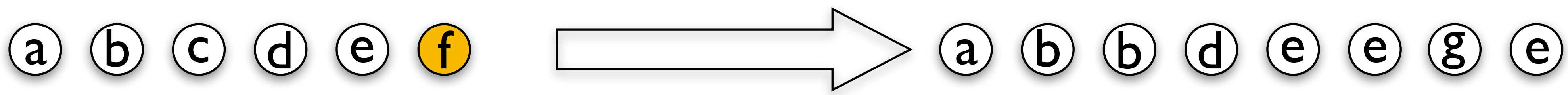

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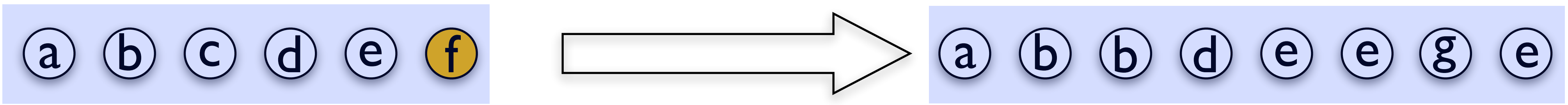

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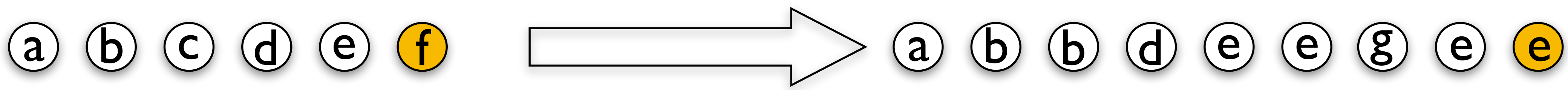
$O(n^2)$ time



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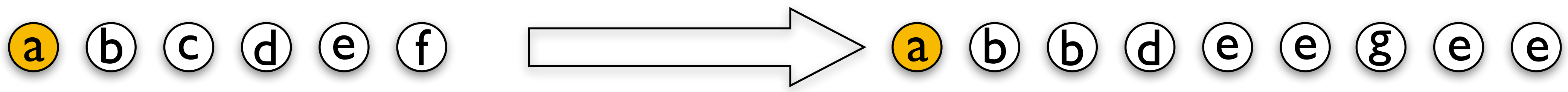

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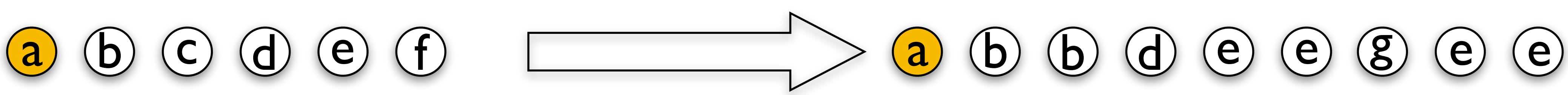
$O(n^2)$ time

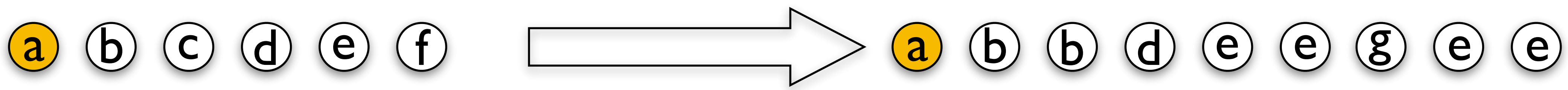
$O(n^4)$ time



Tree Edit Distance Cannot be Computed in Strongly Subcubic Time (unless APSP can)

String Edit Distance Cannot be Computed in Strongly Subquadratic Time (unless SETH is false)

[Backurs,Indyk, STOC'15]



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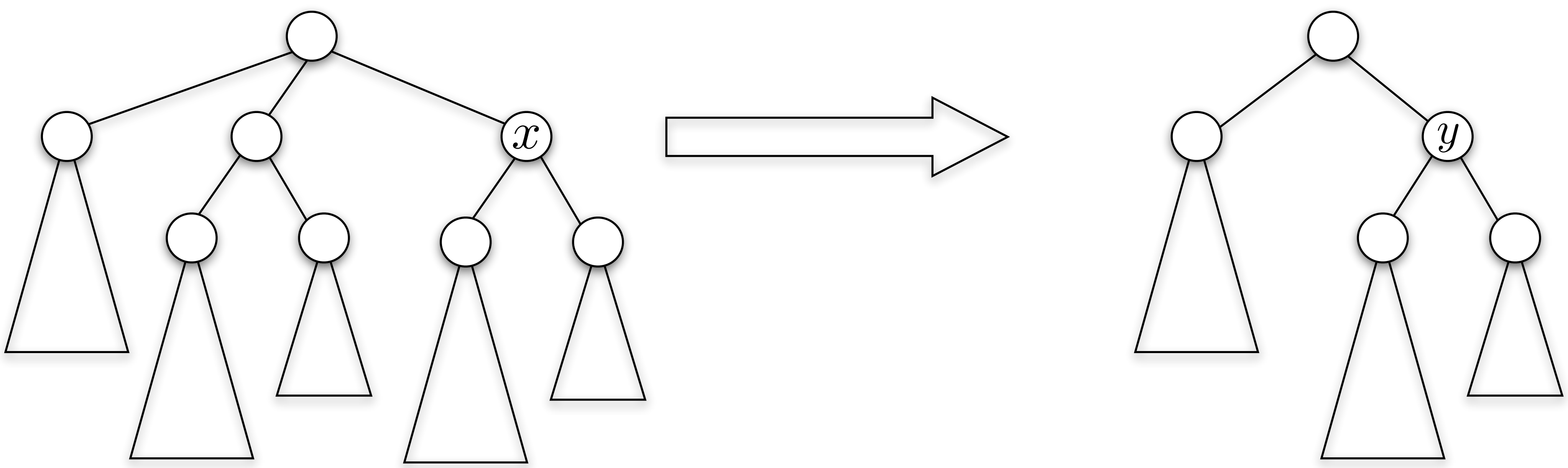

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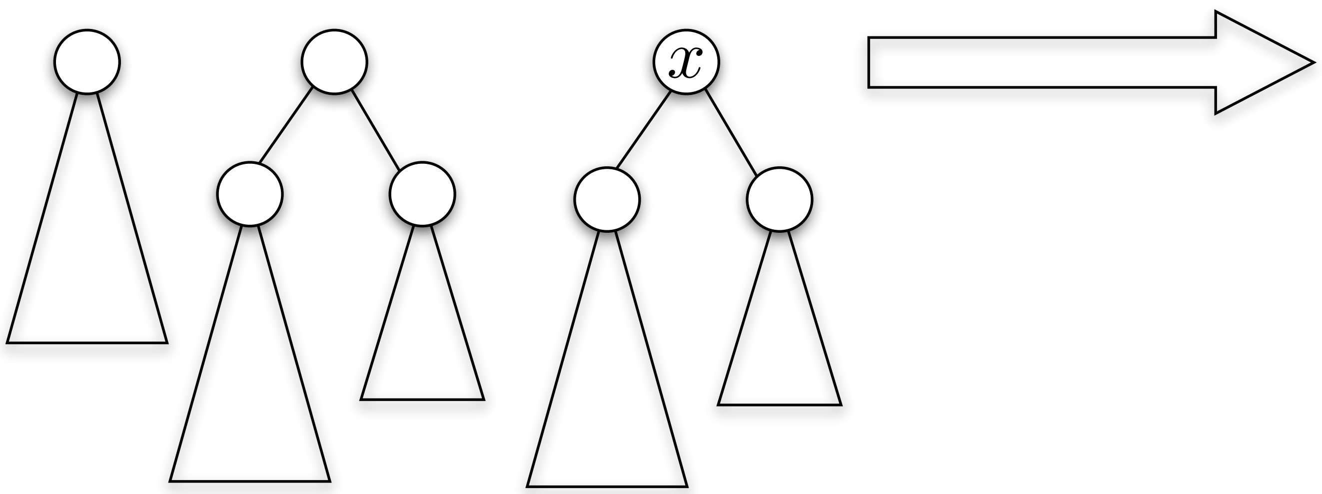

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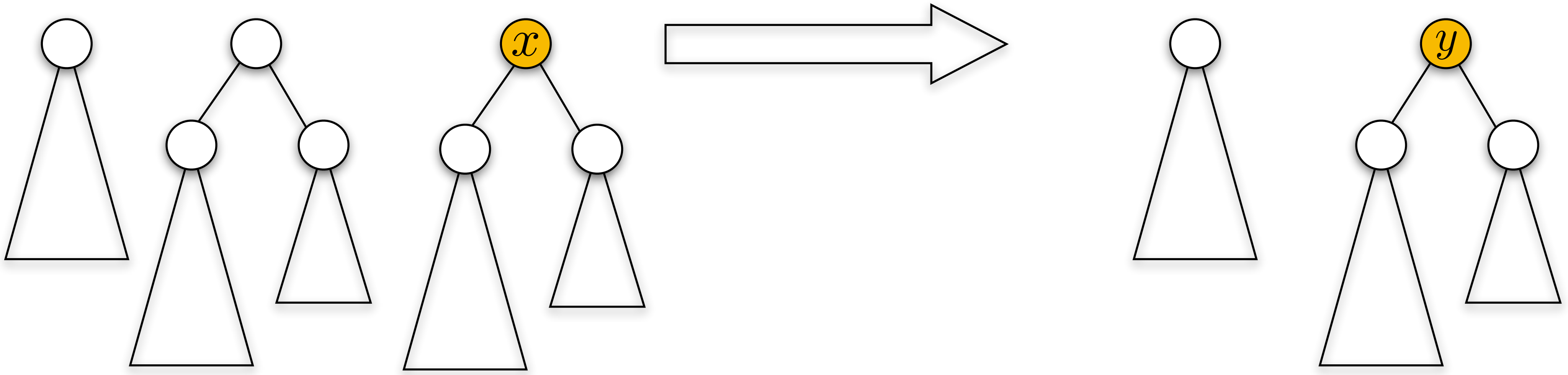

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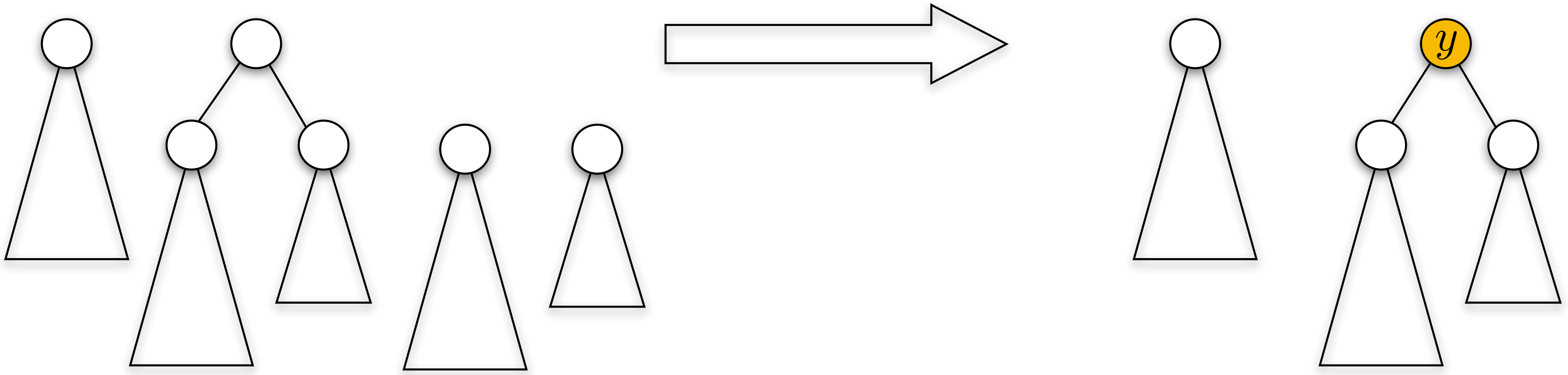

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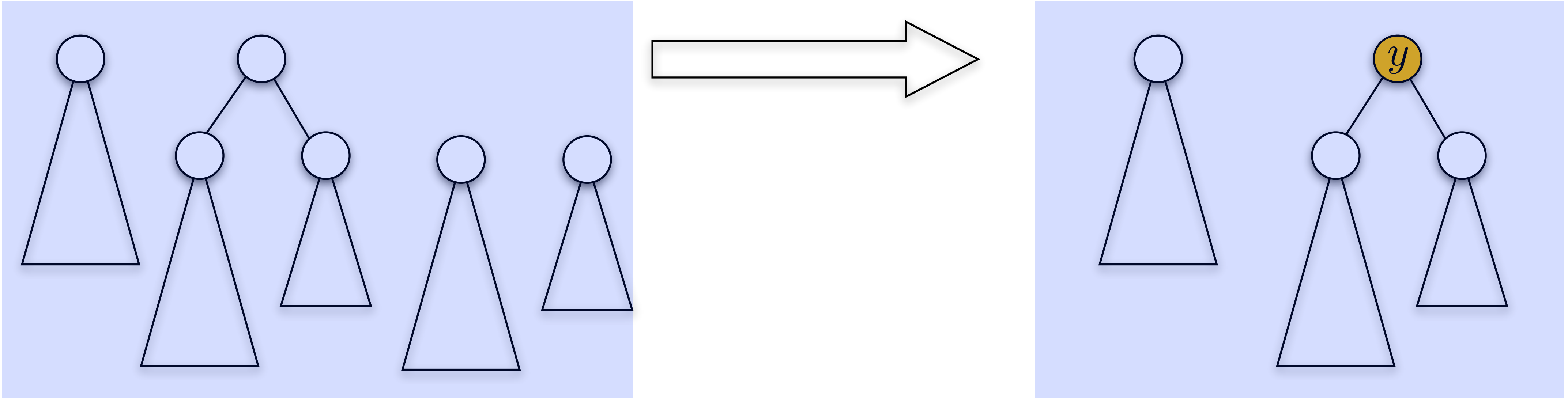

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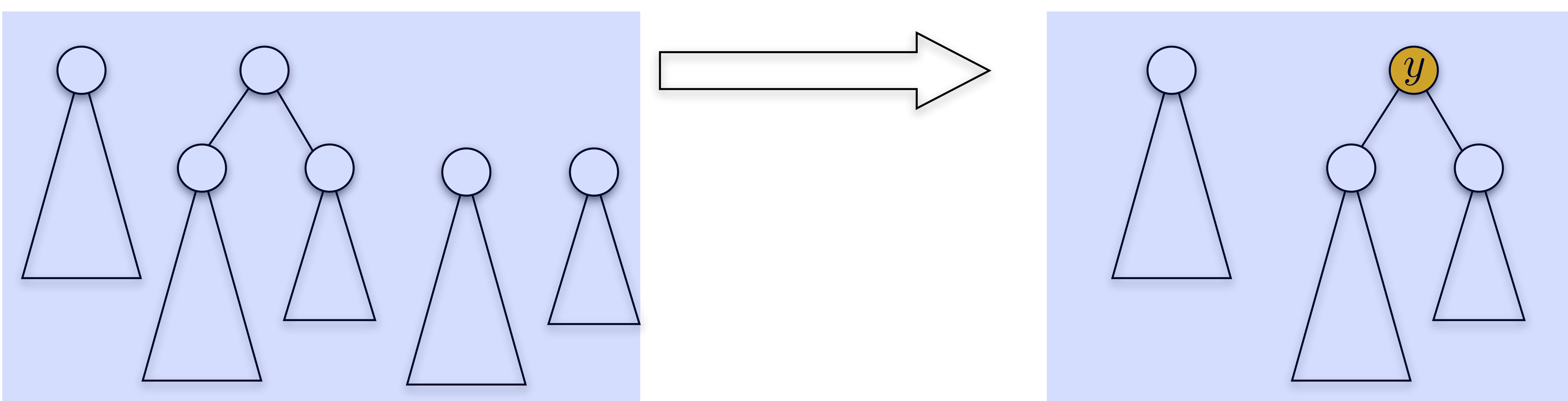

prefix in postorder traversal

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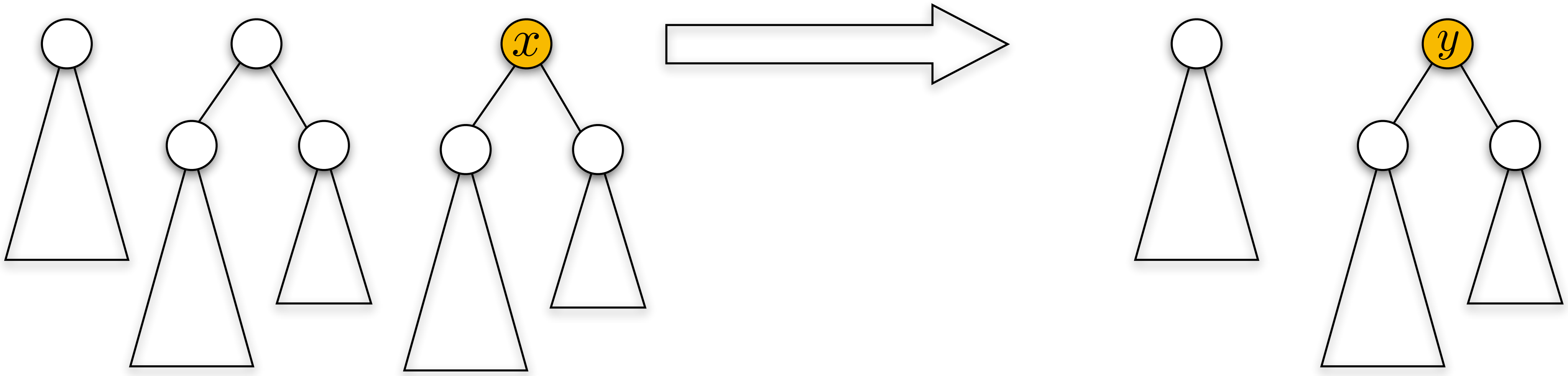

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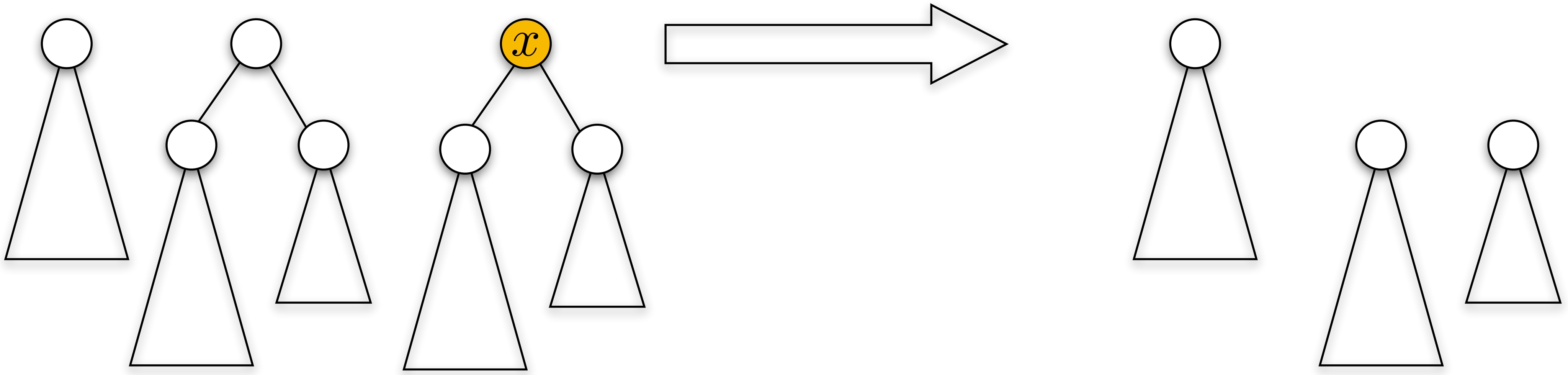

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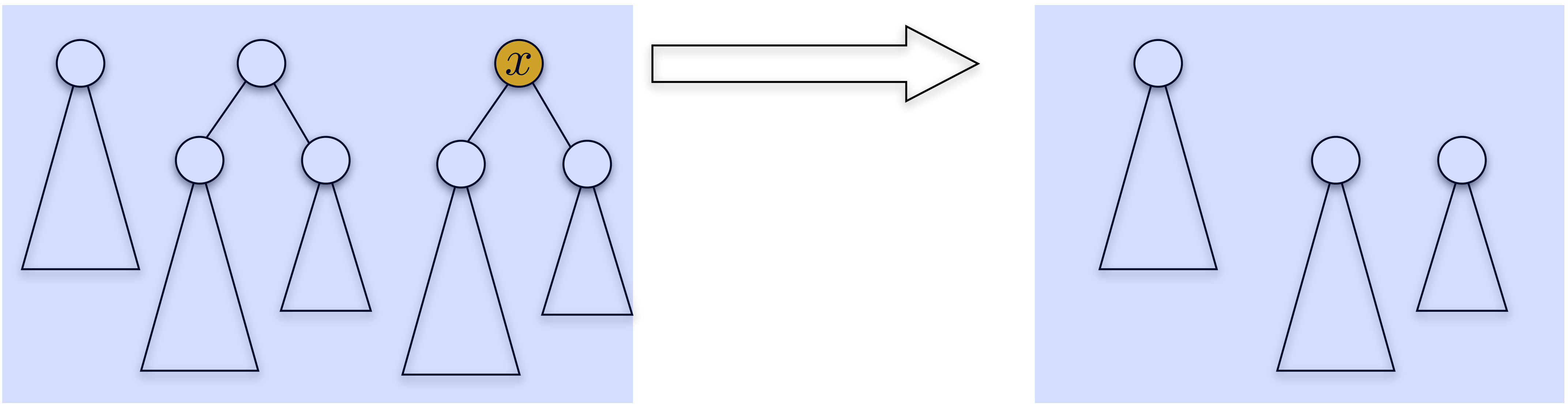

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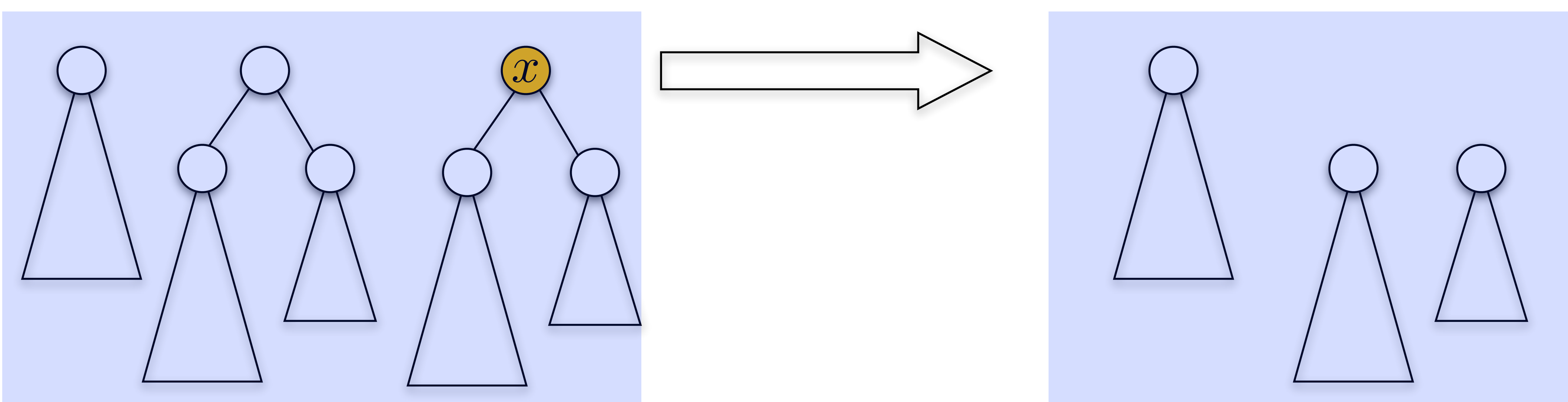

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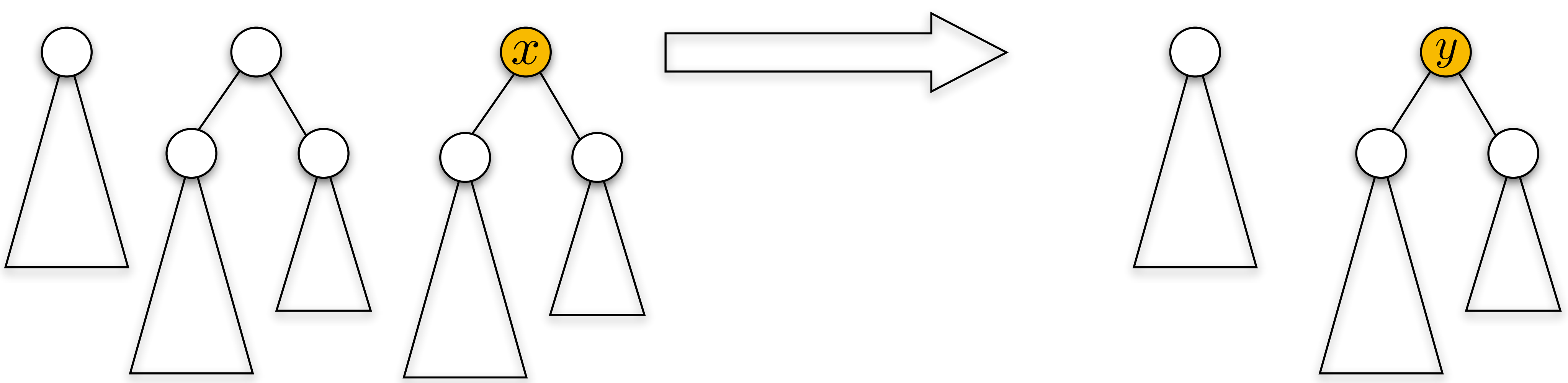

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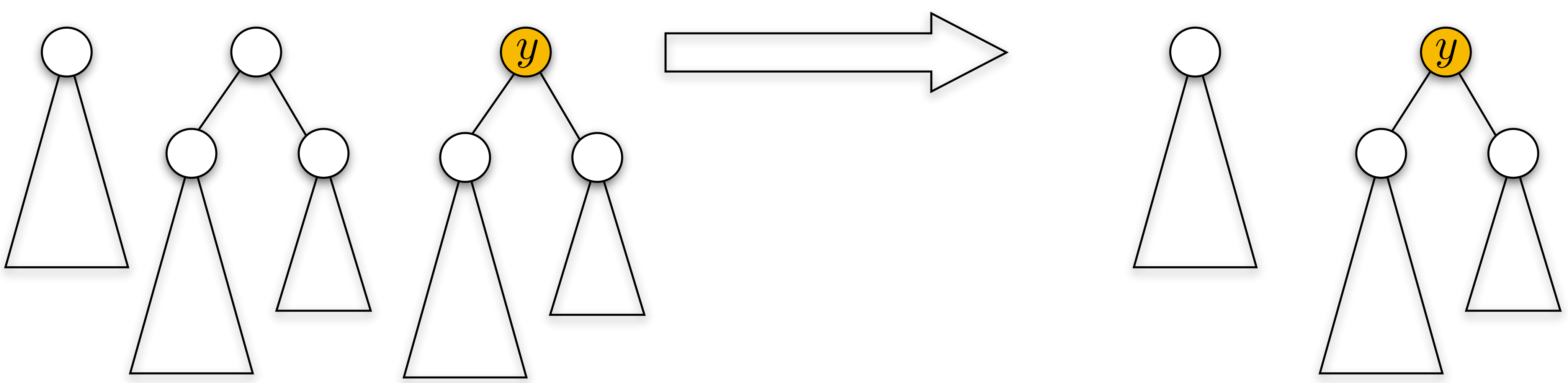

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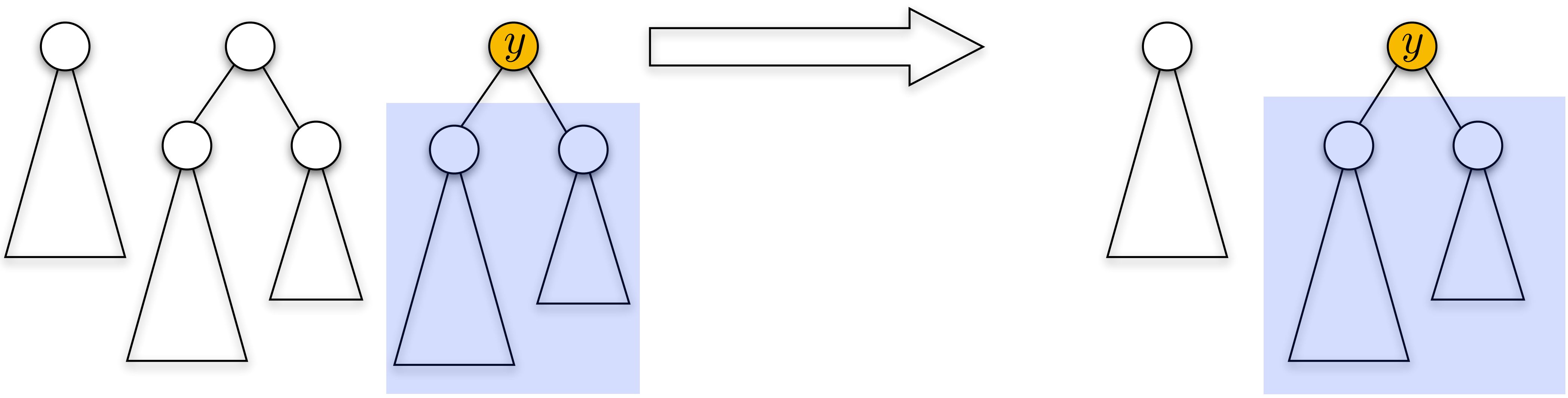

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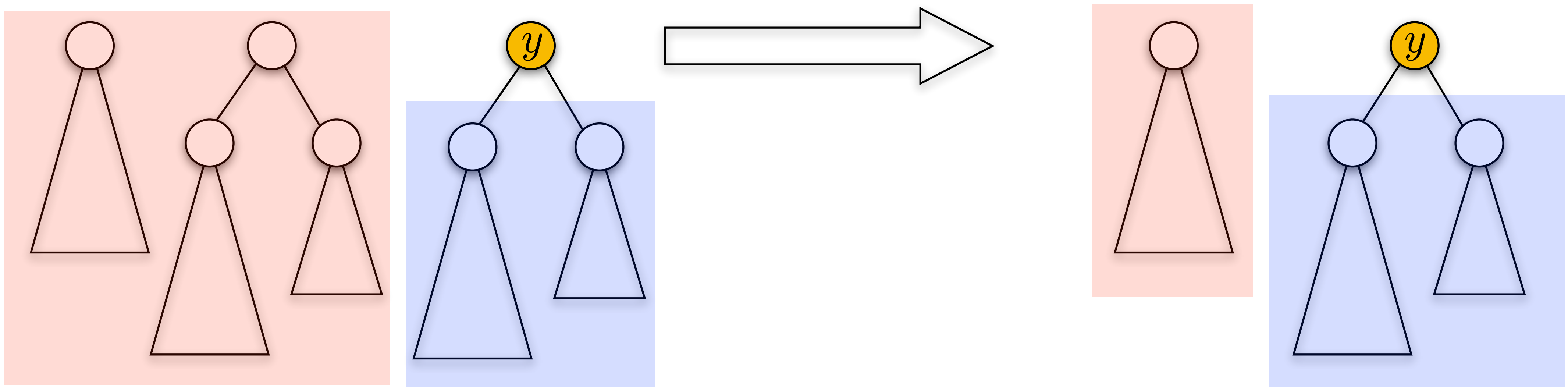

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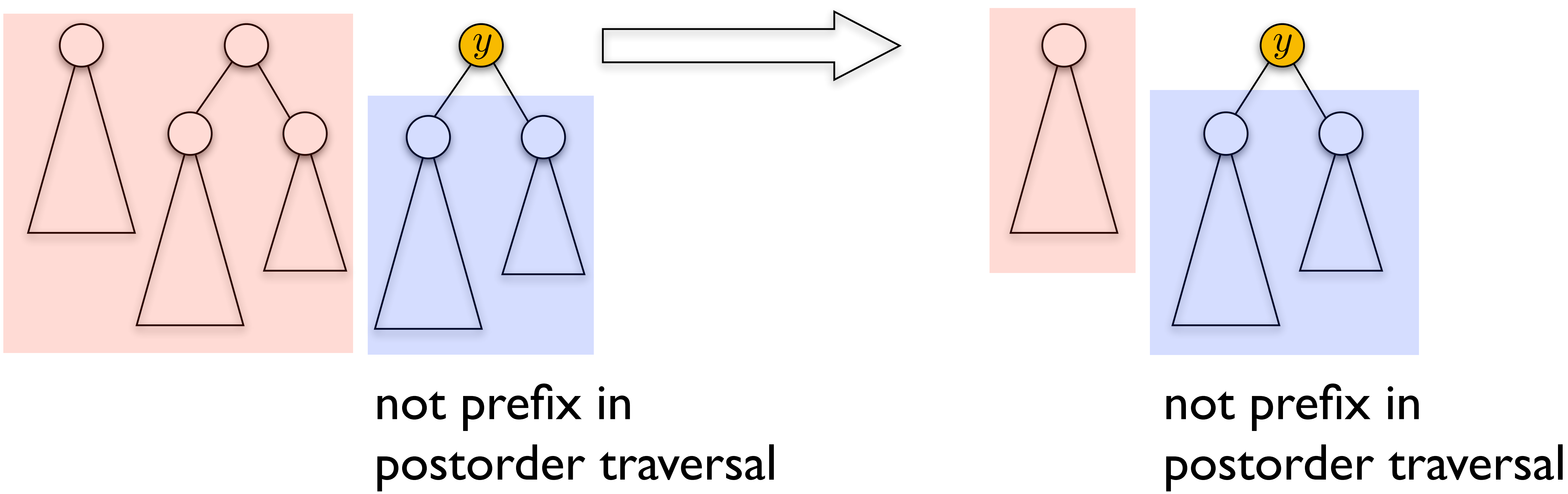

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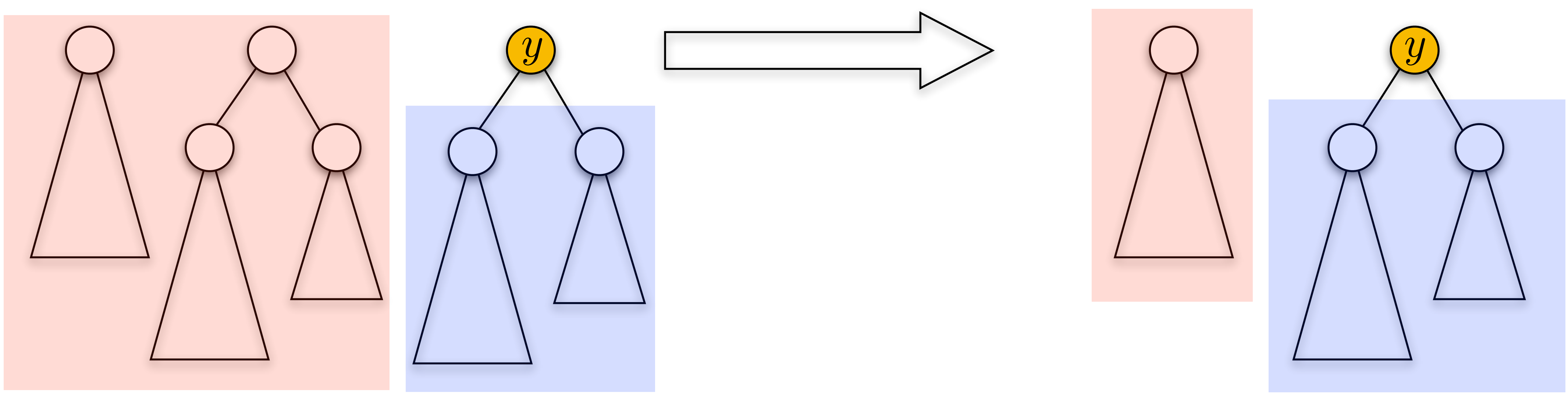
$O(n^4)$ time [Shasha Zhang 1989]



not prefix in
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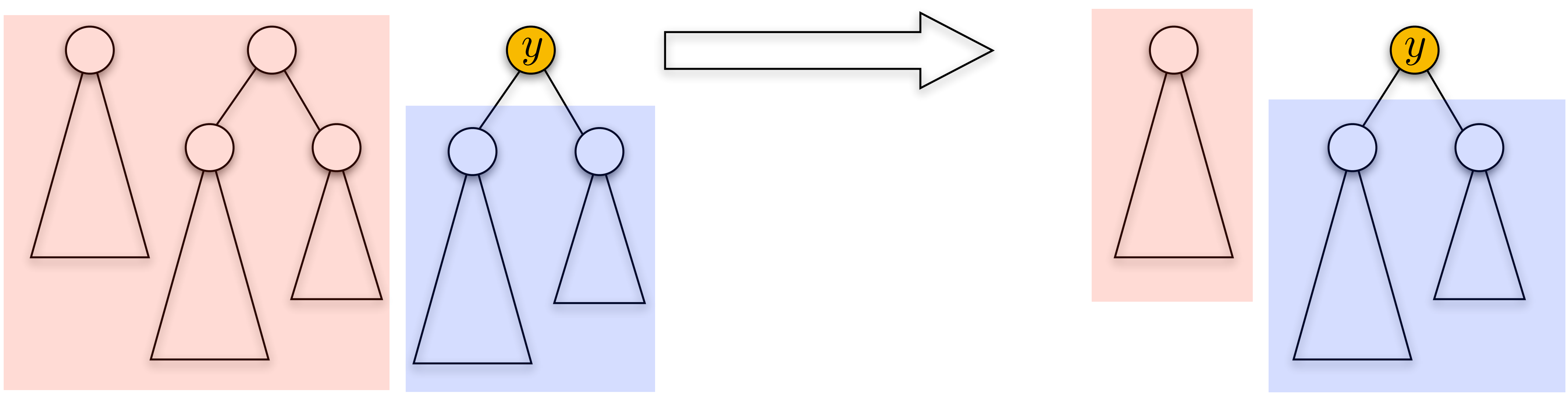
$O(n^3 \log n)$ time [Klein 1998]



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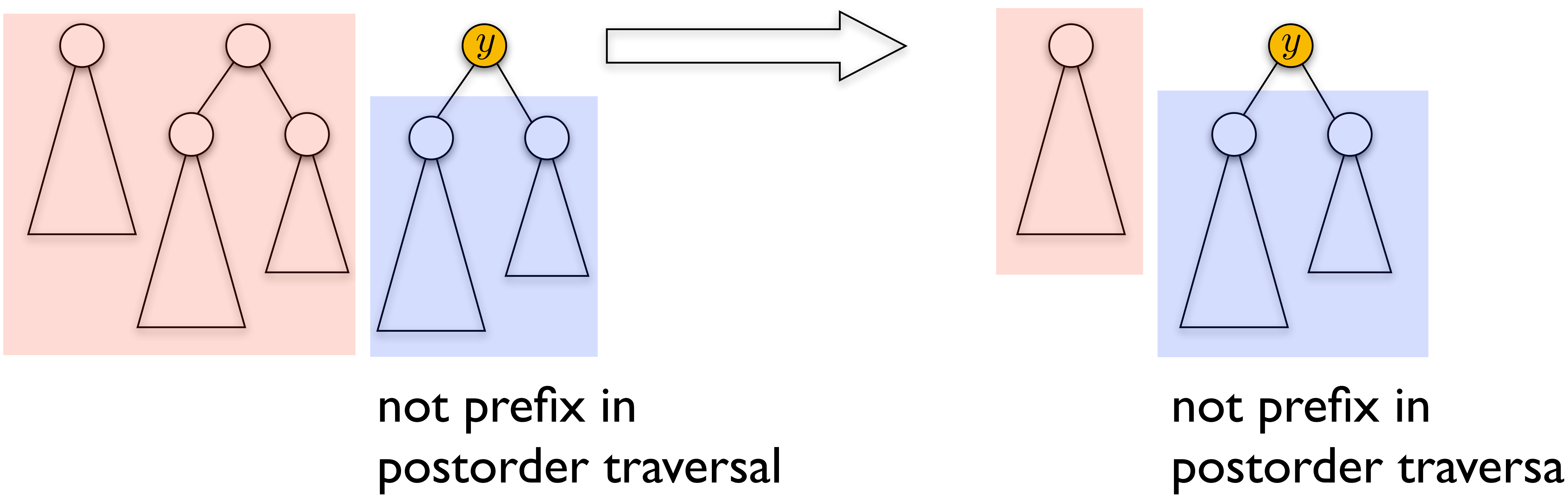

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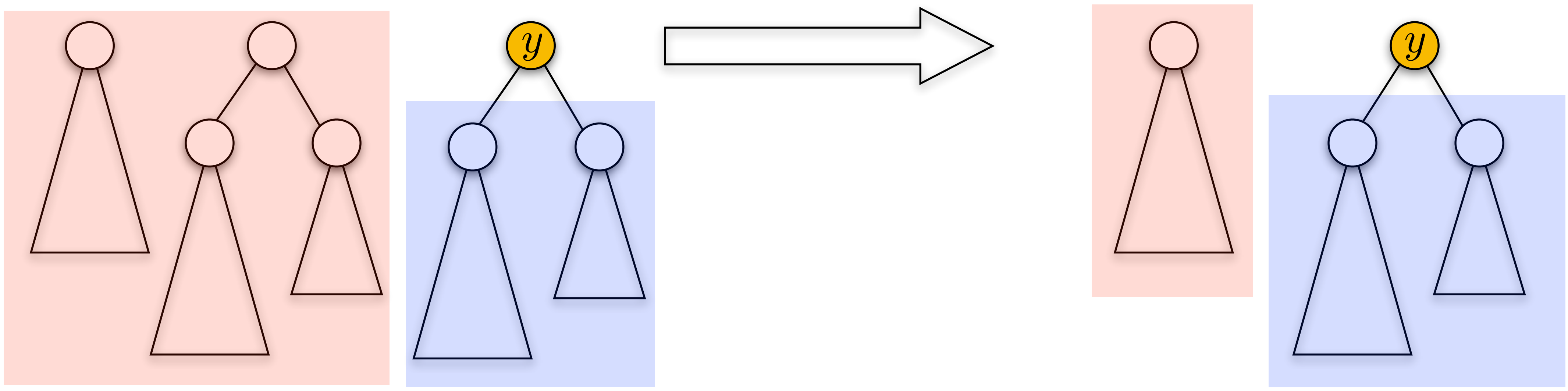

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Conjecture (APSP):

For any $\varepsilon > 0$ there exists $c > 0$, such that All Pairs Shortest Paths on n node graphs with edge weights in $\{1, \ldots, n^c\}$ cannot be solved in $O(n^{3-\varepsilon})$ time.

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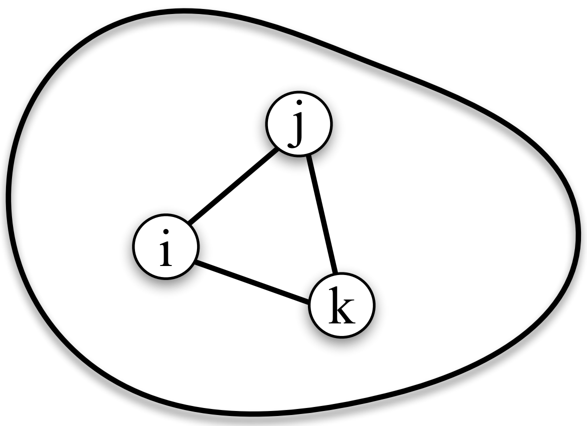
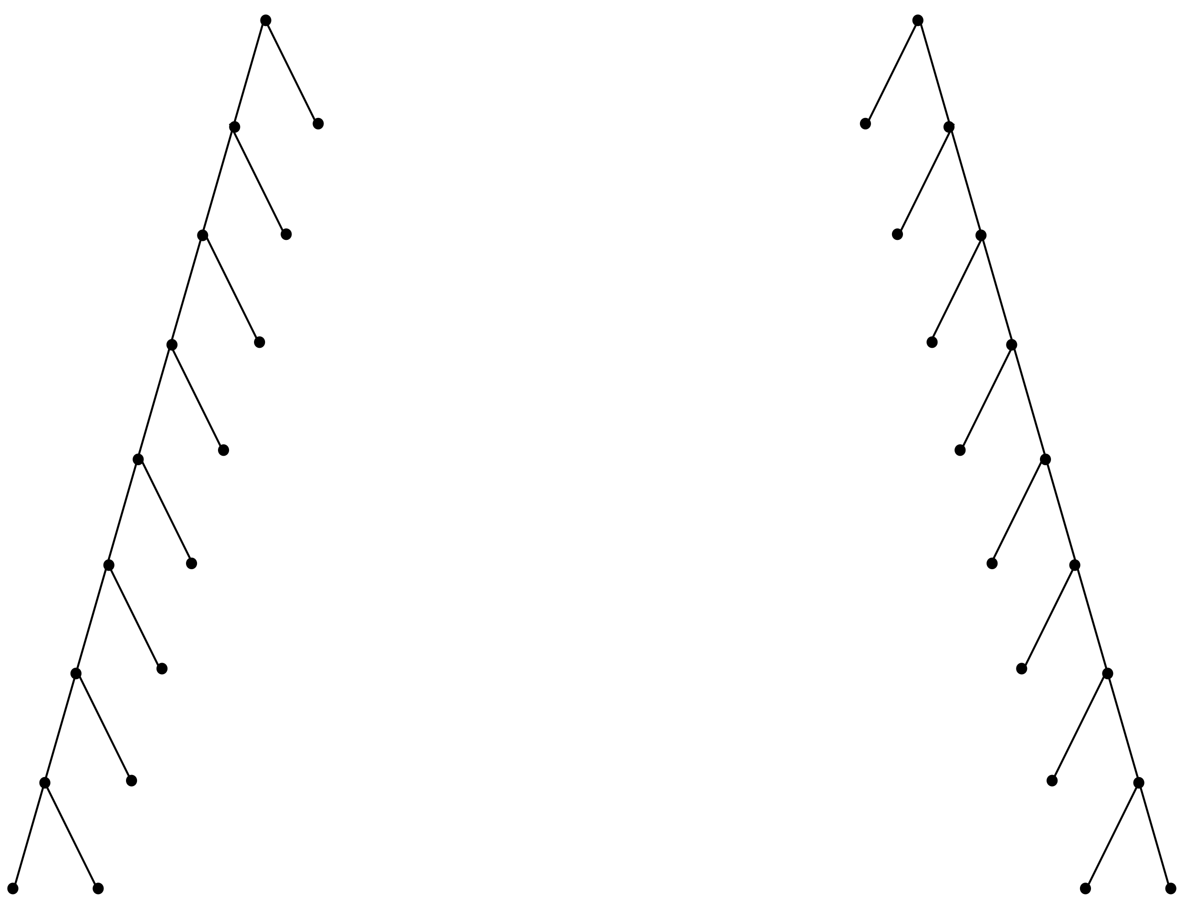
Equivalent to negative triangle detection [Vassilevska-Williams,Williams 2010]



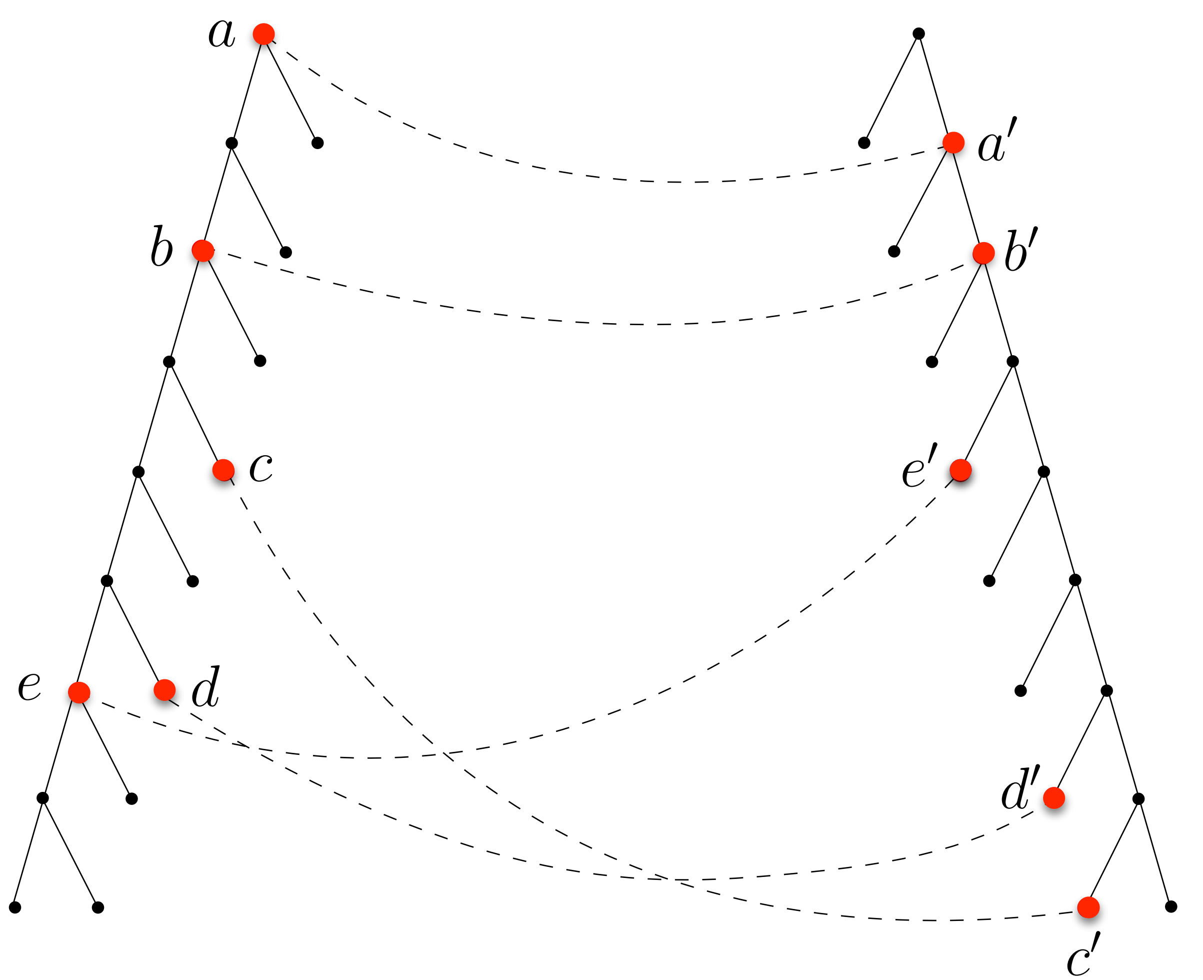
$$w(i,j)+w(j,k)+w(k,i) < 0$$

The Hard Instance of TED



The Hard Instance of TED

To specify a solution to TED it is enough to say which nodes are matched to which (the rest are deleted)



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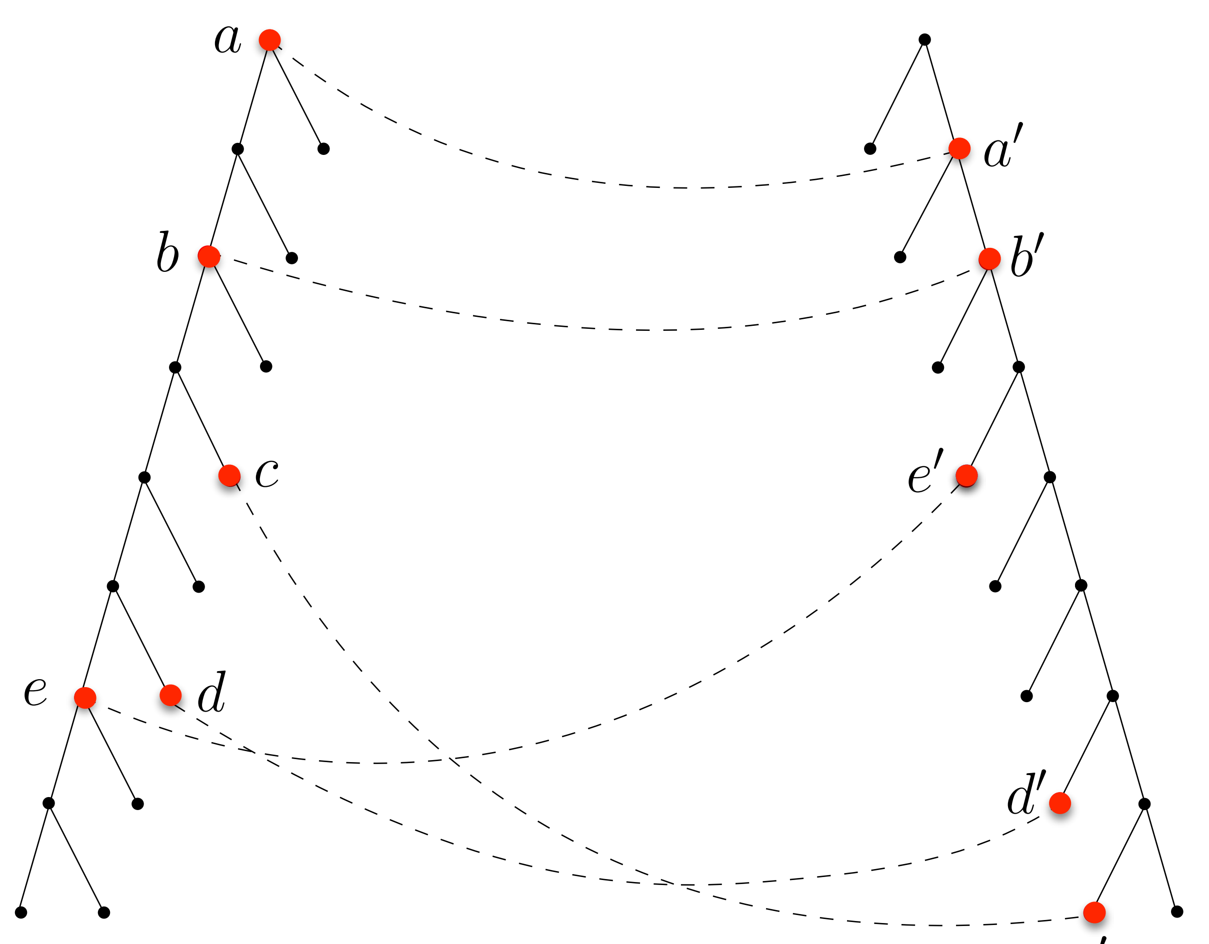
To specify a solution to TED it is enough to say which nodes are matched to which (the rest are deleted)

The matched pairs must have the same left-right and ancestor-descendant relations



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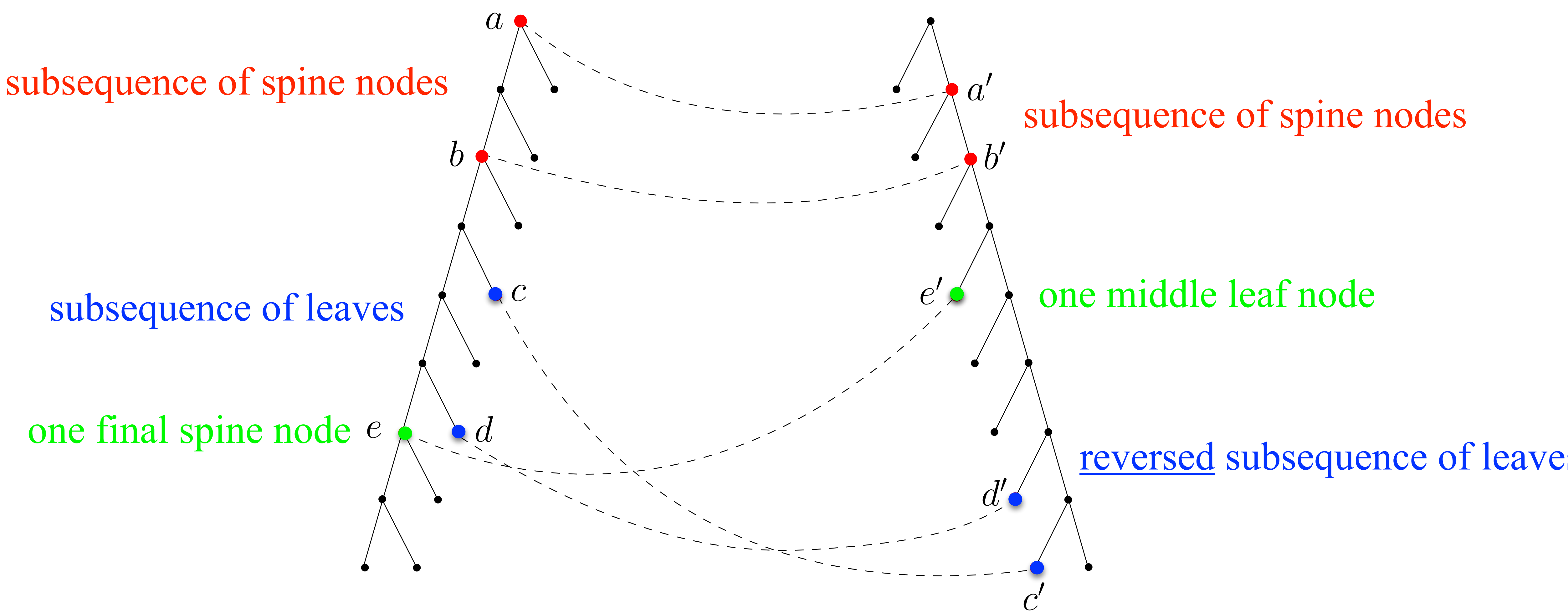

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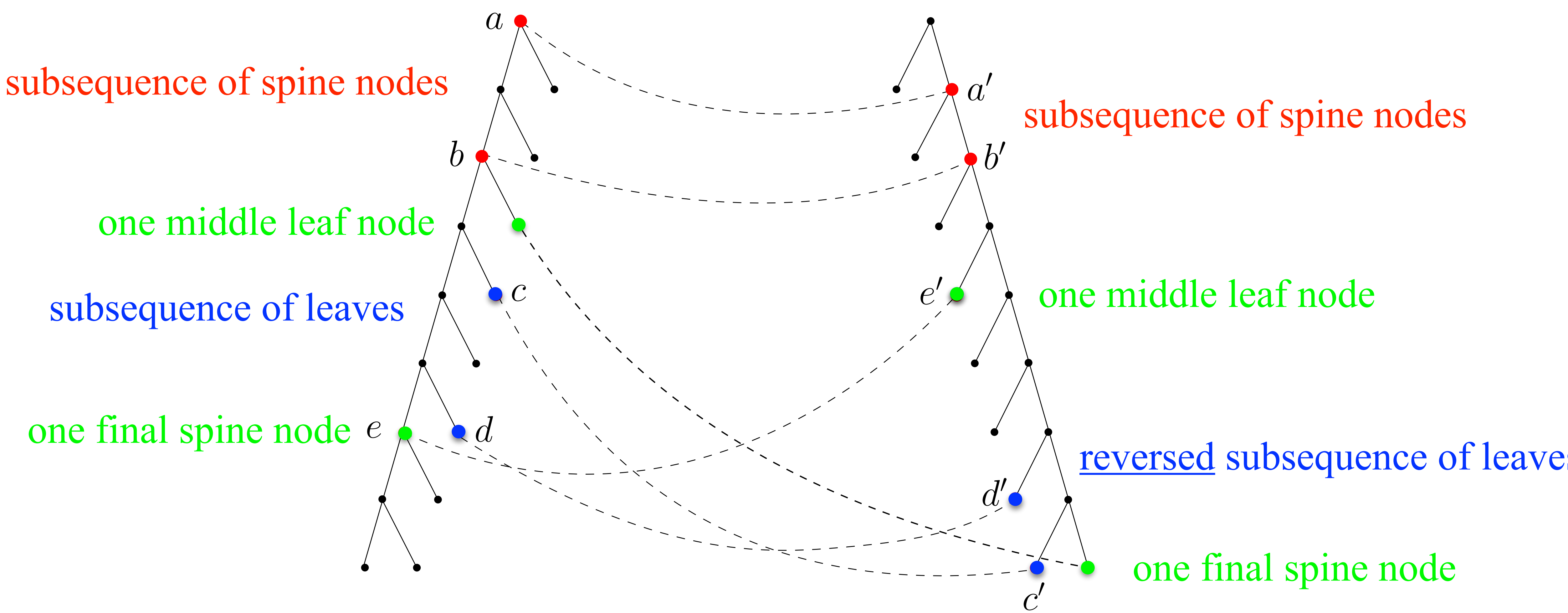

APSP ➜ TED

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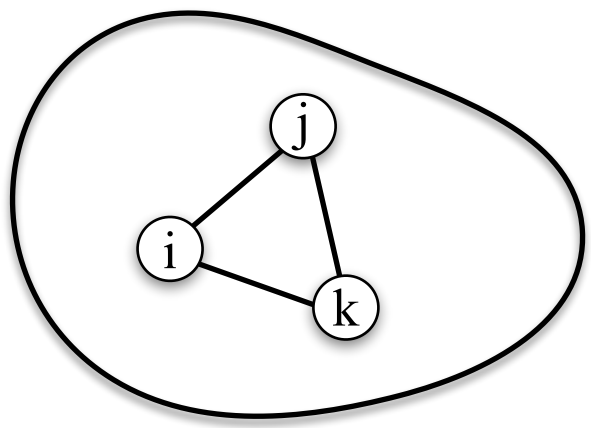

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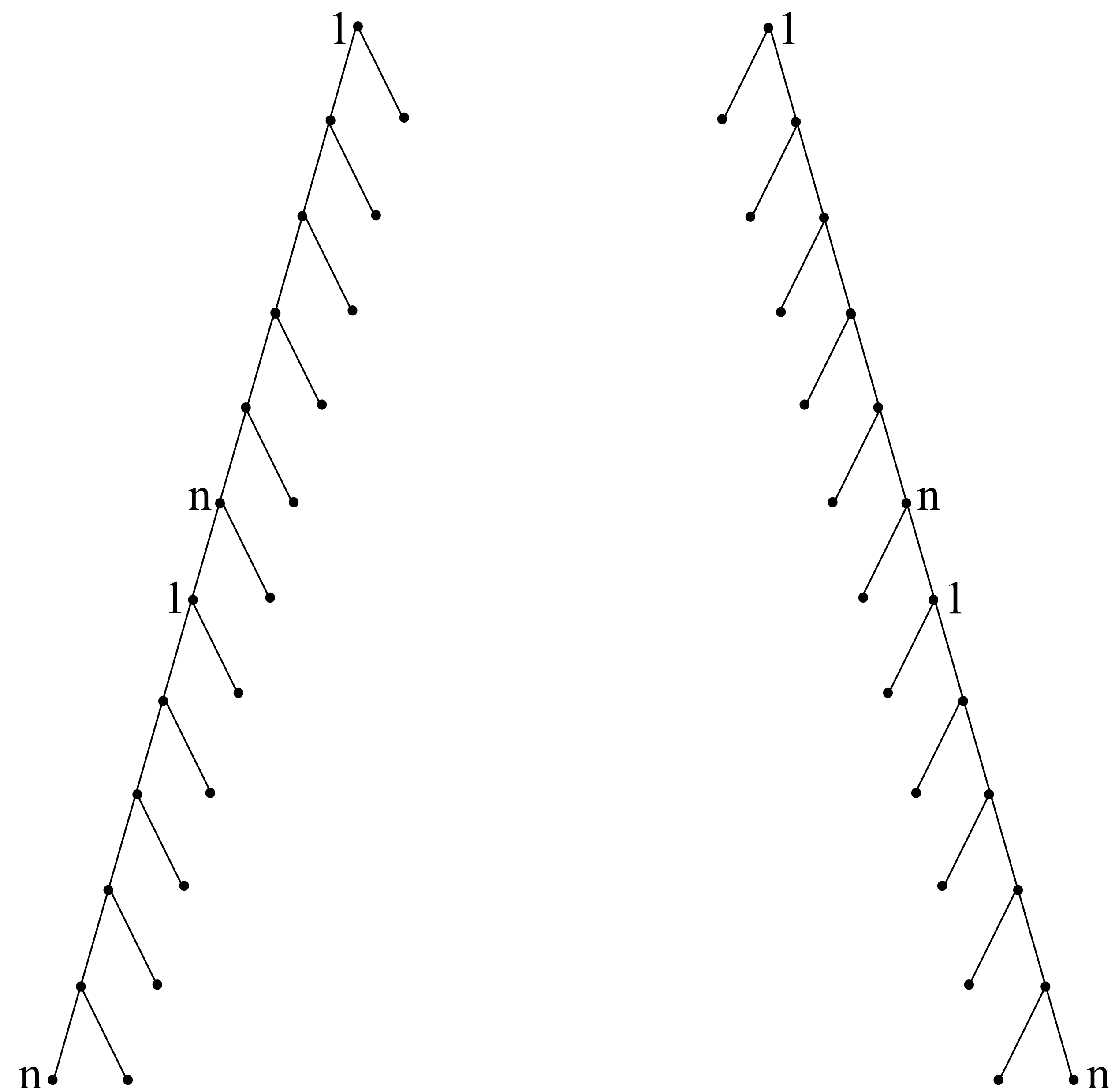

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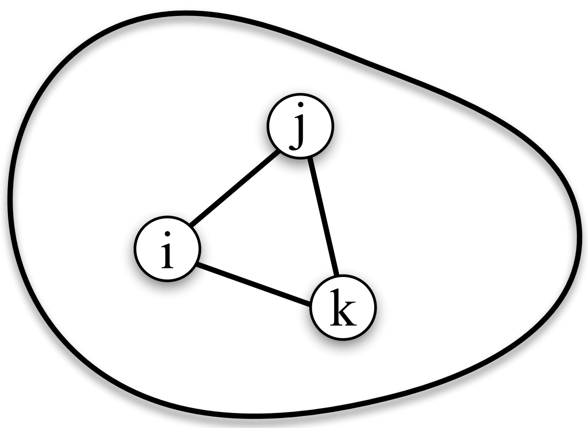

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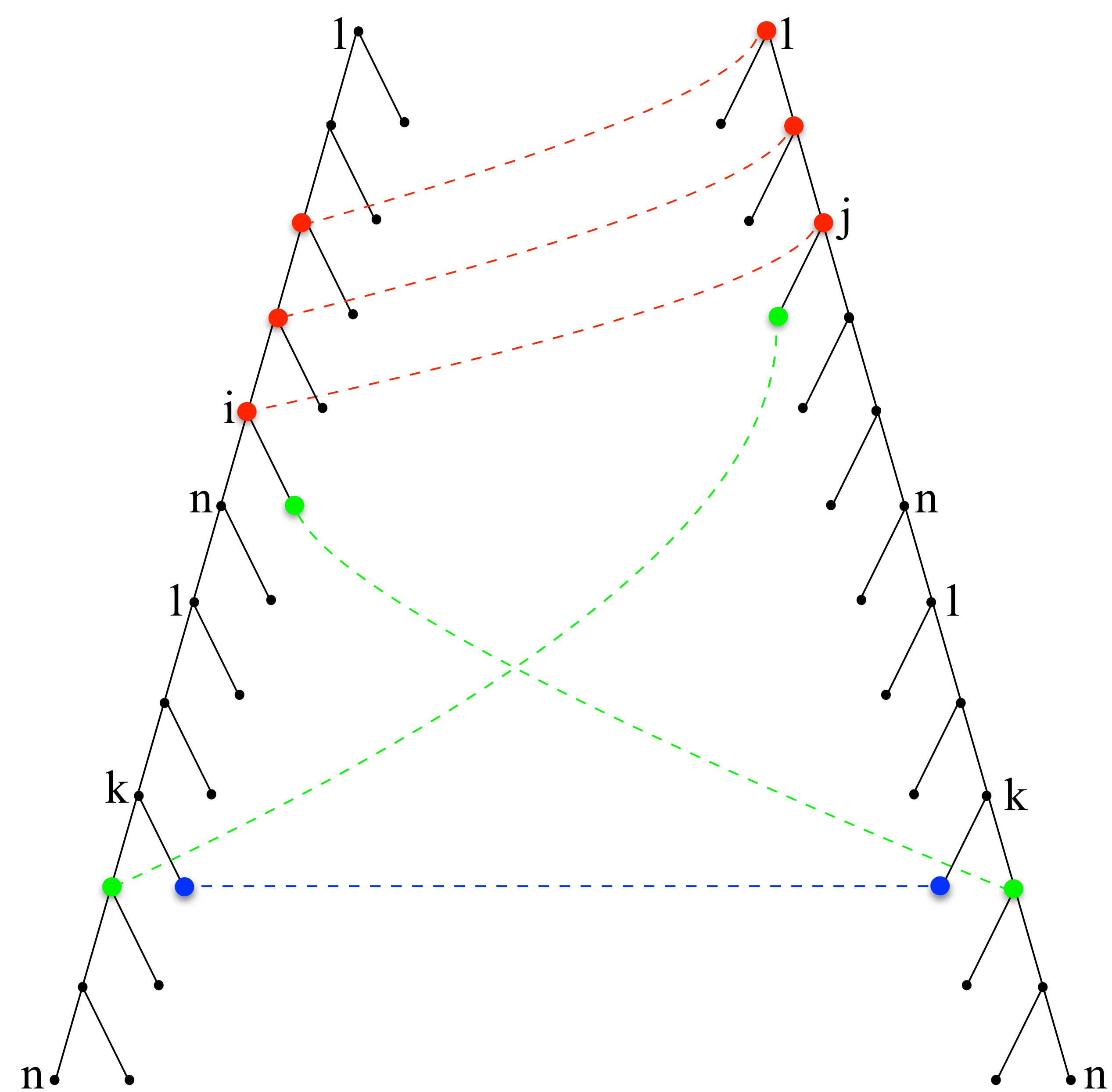

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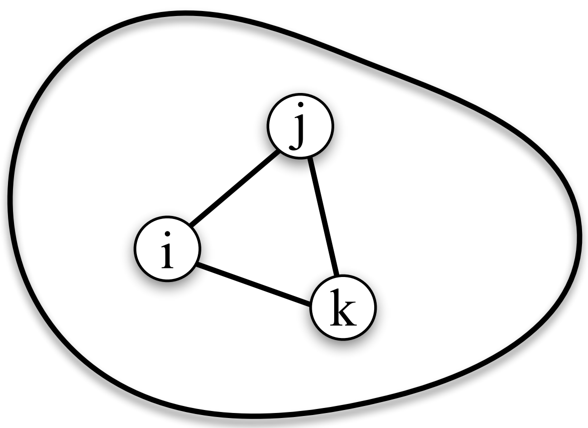

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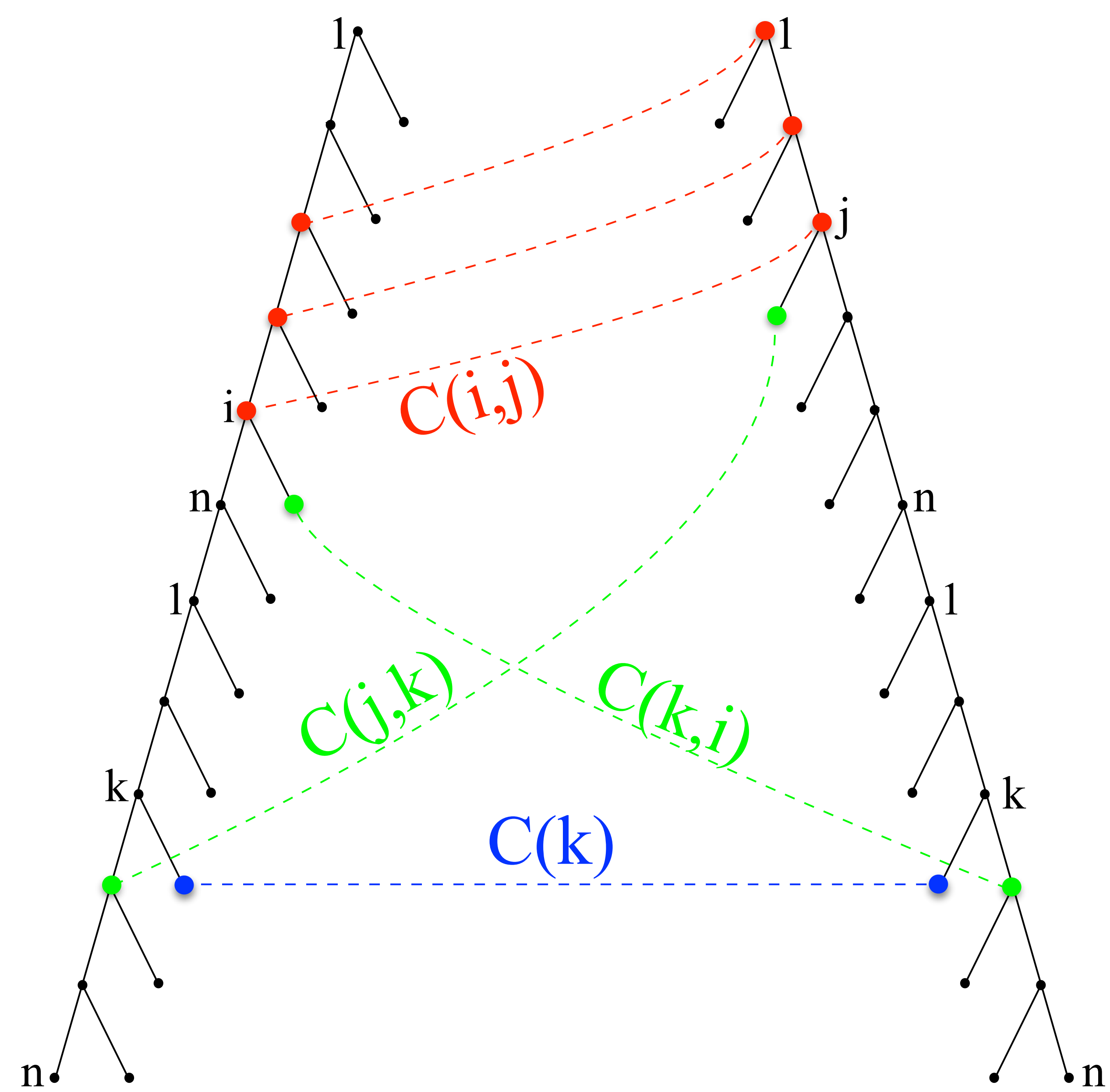

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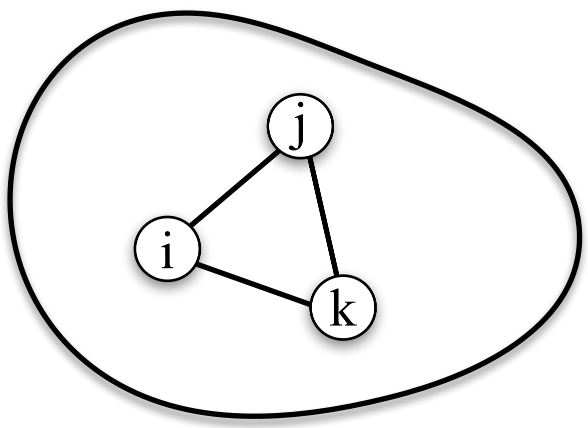

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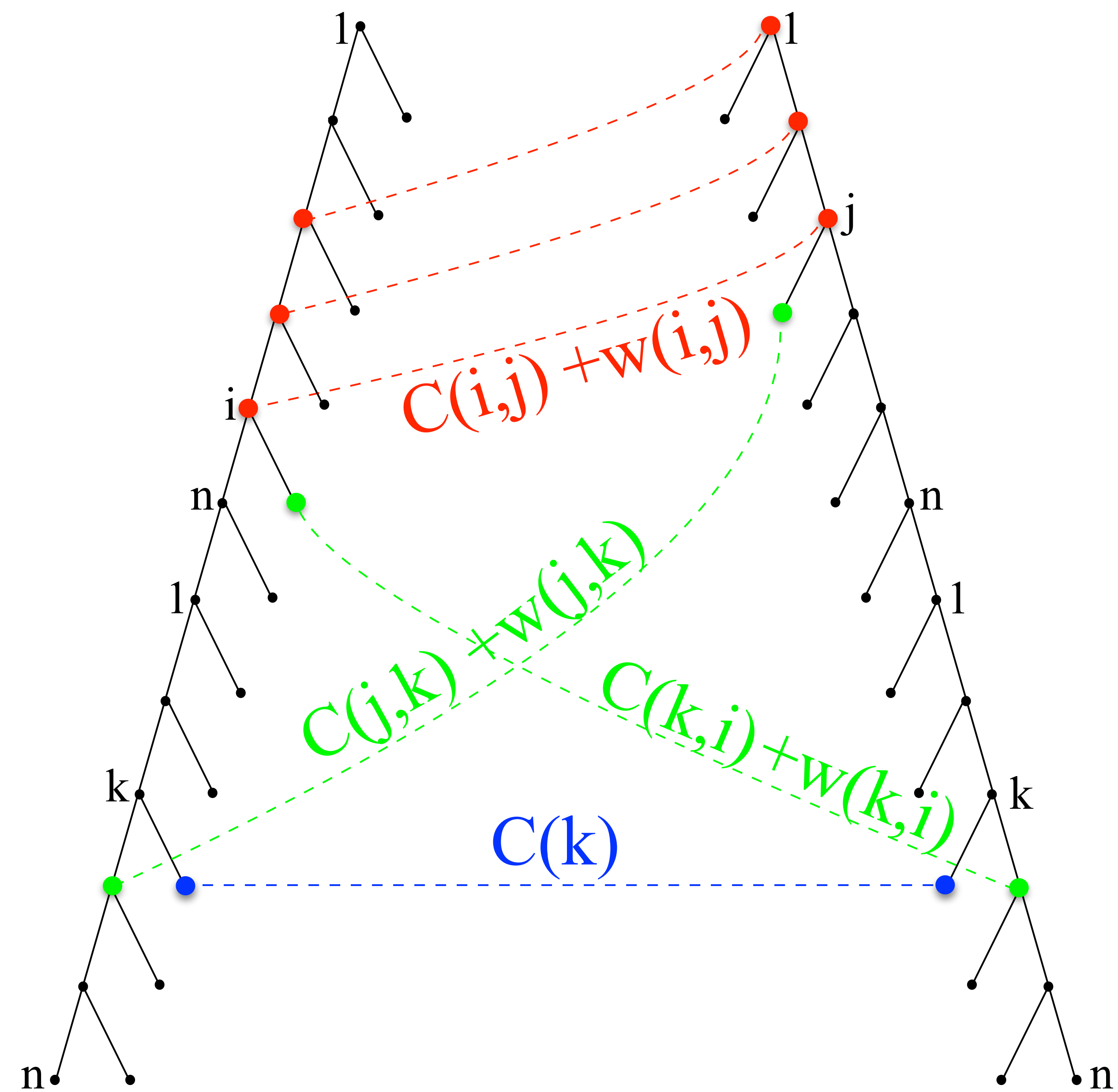
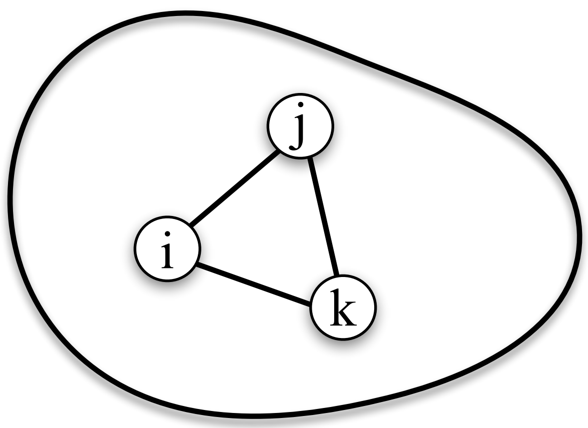


APSP ➜ TED

Large alphabet: $|\Sigma| = \Theta(n)$



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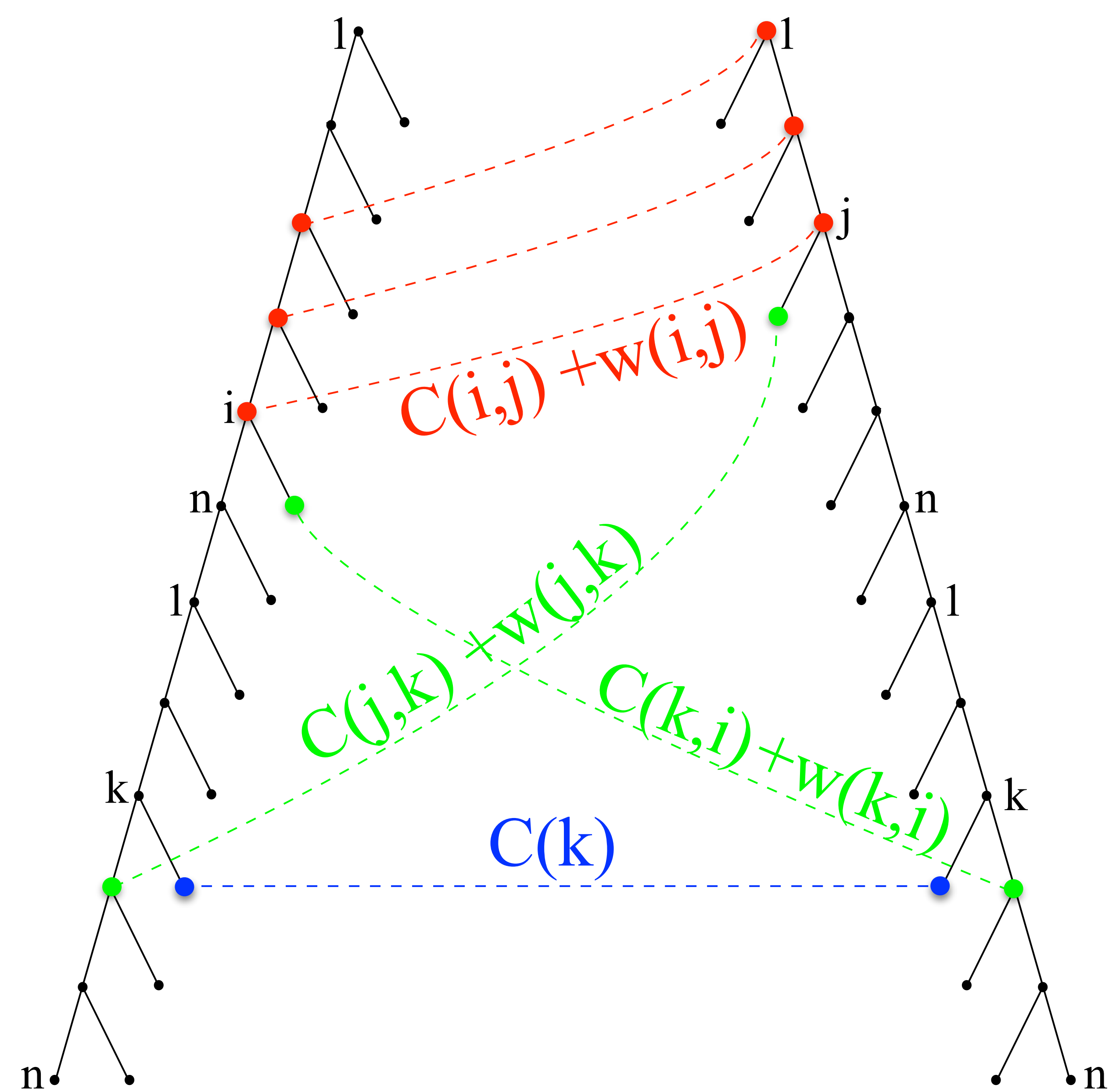

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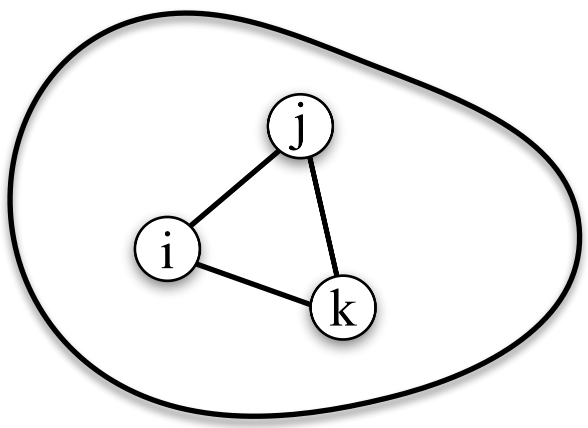

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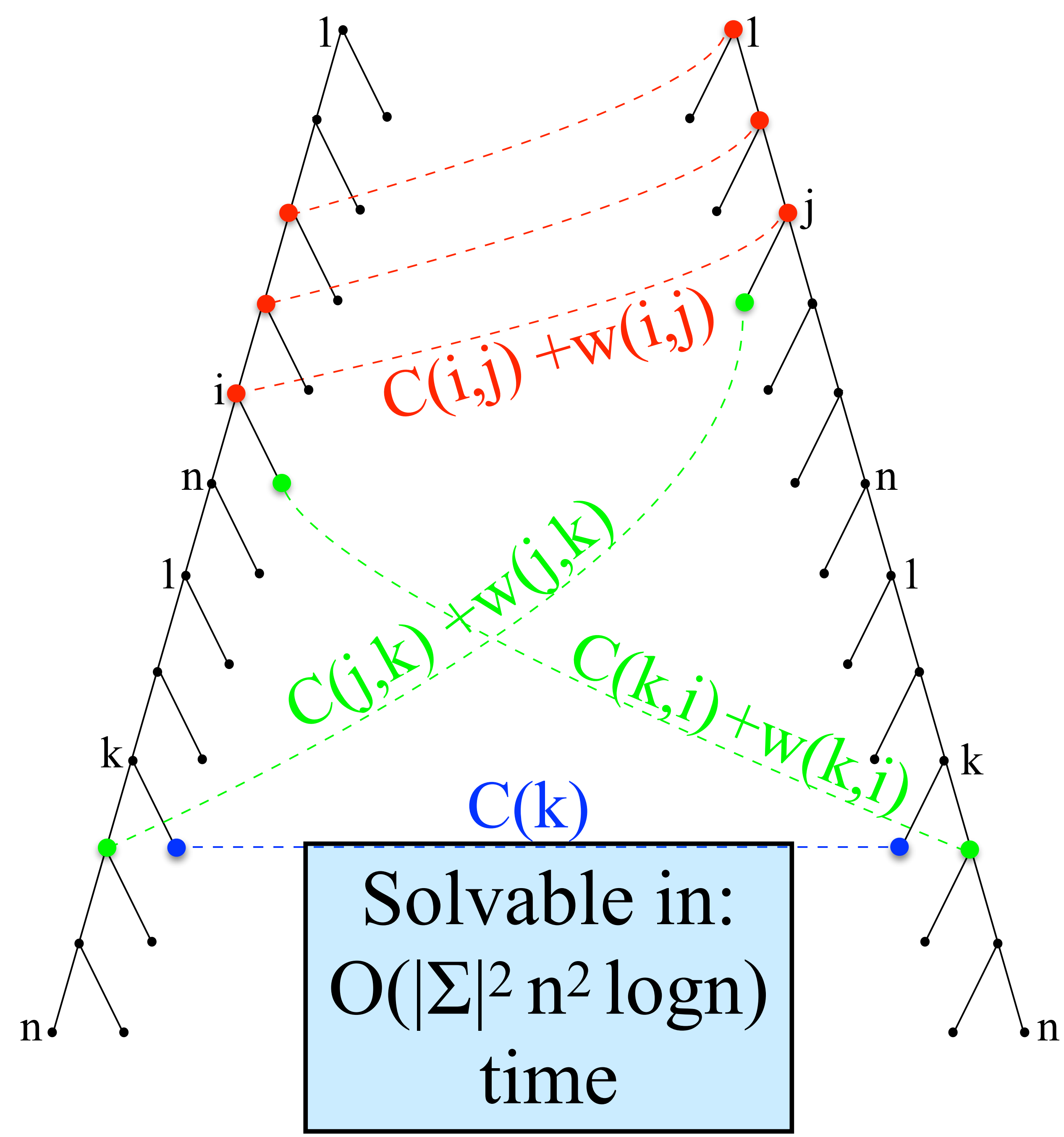
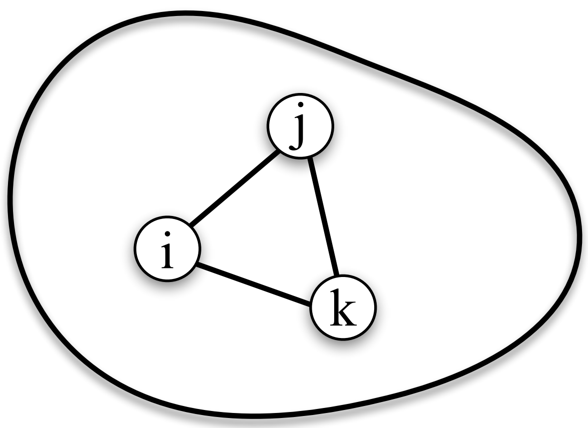


APSP ➜ TED

Small alphabet: $|\Sigma| = O(1)$



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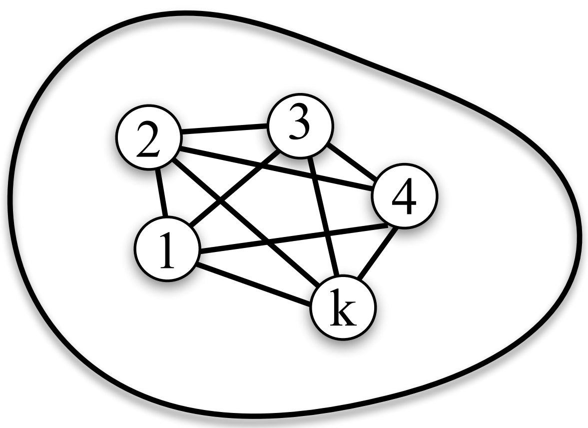
Max-weight k-Clique ➜ TED

Small alphabet: $|\Sigma| = O(1)$

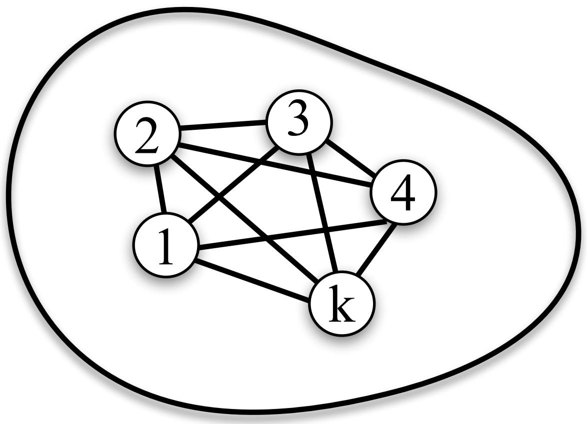


Conjecture (Max-weight k-Clique):

For any $\varepsilon > 0$ there exists $c > 0$, such that for any $k \geq 3$ finding a maximum weight k-Clique in graphs with edge weights in $\{1, \ldots, n^{ck}\}$ cannot be solved in $O(n^{k(1-\varepsilon)})$ time.

Max-weight k-Clique ➜ TED

Small alphabet: $|\Sigma| = O(1)$



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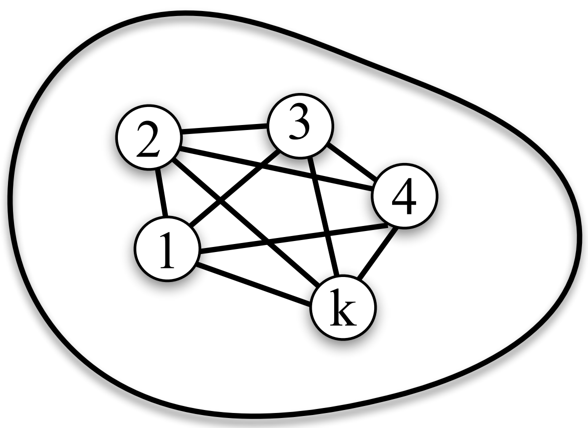

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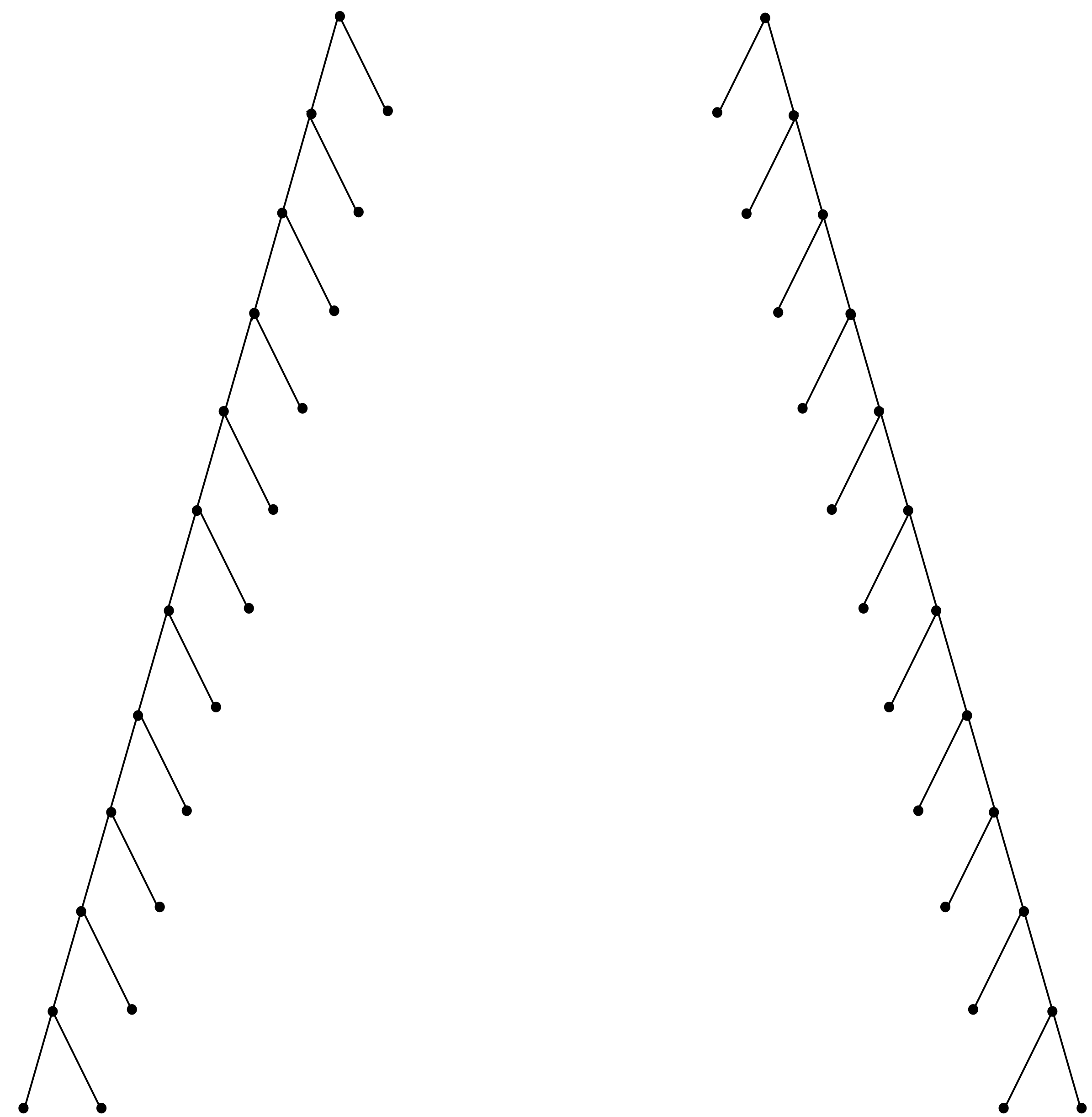

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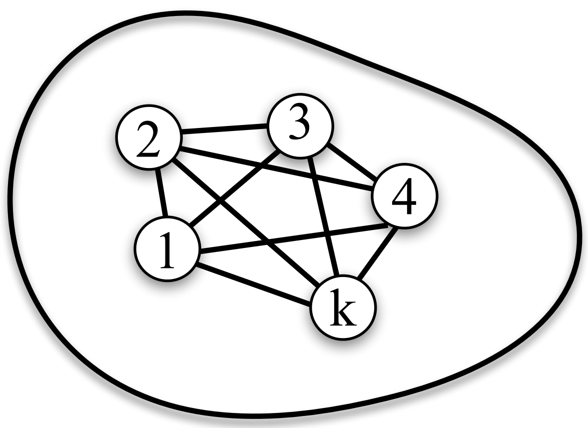

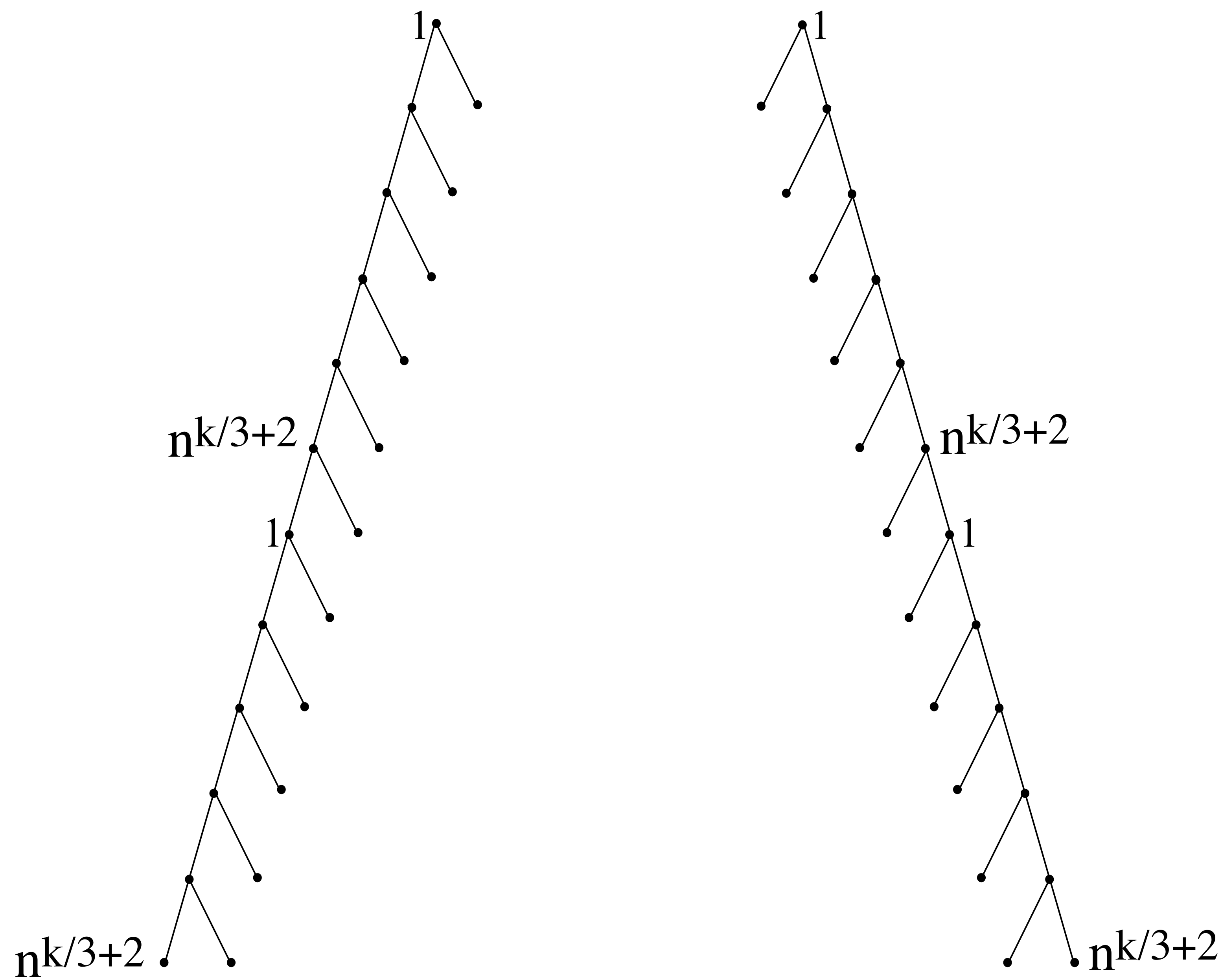

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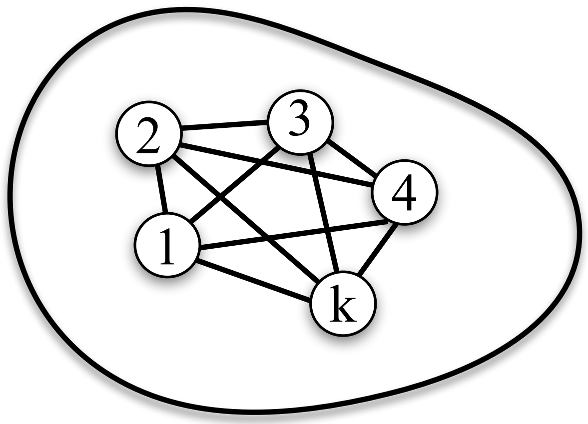

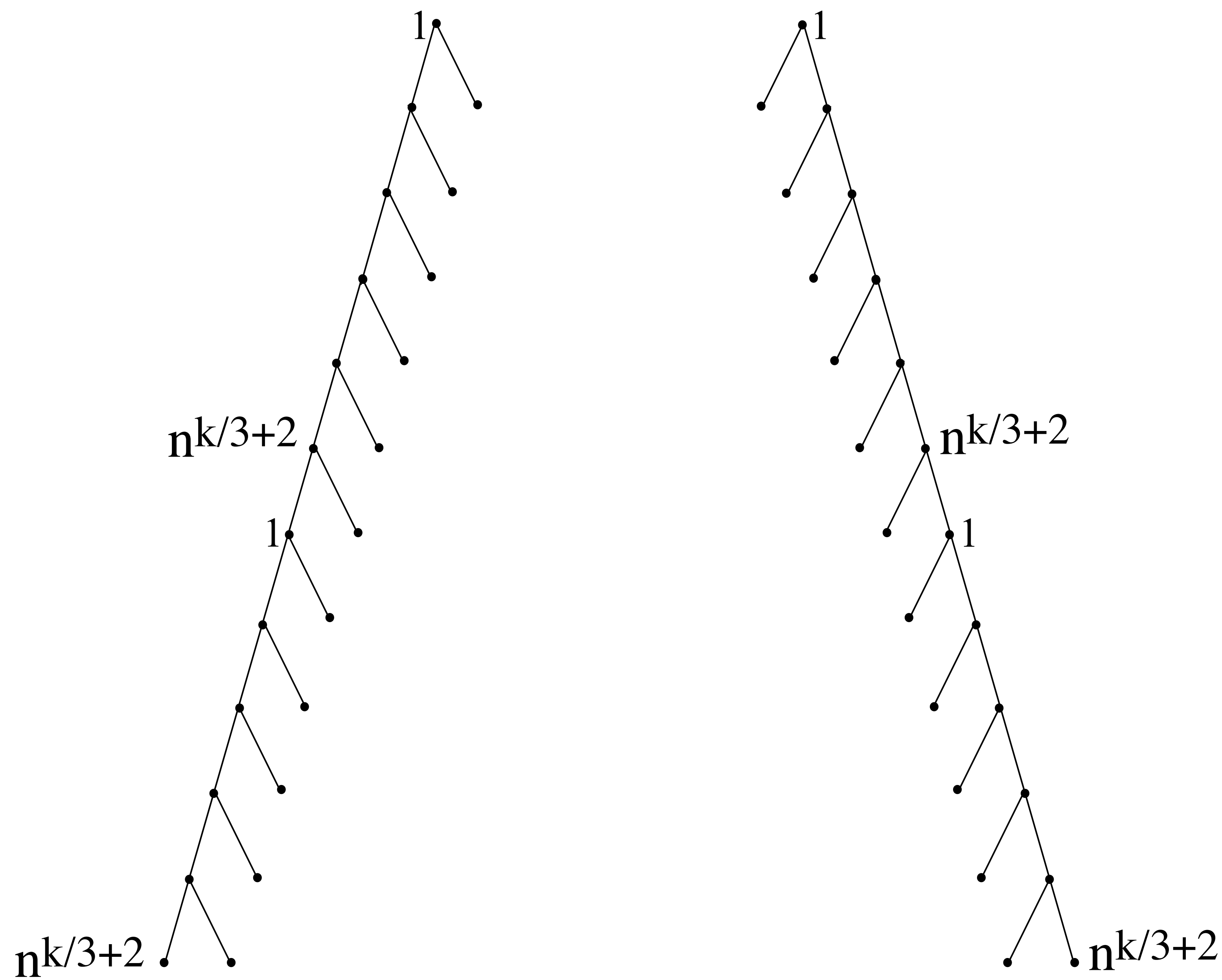

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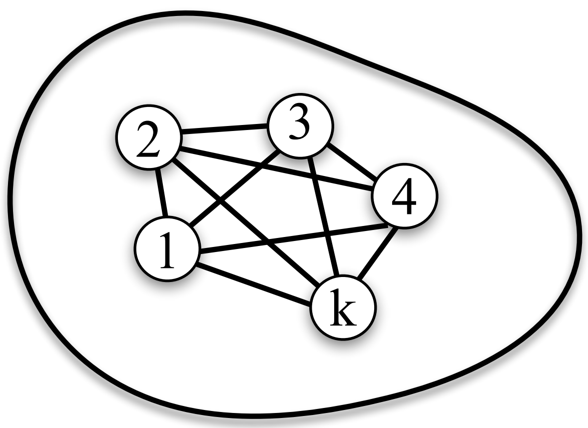

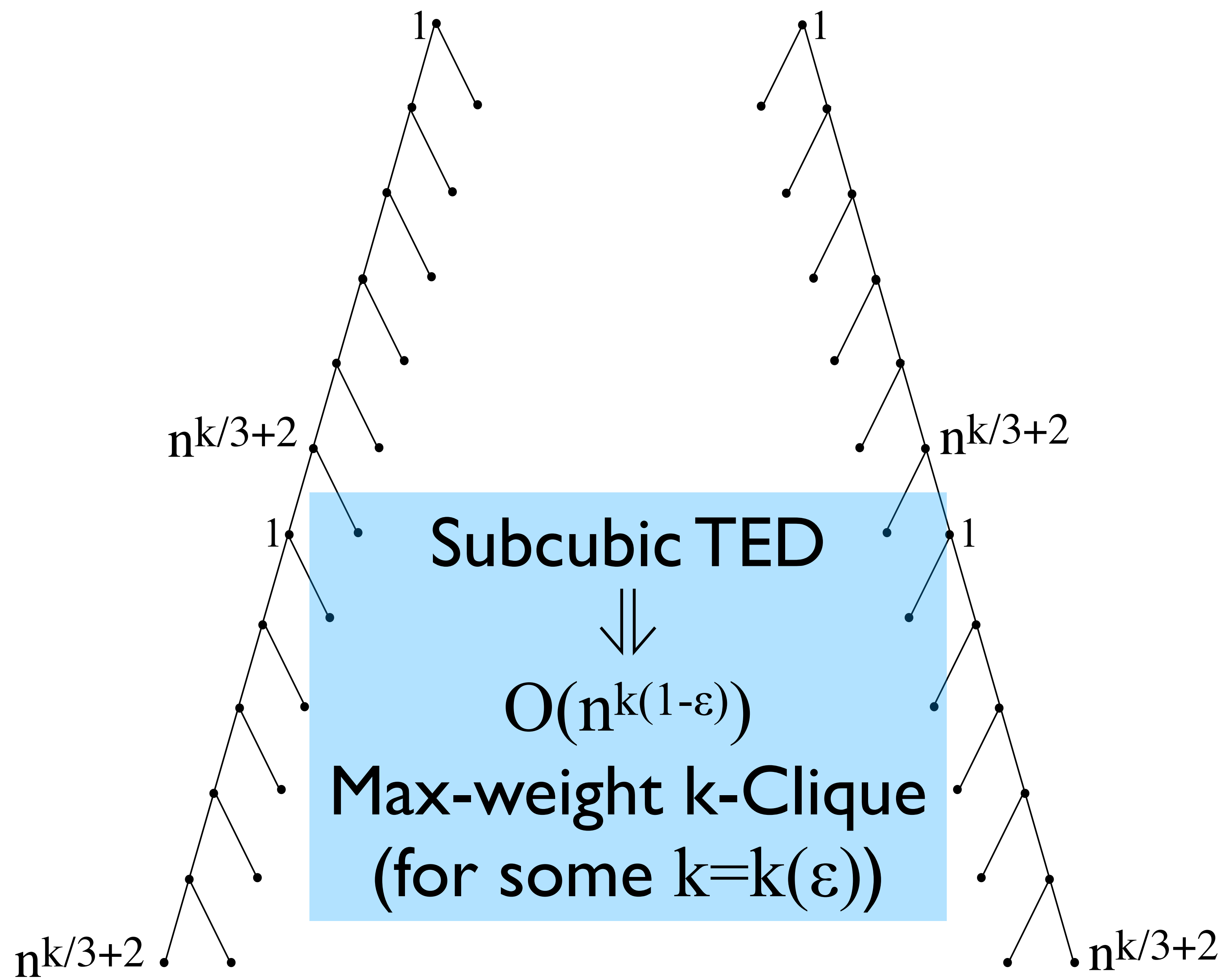

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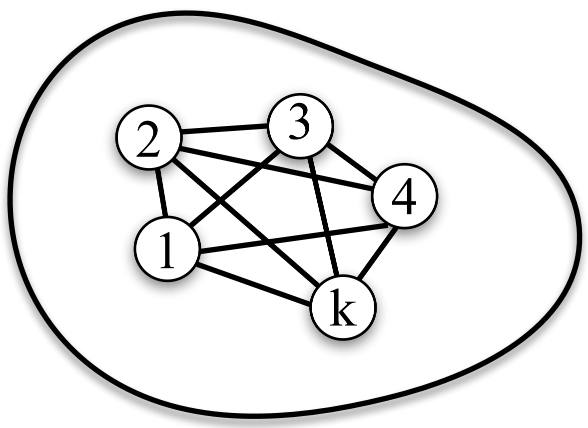
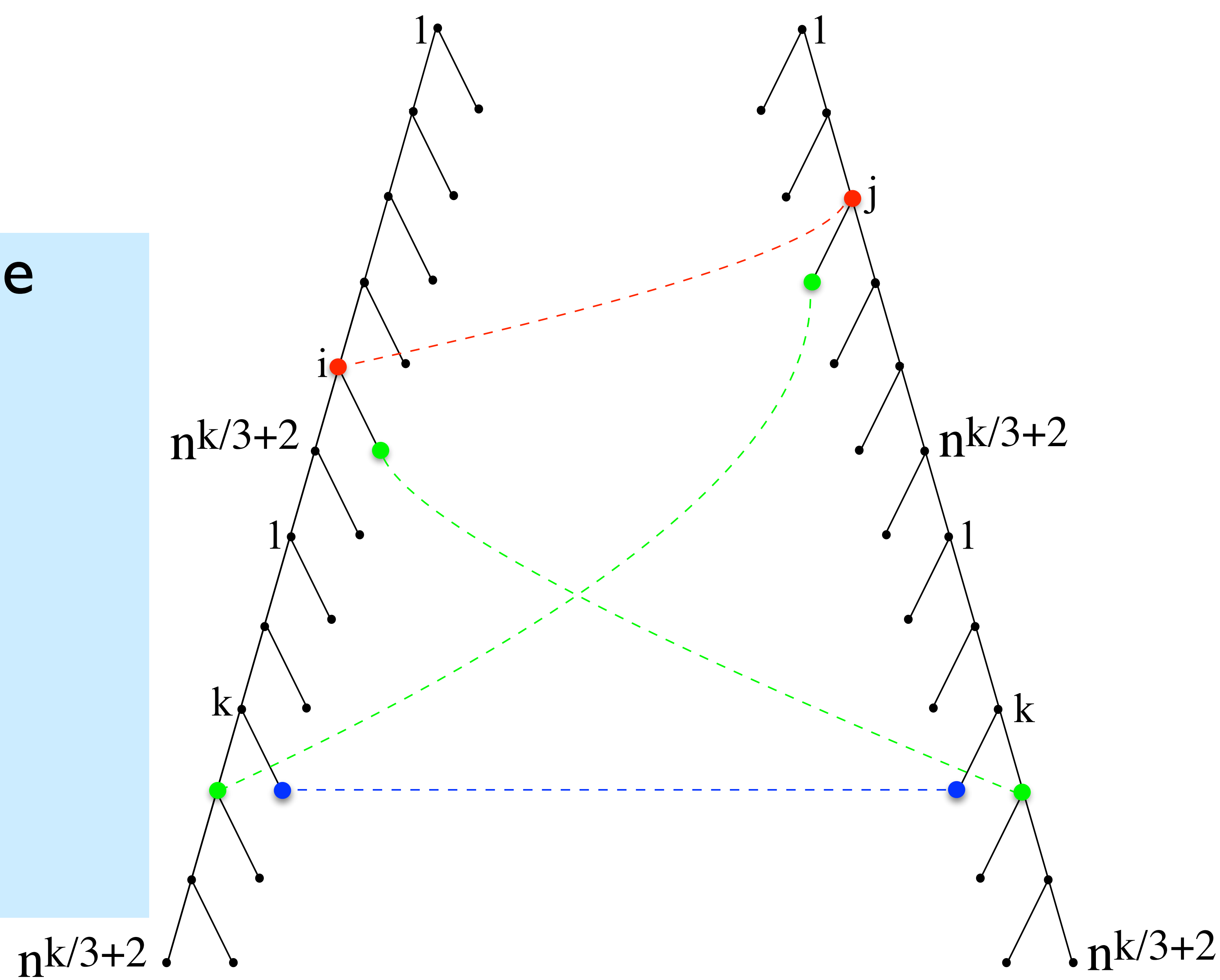


Each k/3 clique is a spine node



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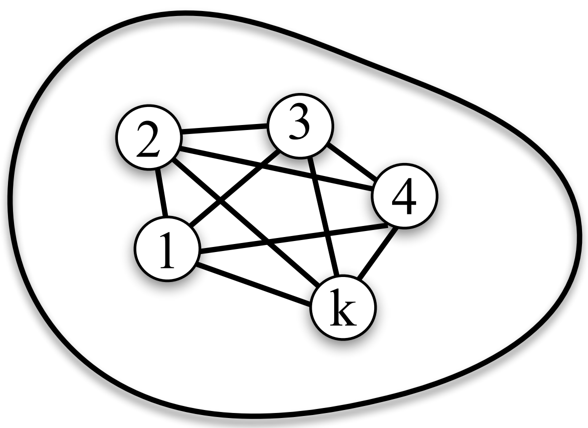

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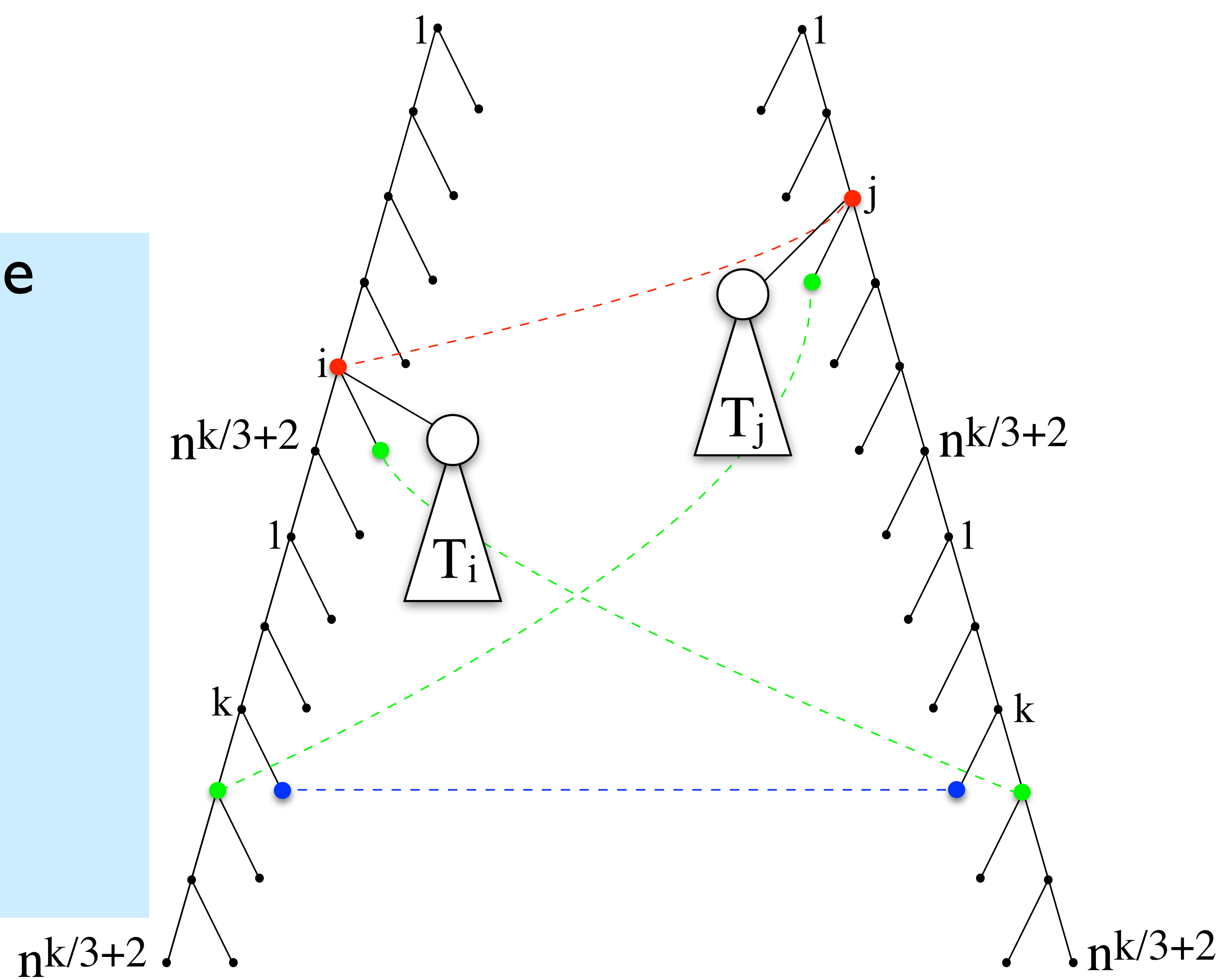

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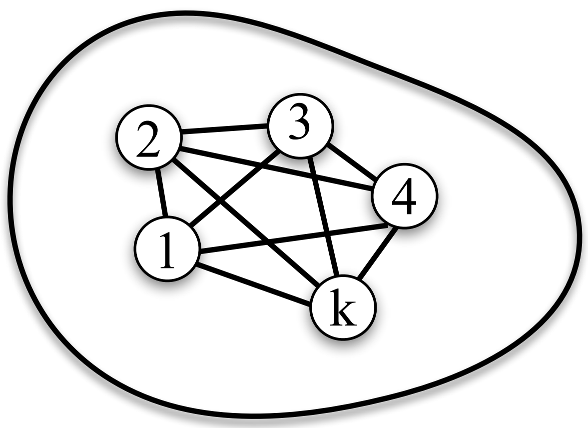

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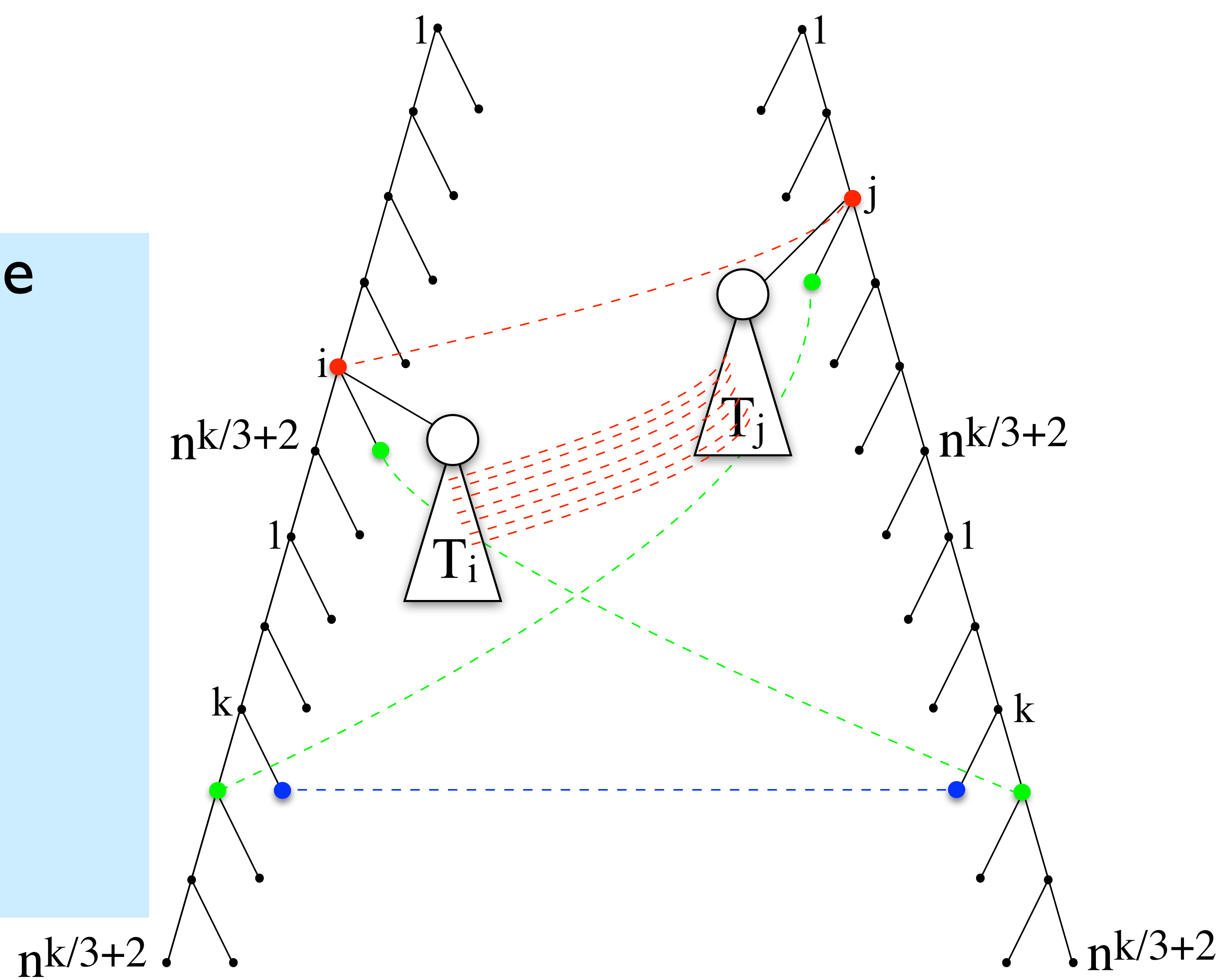

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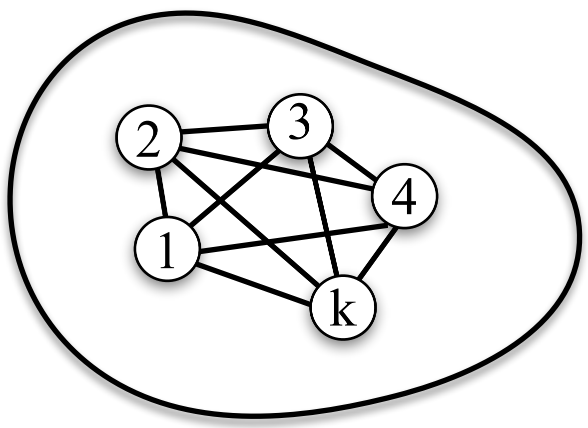

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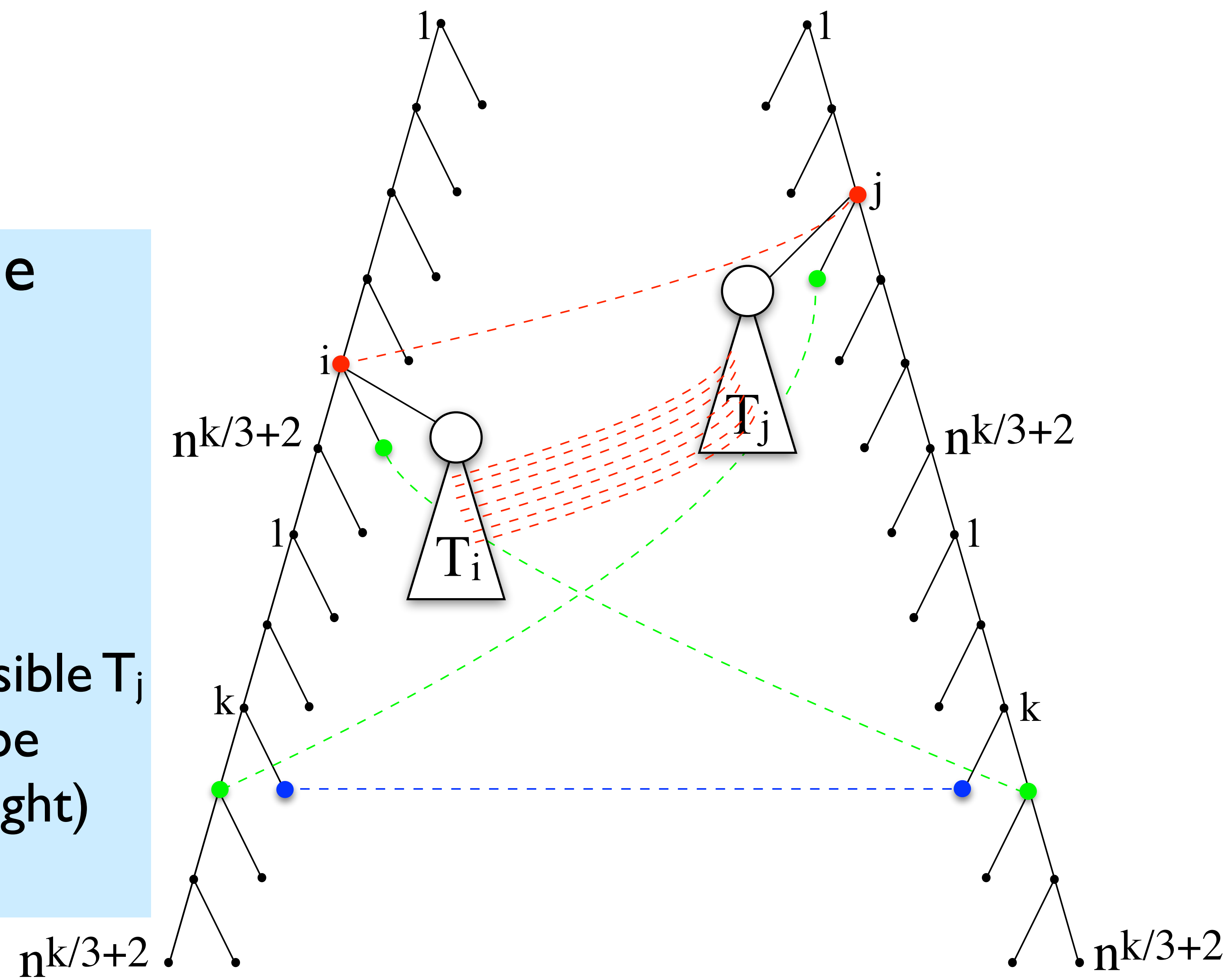

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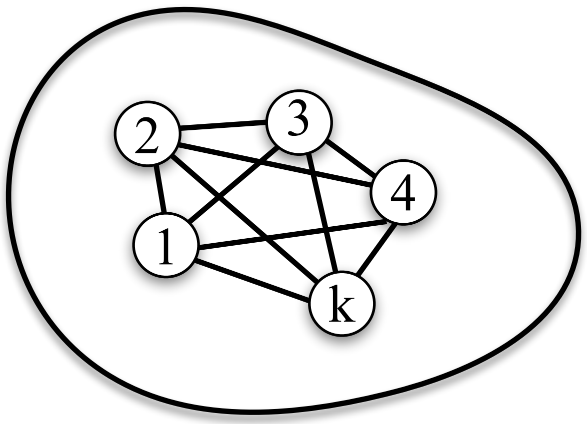

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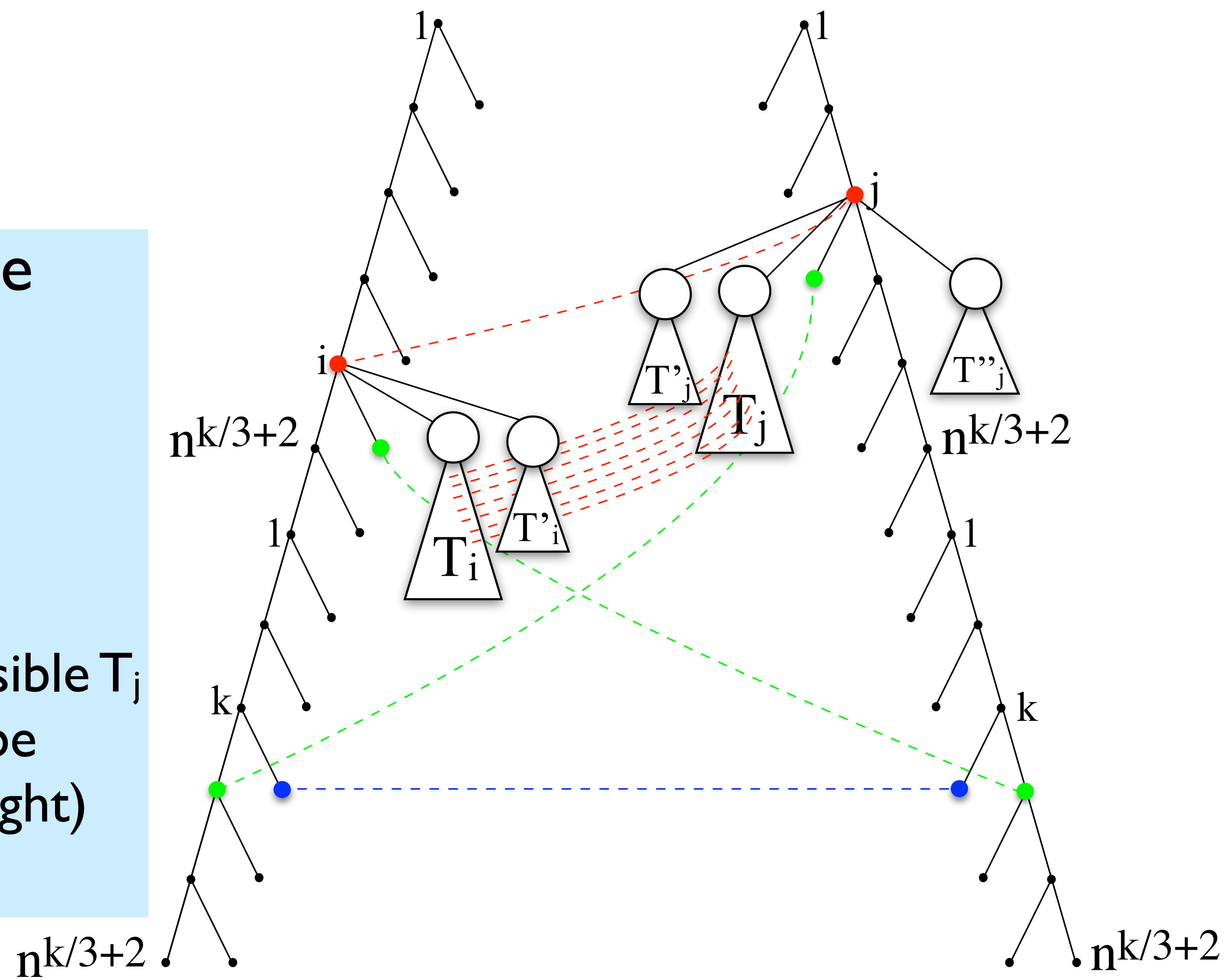
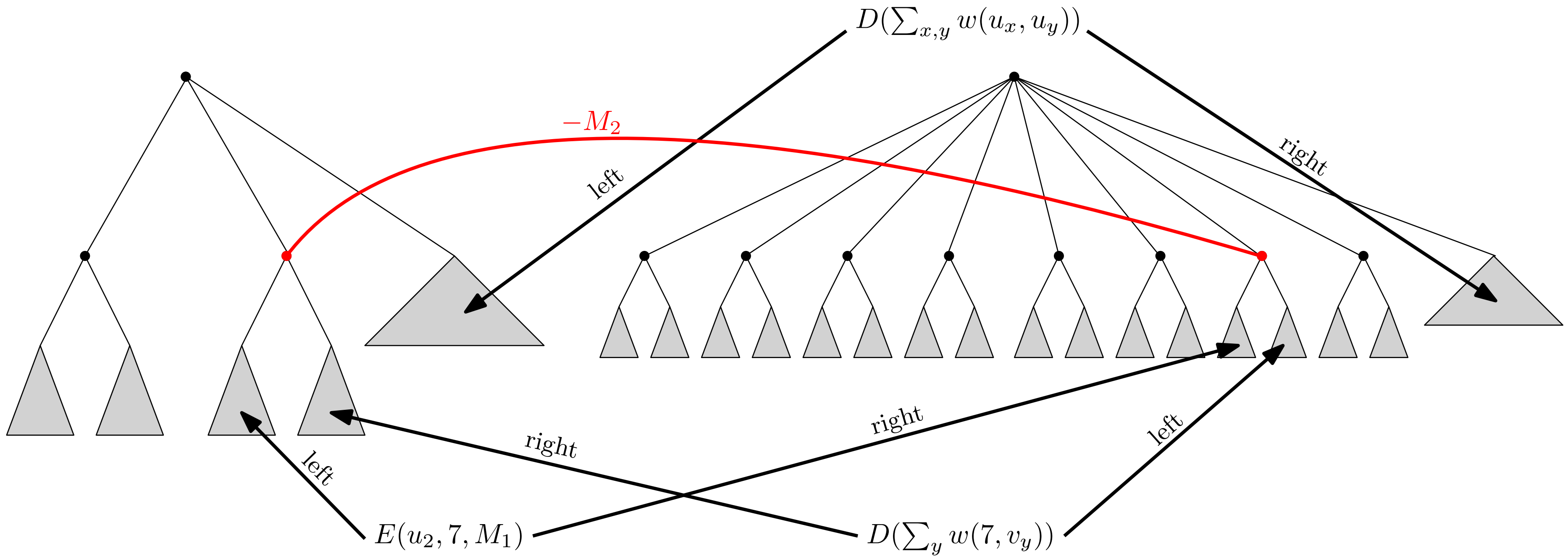
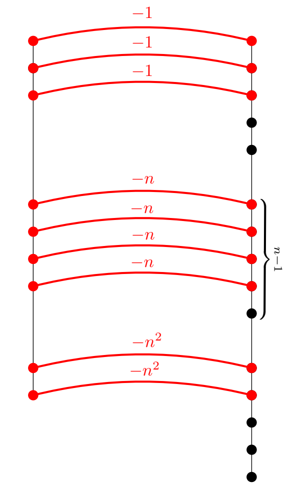
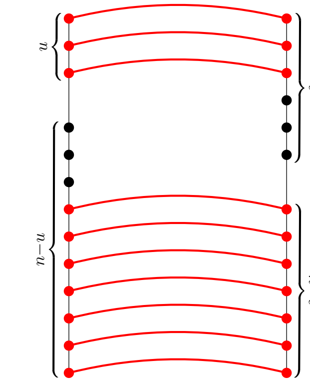








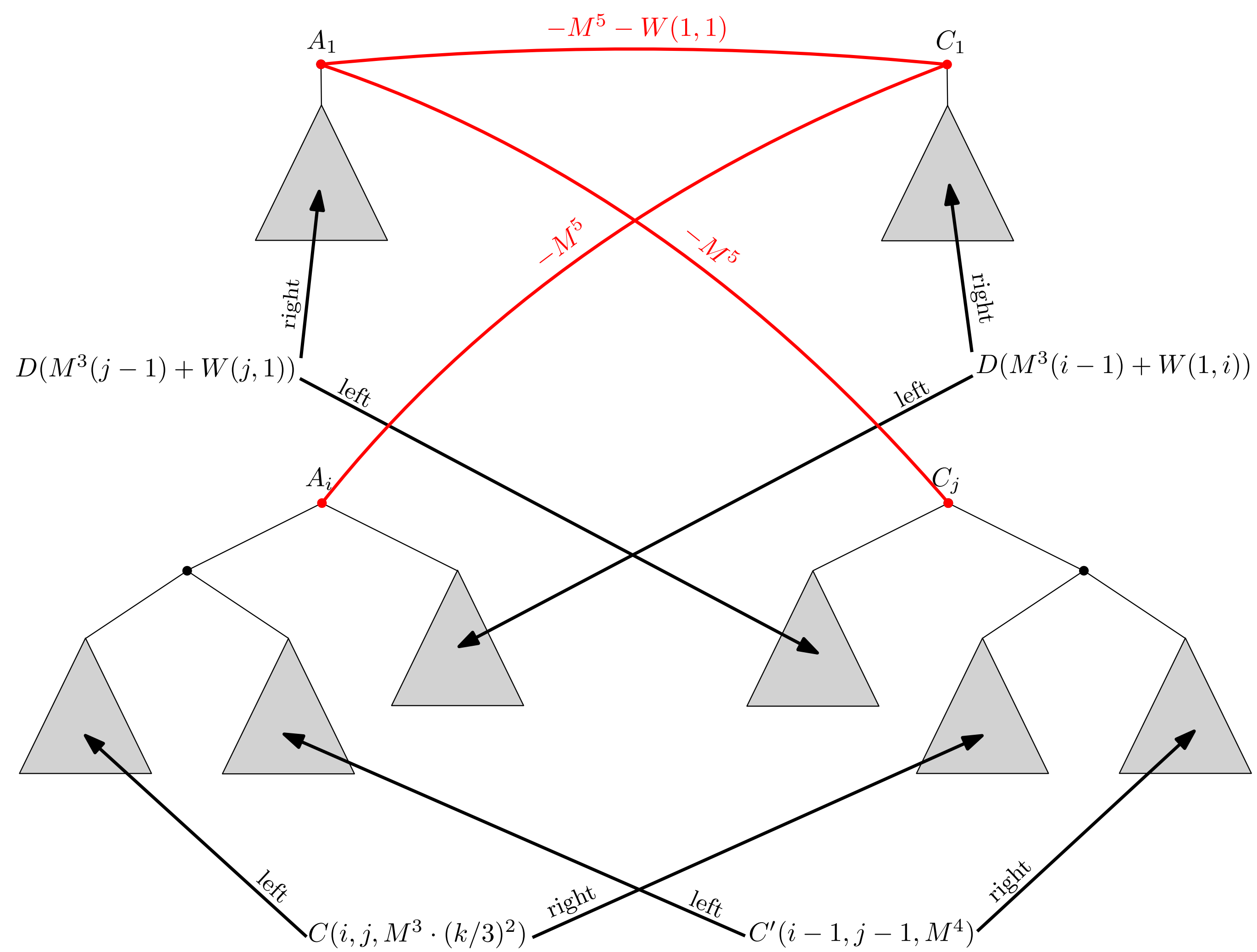
1. $c_{\text{match}}(A'_i, D_{z'}) = -M^6 - M^3(N-i) - W(i, z')$ for every $i = 1, 2, \ldots, N$ and $z' = 1, 2, \ldots, N$,
2. $c_{\text{match}}(B_z, C'_j) = -M^6 - M^3(N-j) - W(z, j)$ for every $z = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, N$.
3. $c_{\text{match}}(A_i, C_j) = -M^2 - W(j, i) + W(j-1, i-1)$ for every $i = 2, 3, \ldots, N$ and $j = 2, 3, \ldots, N$.
4. $c_{\text{match}}(A_i, C_1) = -M^5 - M^3(i-1) - W(1, i)$ for every $i = 1, 2, \ldots, N$,
5. $c_{\text{match}}(A_1, C_j) = -M^5 - M^3(j-1) - W(j, 1)$ for every $j = 1, 2, \ldots, N$.

Lemma 5. *For sufficiently large M, the total cost of an optimal matching is*

$$-M^8 \cdot 2 - M^7 \cdot 2(N-1) - M^6 \cdot 2 - M^5 - M^3 \cdot 2N + M^2 - \max_{i,j,z}\{W(i,z) + W(z,j) + W(j,i)\}.$$

Proof. Consider i, j, z maximizing $W(i,z) + W(z,j) + W(j,i)$. We may assume that $i \geq j$. Then, it is possible to choose the following matching:

1. b_k to c'_j with cost $-M^8$,
2. some nodes from the copy of I being the left child of c'_j to some spine nodes below b_z with total cost $-M^7(N-z)$,
3. a'_i to d_k with cost $-M^8$,
4. some nodes from the copy of I being d of a'_i to some spine nodes below d_z with total cost $-M^7(N-z)$,
5. b'_1 to d'_{z-1}, b'_2 to d'_{z-2}, …, b'_{z-1} to d'_1 with cost $-M^7 \cdot 2$ each,
6. a_i to c_j, a_{i-1} to c_{j-1}, …, a_{i-j+1} to c_1 with cost $-M^3 \cdot 2 + M^2$ each,
7. A'_i to D_z with cost $-M^6 - M^3(N-i) - W(i,z)$,
8. B_z to C'_j with cost $-M^6 - M^3(N-j) - W(z,j)$,
9. A_i to C_j, A_{i-1} to C_{j-1}, …, A_{i-j+2} to C_2 with costs $-M^2 - W(j,i) + W(j-1,i-1)$, $-M^2 - W(j-1,i-1) + W(j-2,i-2)$, …, $-M^2 - W(2, i-j+2) + W(1, i-j+1)$.
10. A_{i-j+1} to C_1 with cost $-M^5 - M^3(i-j) - W(1, i-j+1)$.



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Thank You!