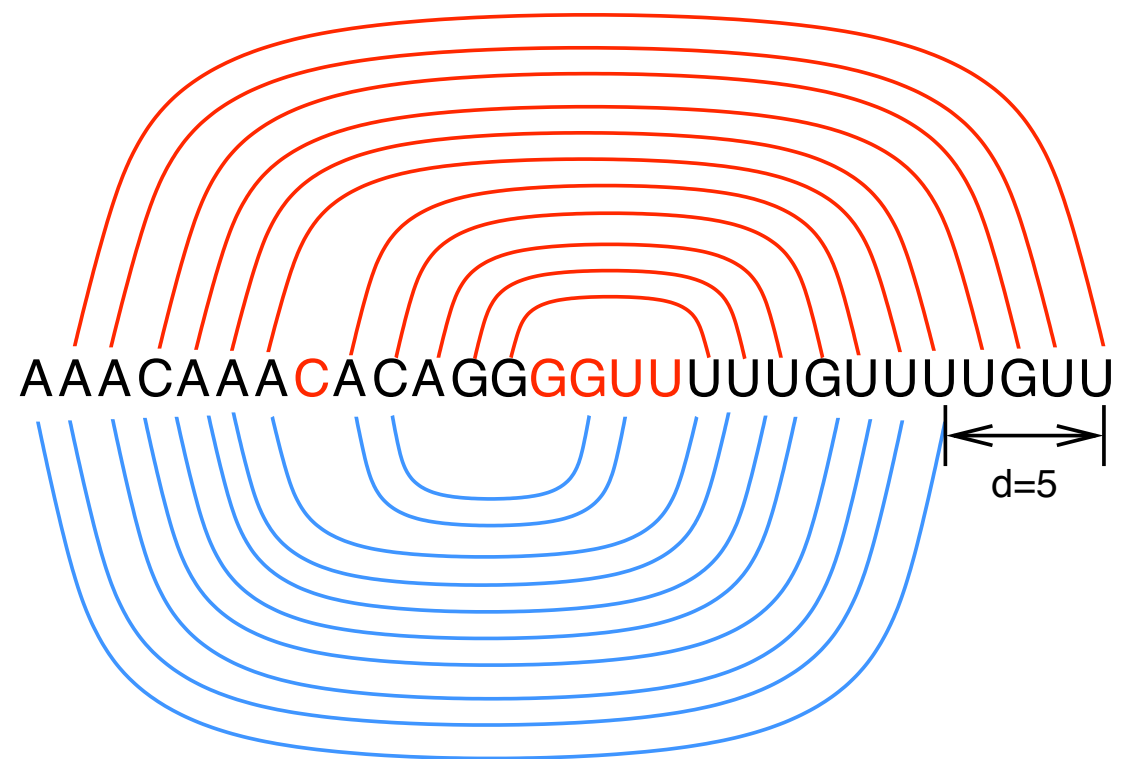


Fast RNA Structure Alignment for Crossing Input Structures



Rolf Backofen
Gad M. Landau
Mathias Möhl
Dekel Tsur
Oren Weimann

String Edit Distance

C o m b i n a t o r i a l O p t i m i s a t i o n

O p t i m a t o r i a l C o m b i n i s a t i o n

String Edit Distance

C o m b i n a t o r i a l O p t i m i s a t i o n

O p t i m a t o r i a l C o m b i n i s a t i o n

String Edit Distance

C o m b i n a t o r i a l O p t i m i s a t i o n

O p t i m a t o r i a l C o m b i n i s a t i o n

String Edit Distance

C o m b i n a t o r i a l O p t i m i s a t i o n

O p t i m a t o r i a l C o m b i n i s a t i o n

String Edit Distance

C o m b i n a t o r i a l O p t i m i s a t i o n

O p t i m a t o r i a l C o m b i n i s a t i o n

Every two prefixes

String Edit Distance

C o m b i n a t o r i a l O p t i m i s a t i o n

O p t i m a t o r i a l C o m b i n i s a t i o n

Every two prefixes  $O(n^2)$

String Edit Distance

C o m b i n a t o r i a l O p t i m i s a t i o n

O p t i m a t o r i a l C o m b i n i s a t i o n

Every two infixes

String Edit Distance

C o m b i n a t o r i a l O p t i m i s a t i o n

O p t i m a t o r i a l C o m b i n i s a t i o n

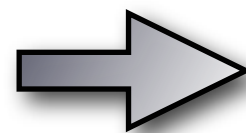
Every two infixes

String Edit Distance

C o m b i n a t o r i a l O p t i m i s a t i o n

O p t i m a t o r i a l C o m b i n i s a t i o n

Every two infixes



$O(n^4)$

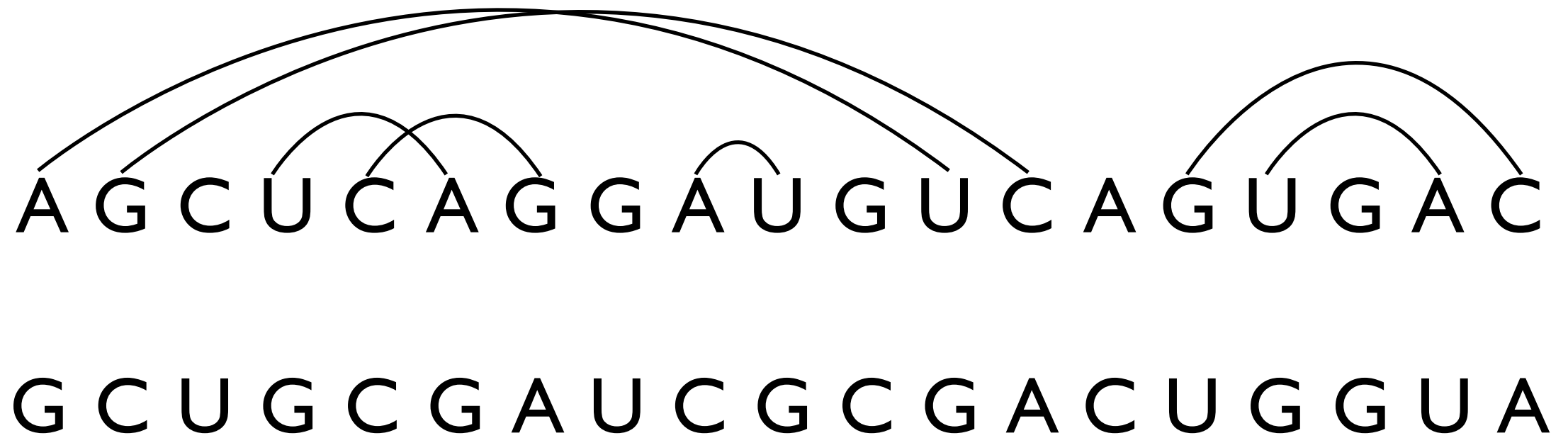


RNA Edit Distance

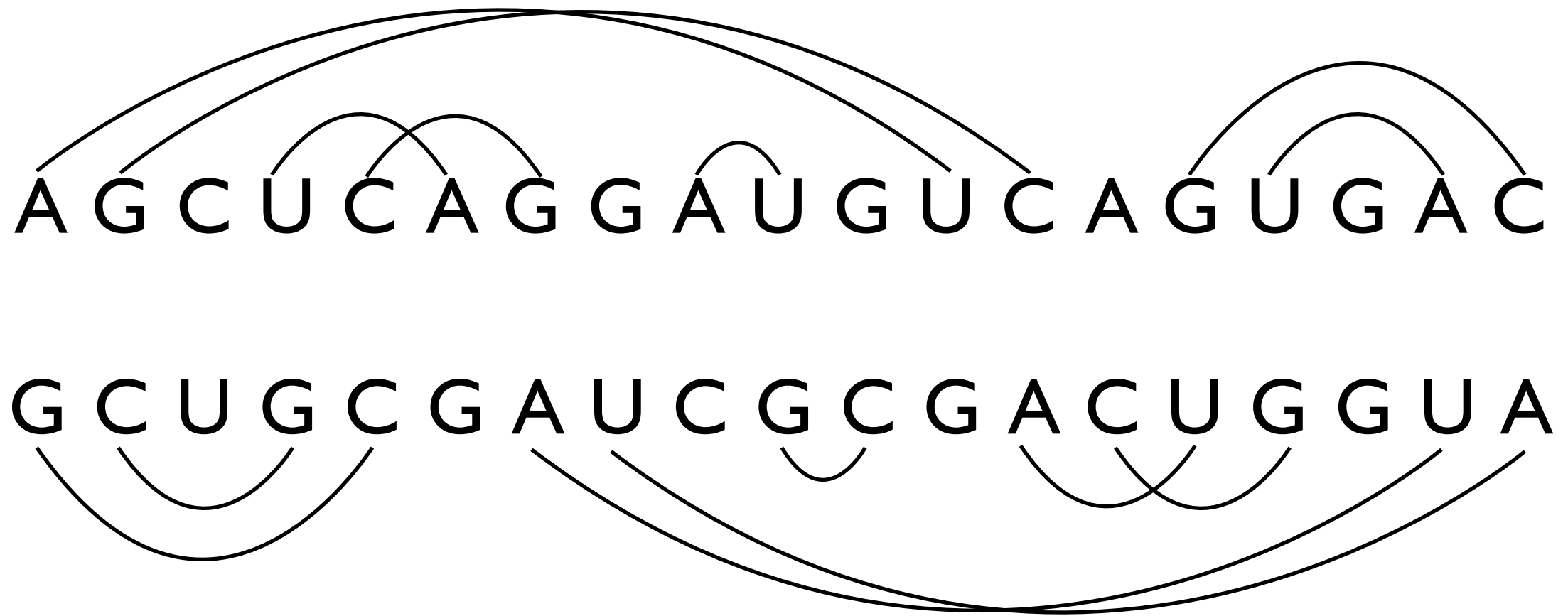
A G C U C A G G A U G U C A G U G A C

G C U G C G A U C G C G A C U G G U A

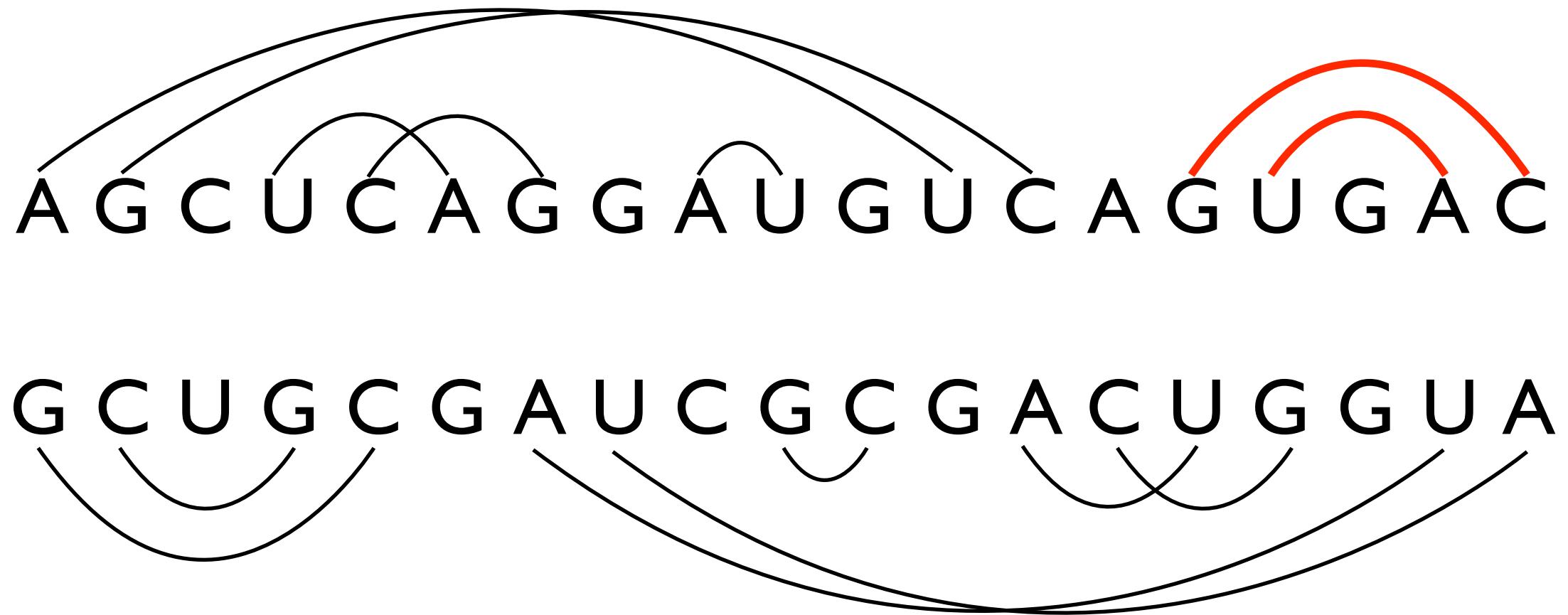
RNA Edit Distance



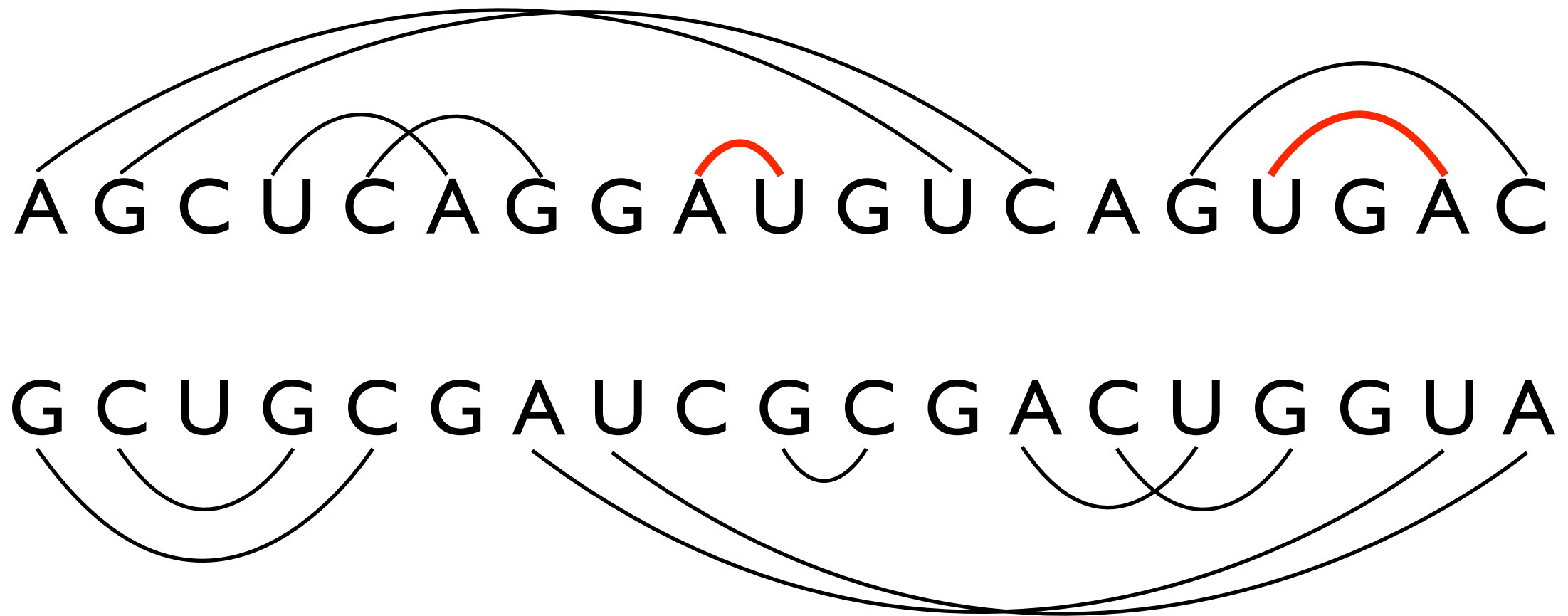
RNA Edit Distance



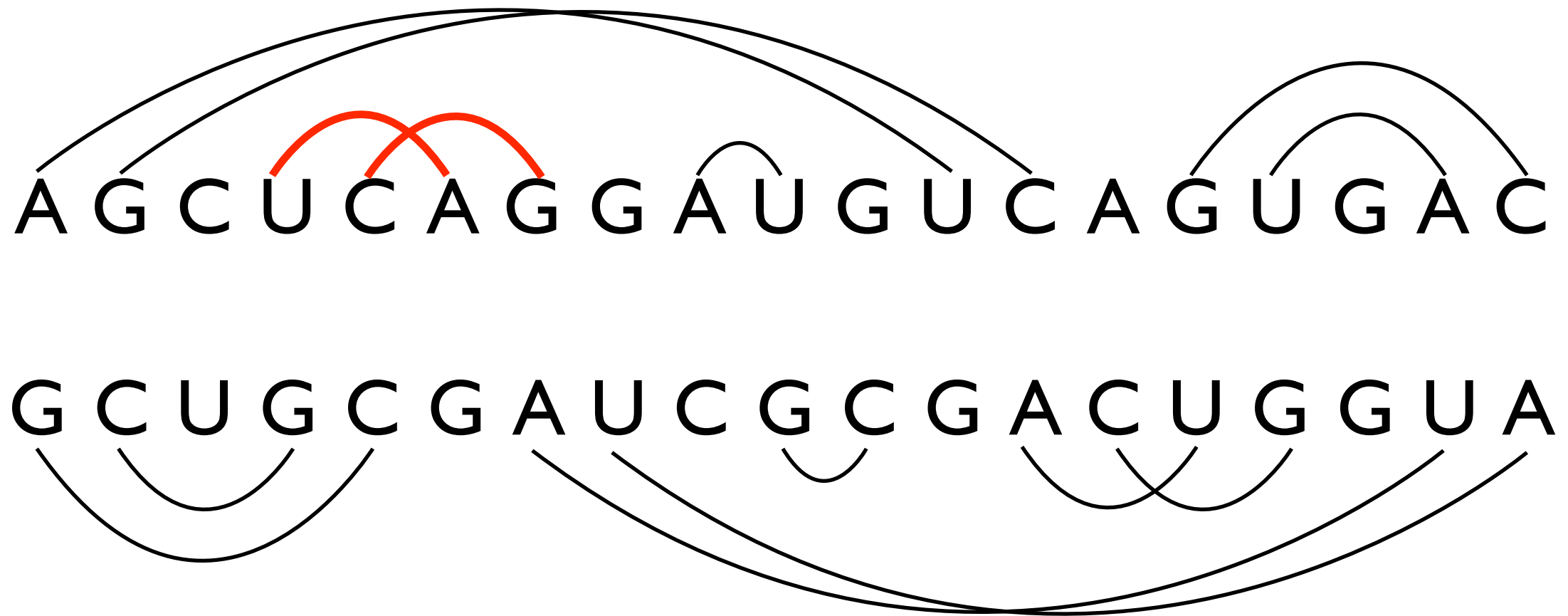
RNA Edit Distance



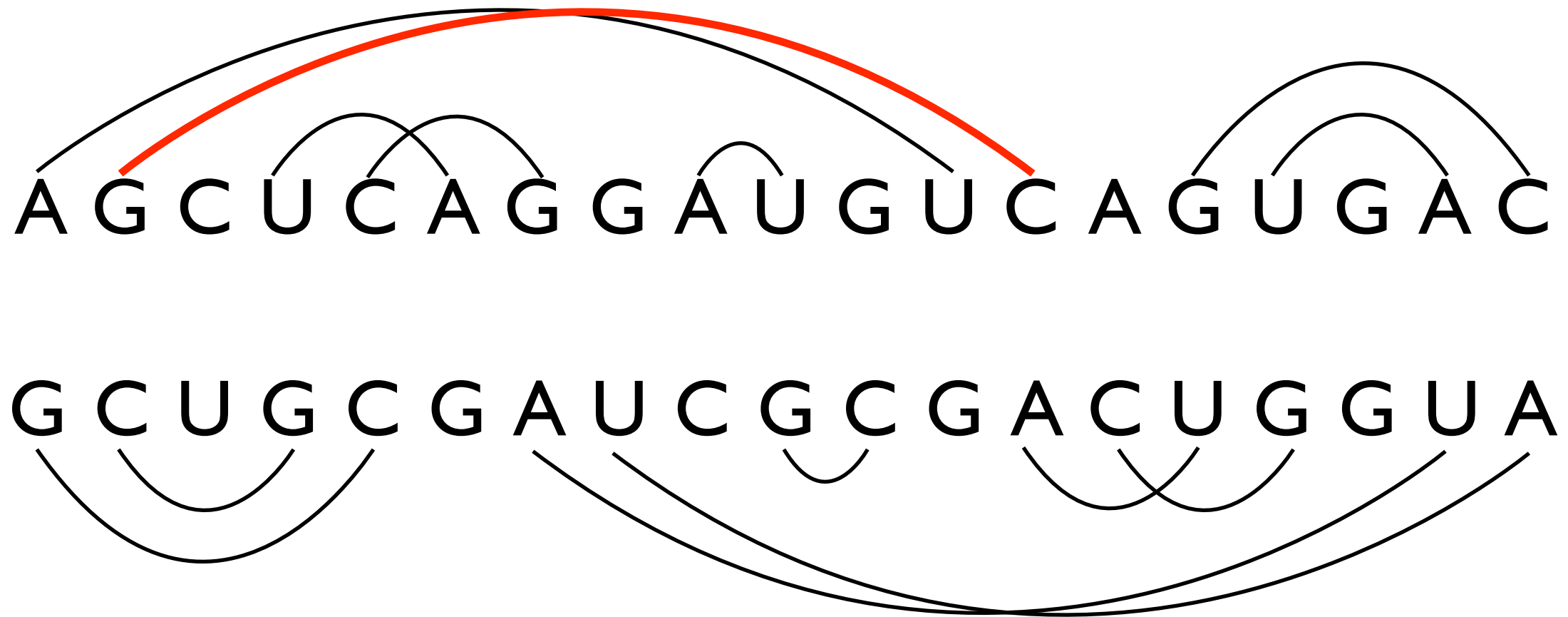
RNA Edit Distance



RNA Edit Distance

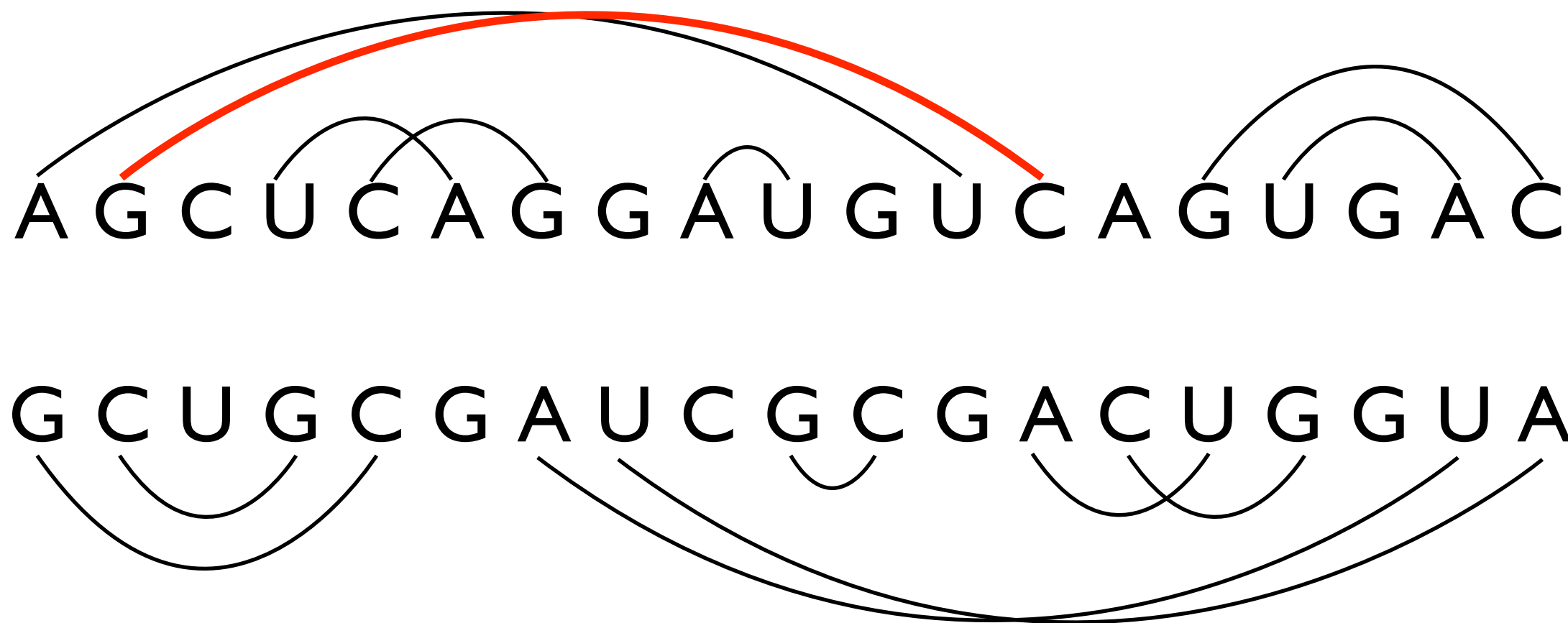


RNA Edit Distance



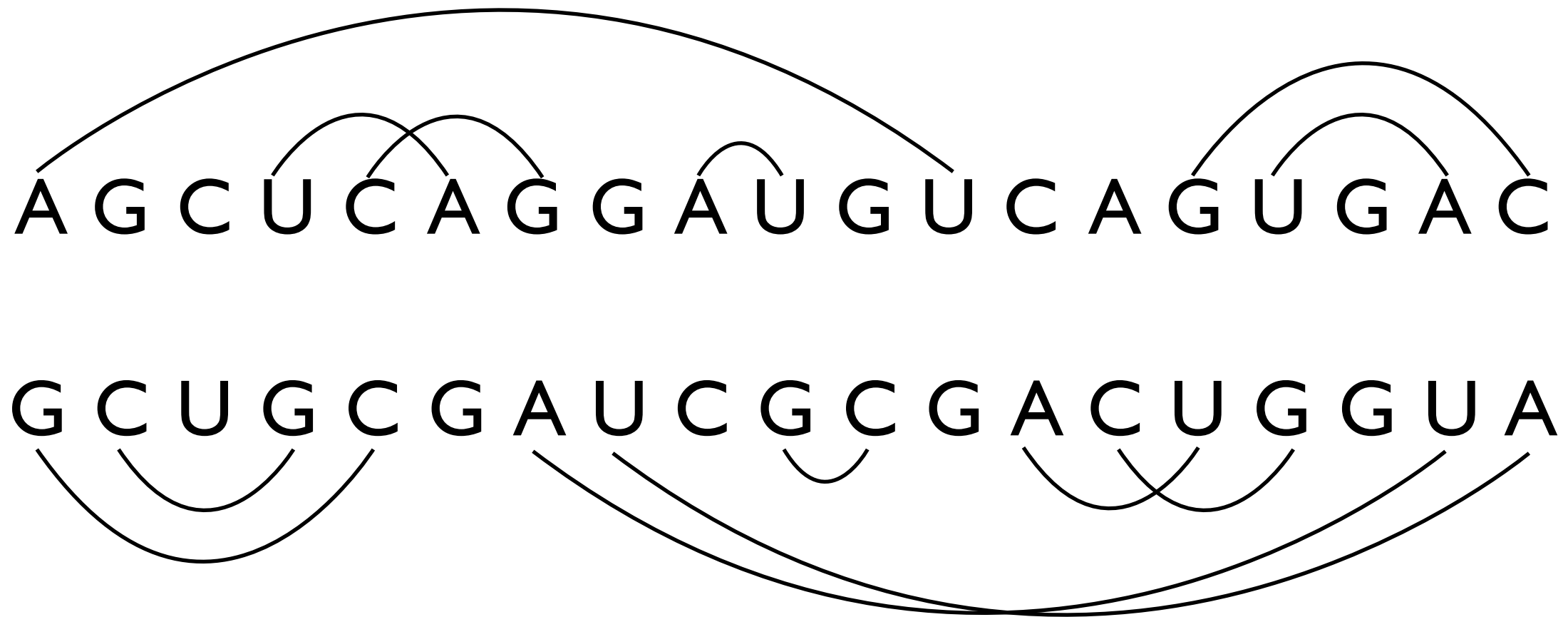
Additional edit operations:

RNA Edit Distance



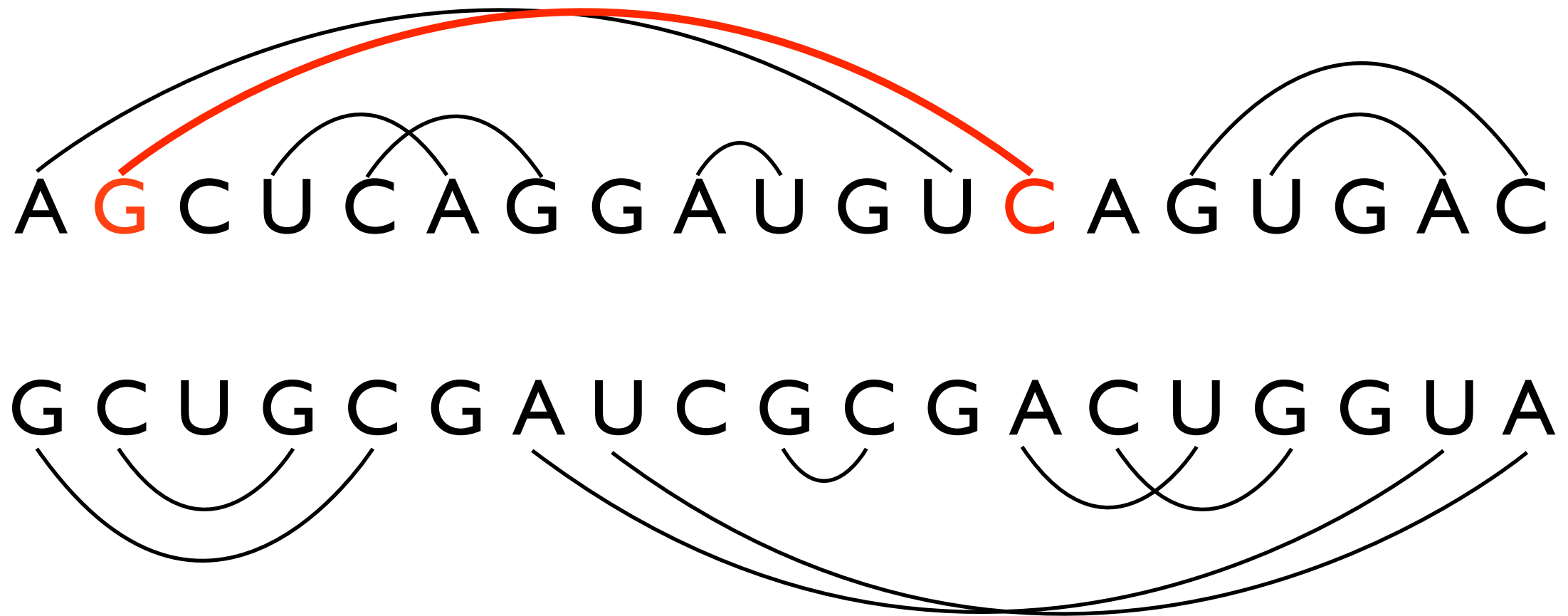
Additional edit operations: Arc Deletion / Insertion

RNA Edit Distance



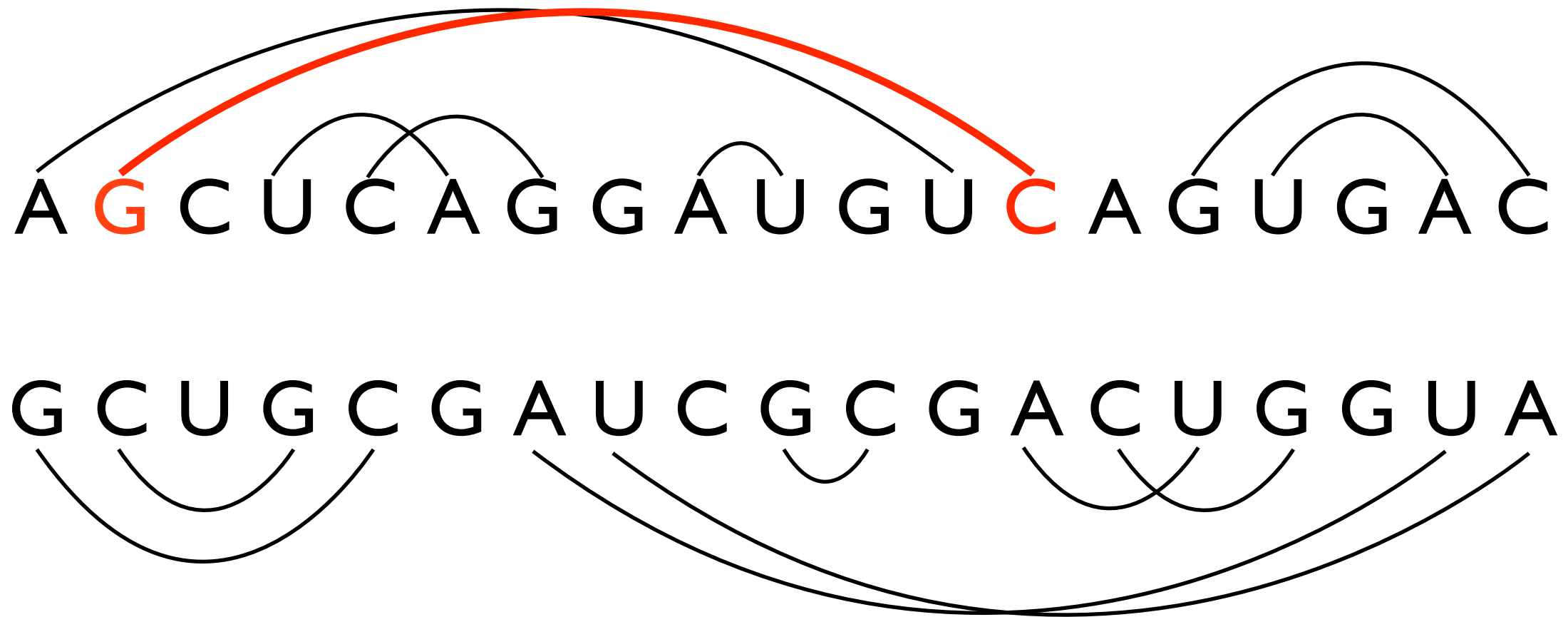
Additional edit operations: Arc Deletion / Insertion

RNA Edit Distance



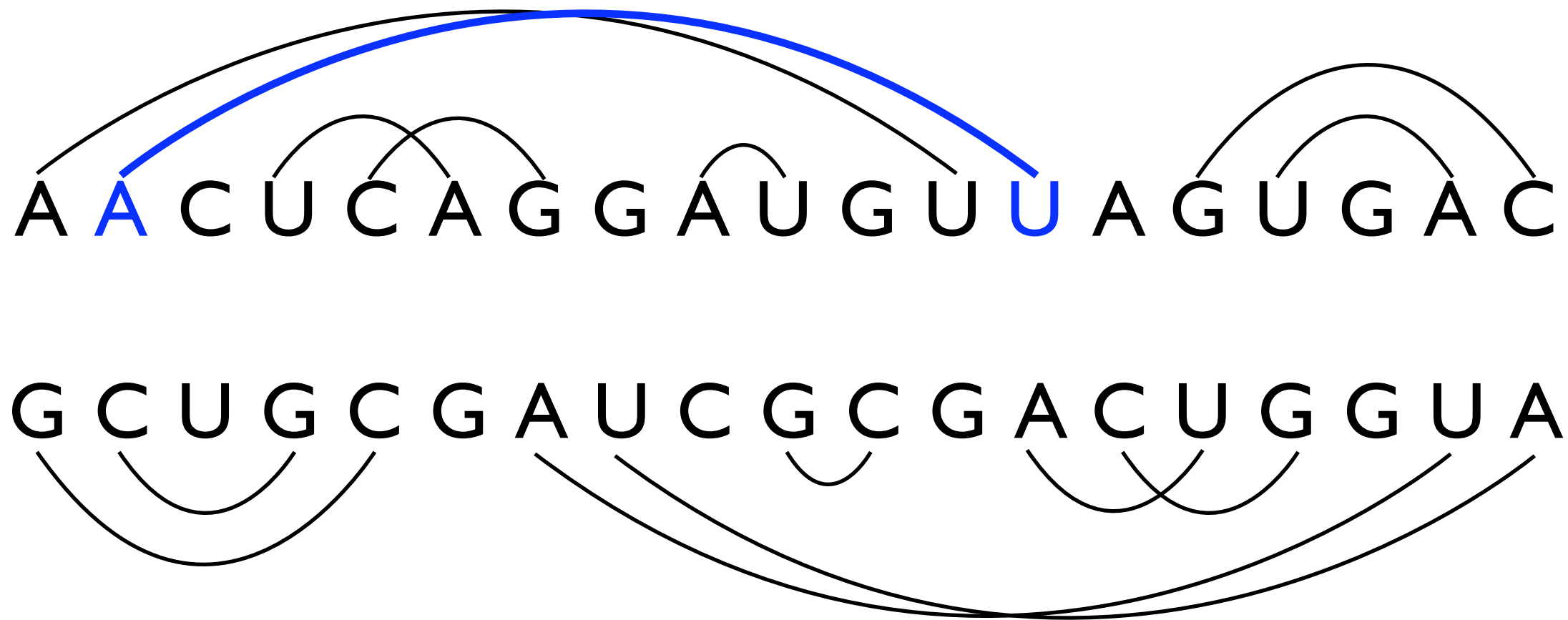
Additional edit operations:

RNA Edit Distance



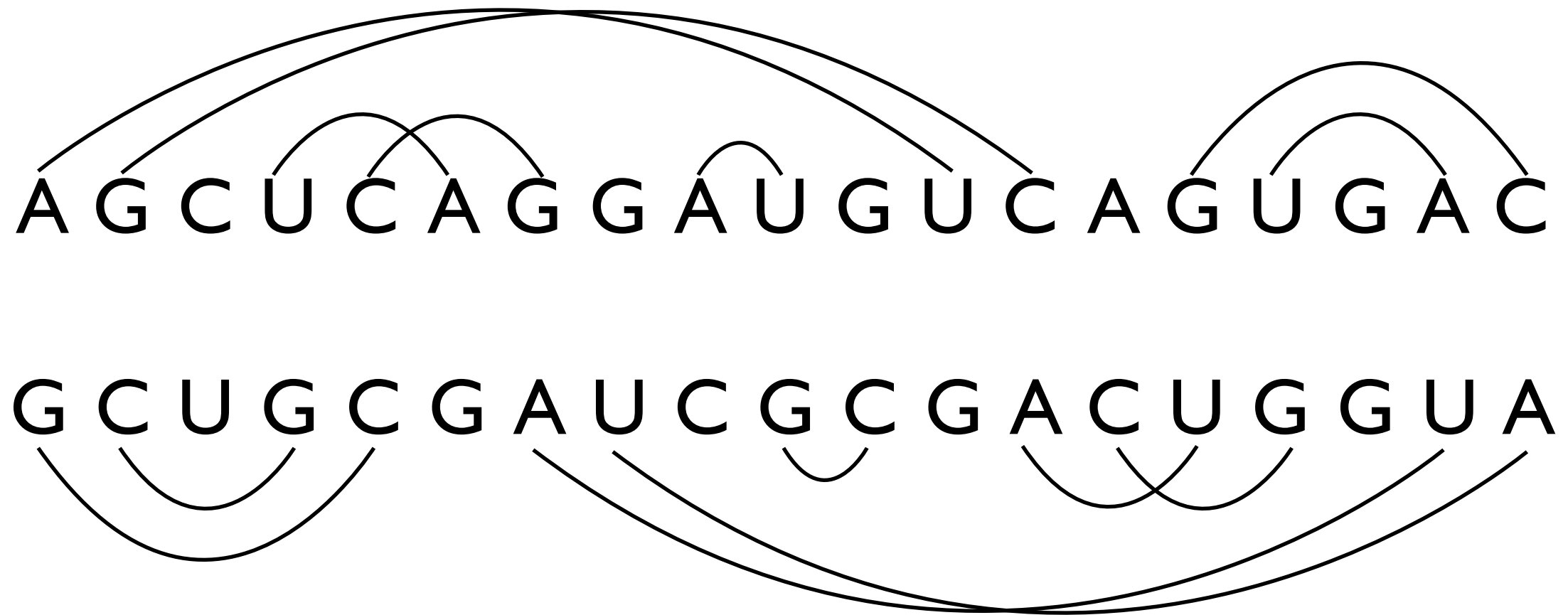
Additional edit operations: Arc Match / Relabel

RNA Edit Distance

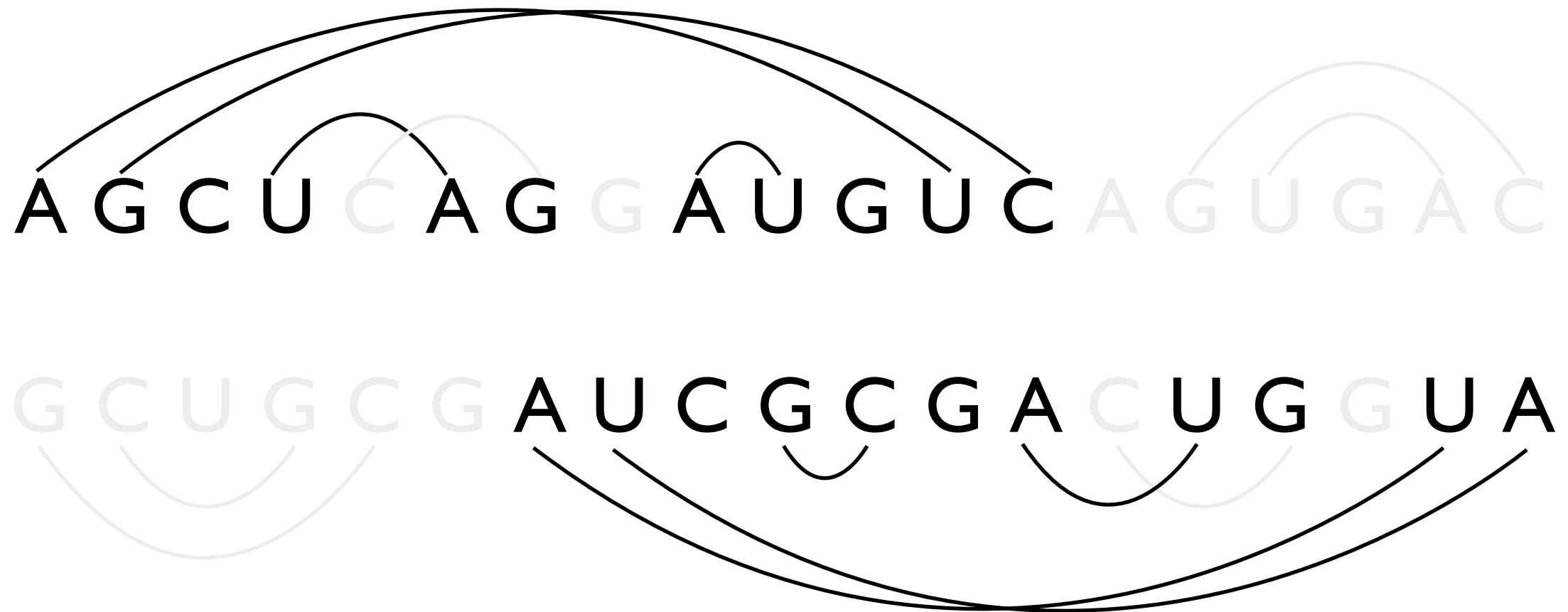


Additional edit operations: Arc Match / Relabel

RNA Edit Distance

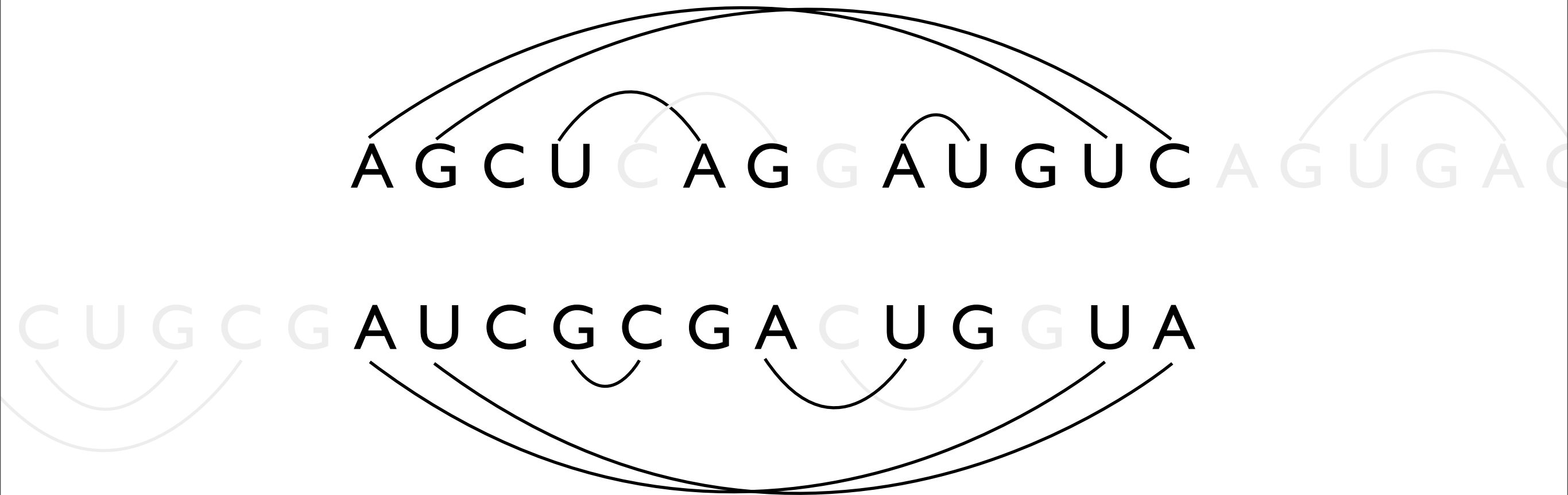


RNA Edit Distance



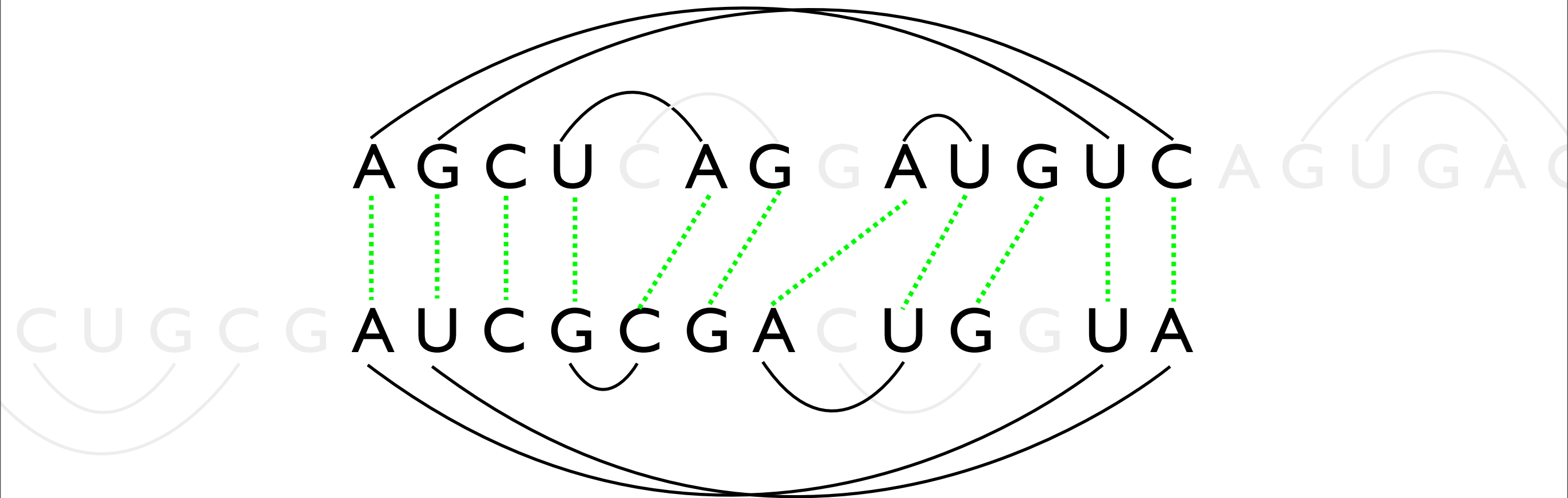
Consensus Structure

RNA Edit Distance



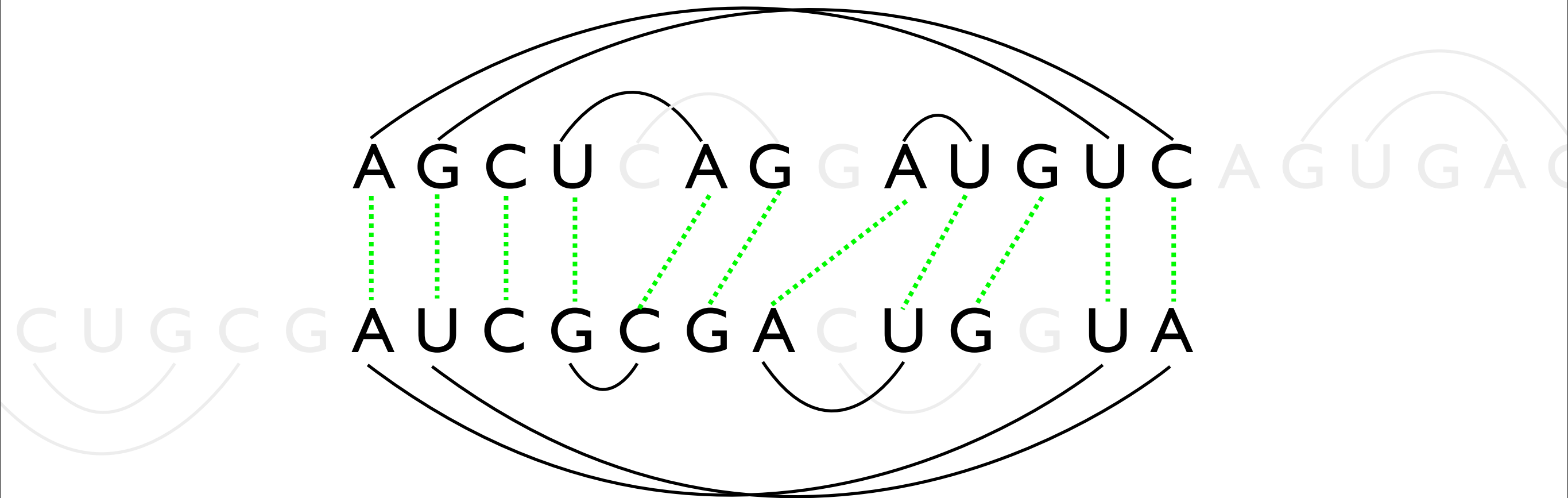
Consensus Structure

RNA Edit Distance



Consensus Structure

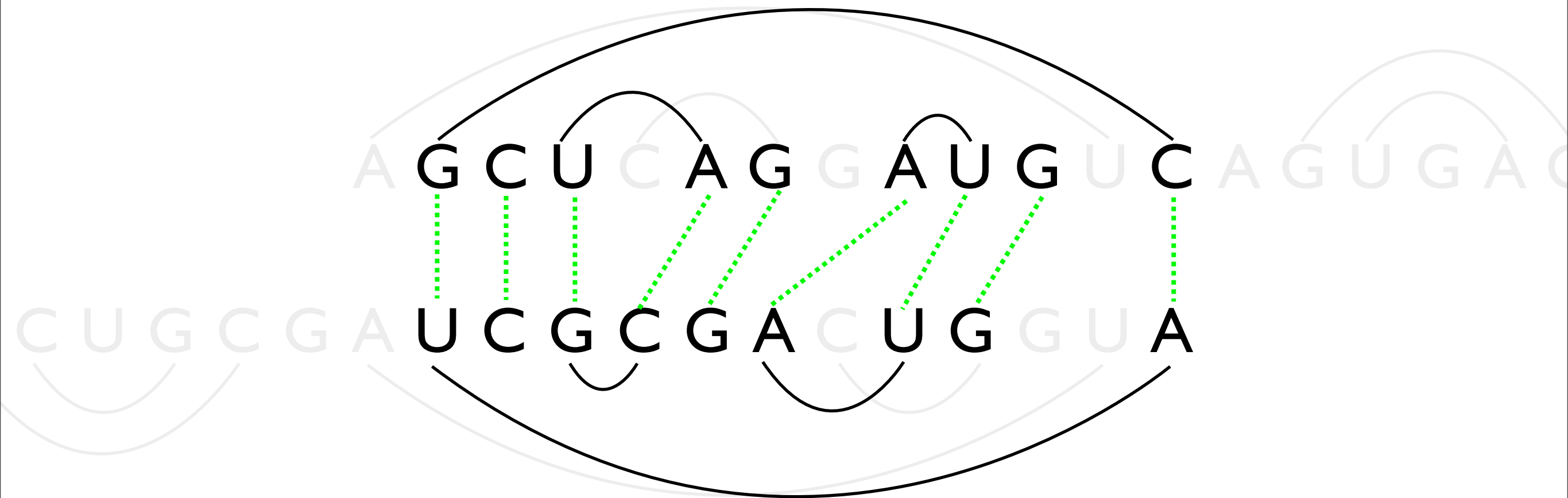
RNA Edit Distance



Arbitrary Consensus Structure: NP-hard



RNA Edit Distance



~~Arbitrary~~ Consensus Structure:
Non-crossing

[Jiang *et al.* 2002]

$O(n^4)$



Non-Crossing Consensus



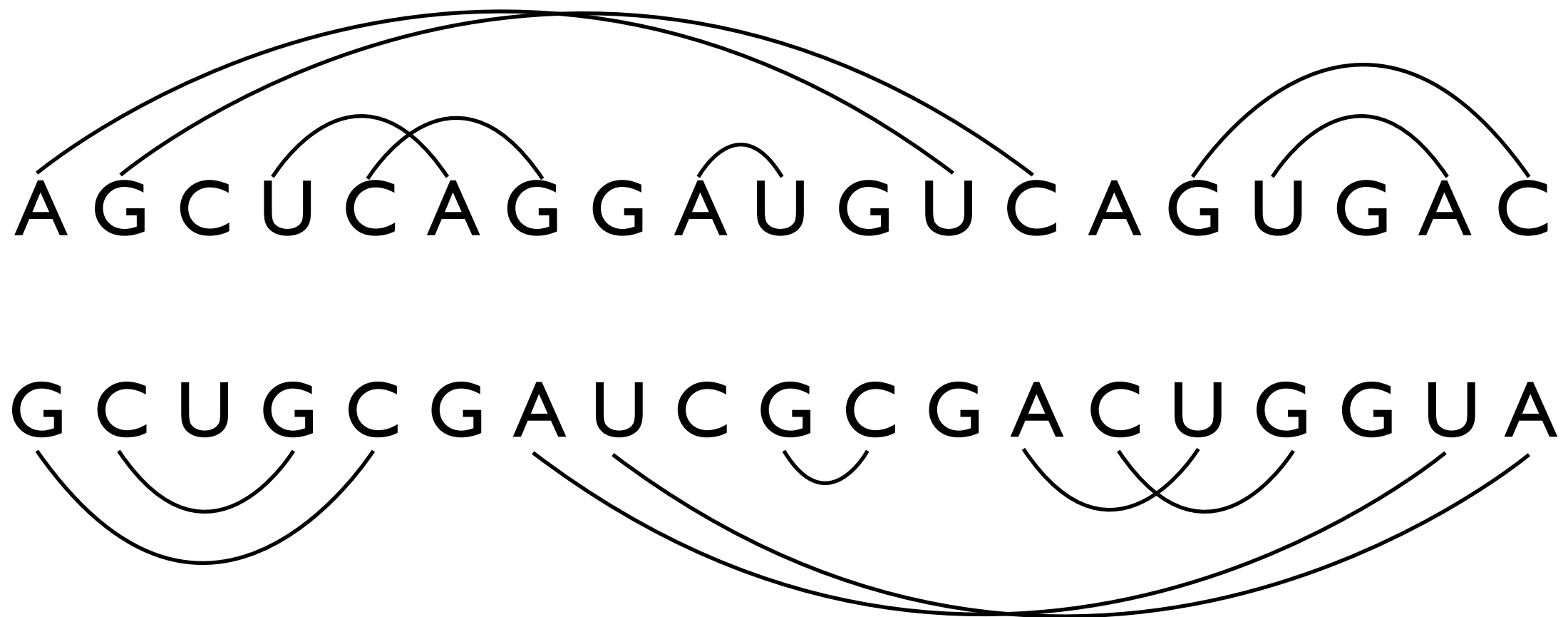
[Jiang *et al.* 2002]

~~Arbitrary~~ Consensus Structure:
Non-crossing

$O(n^4)$



Non-Crossing Consensus



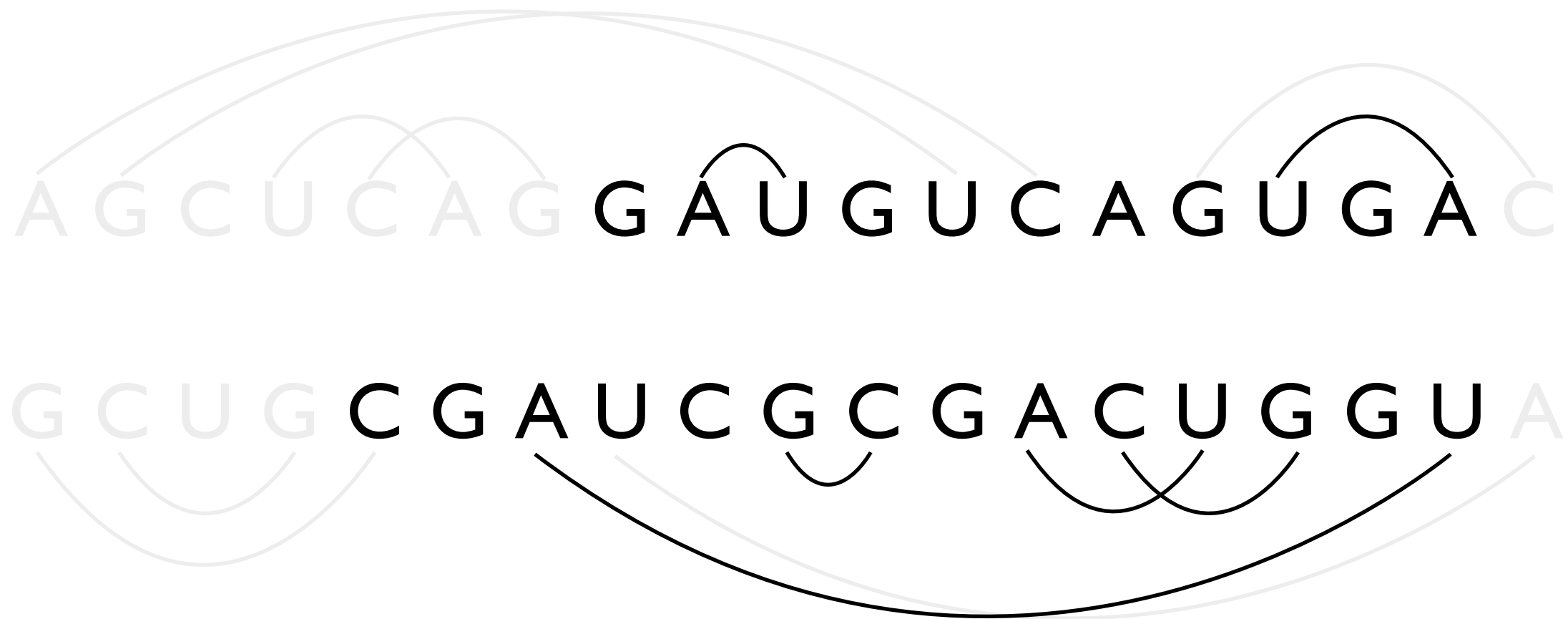
[Jiang *et al.* 2002]

~~Arbitrary~~ Consensus Structure:
Non-crossing

$O(n^4)$

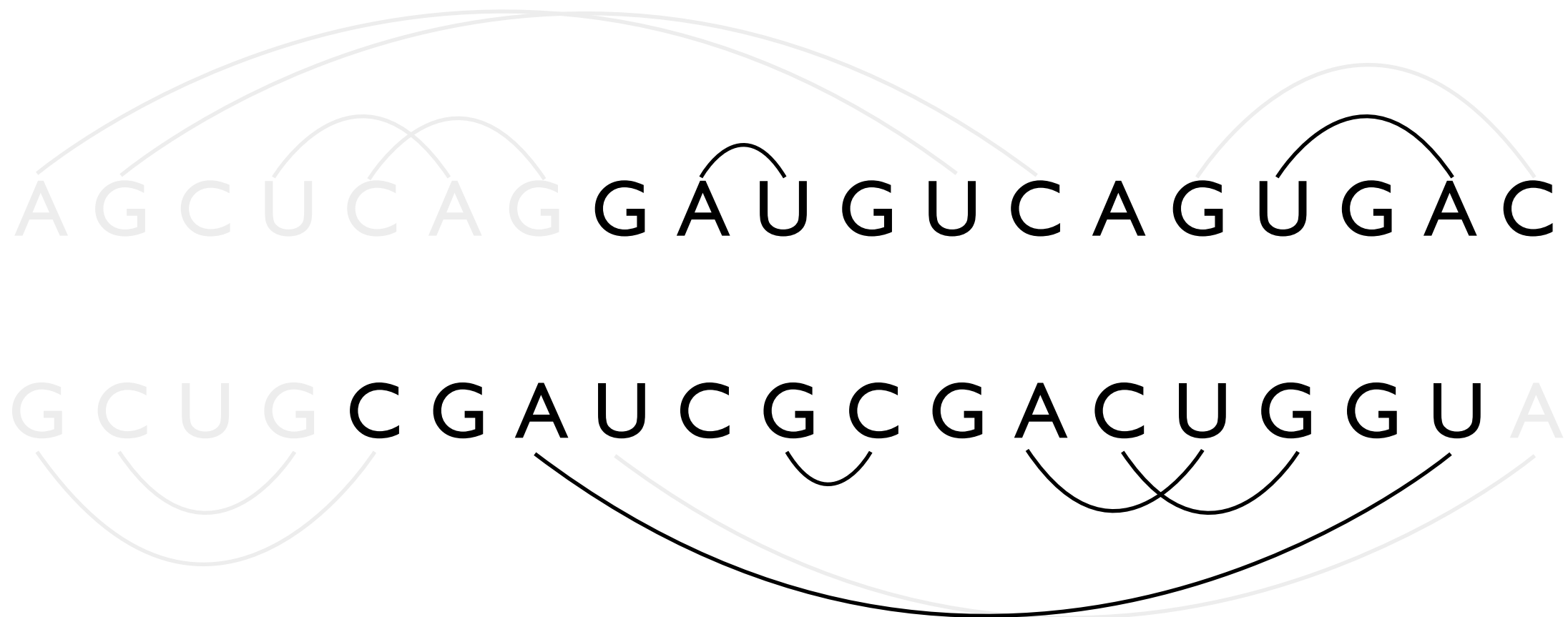


Non-Crossing Consensus



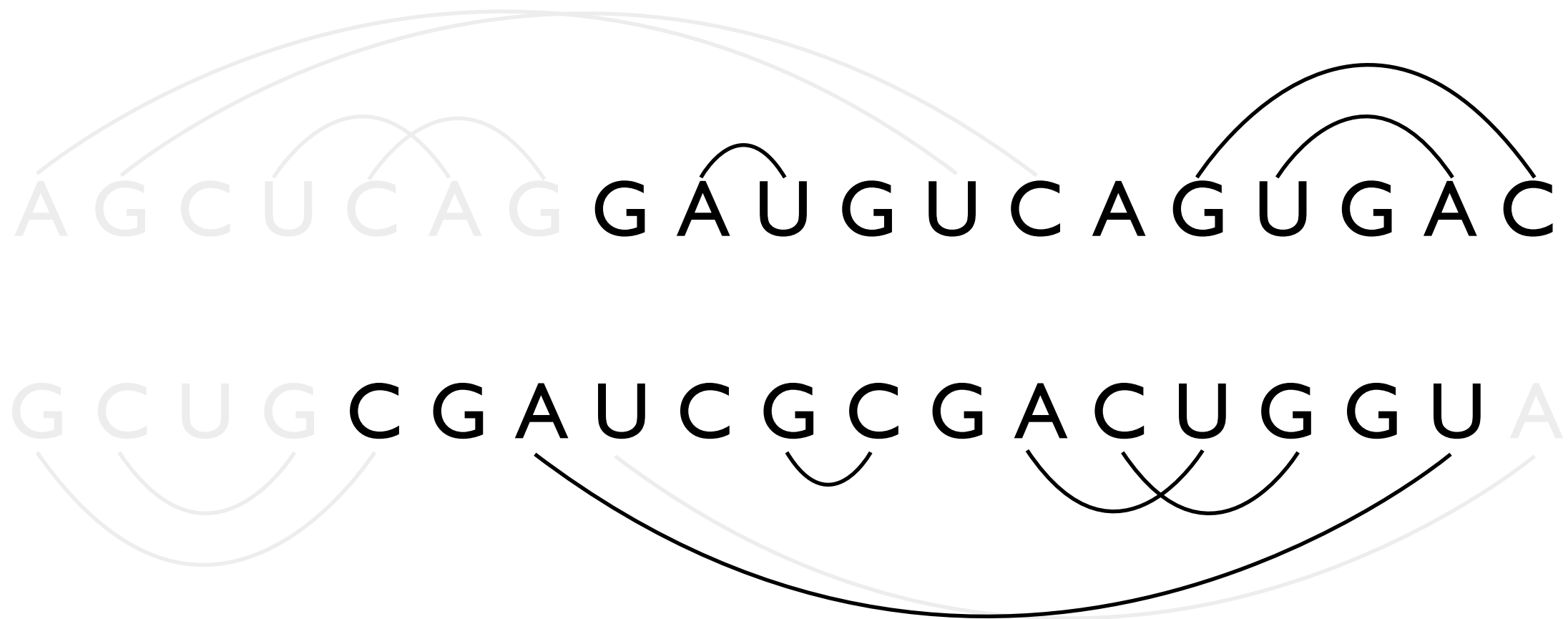
Every two infixes $\rightarrow O(n^4)$

Non-Crossing Consensus



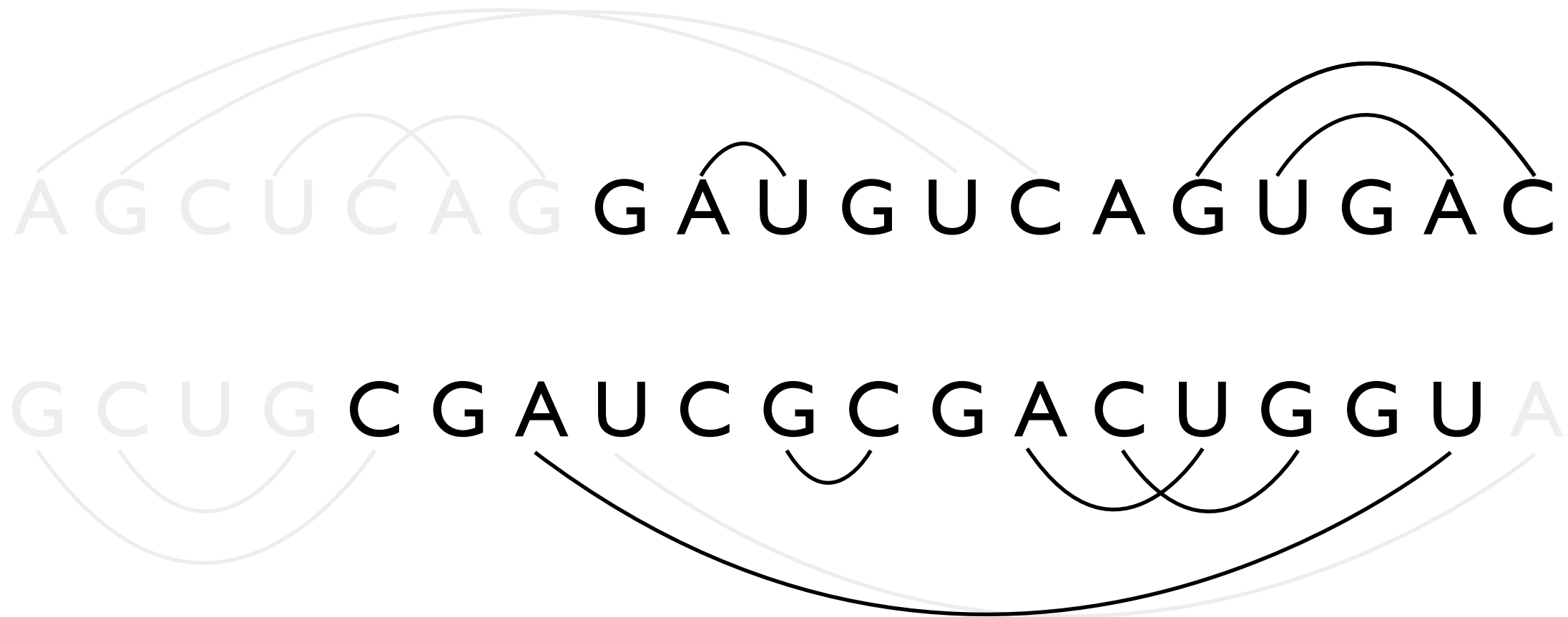
Every two infixes $\rightarrow O(n^4)$

Non-Crossing Consensus



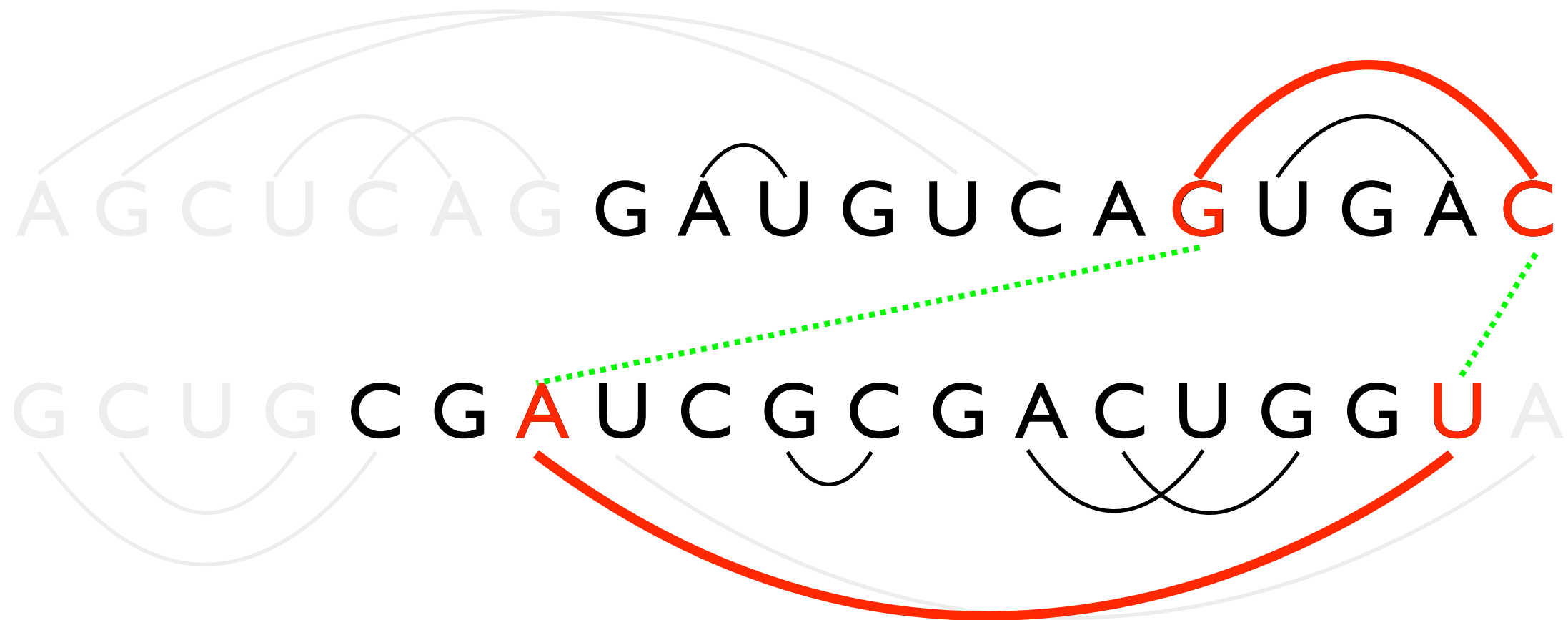
Every two infixes $\rightarrow O(n^4)$

Non-Crossing Consensus



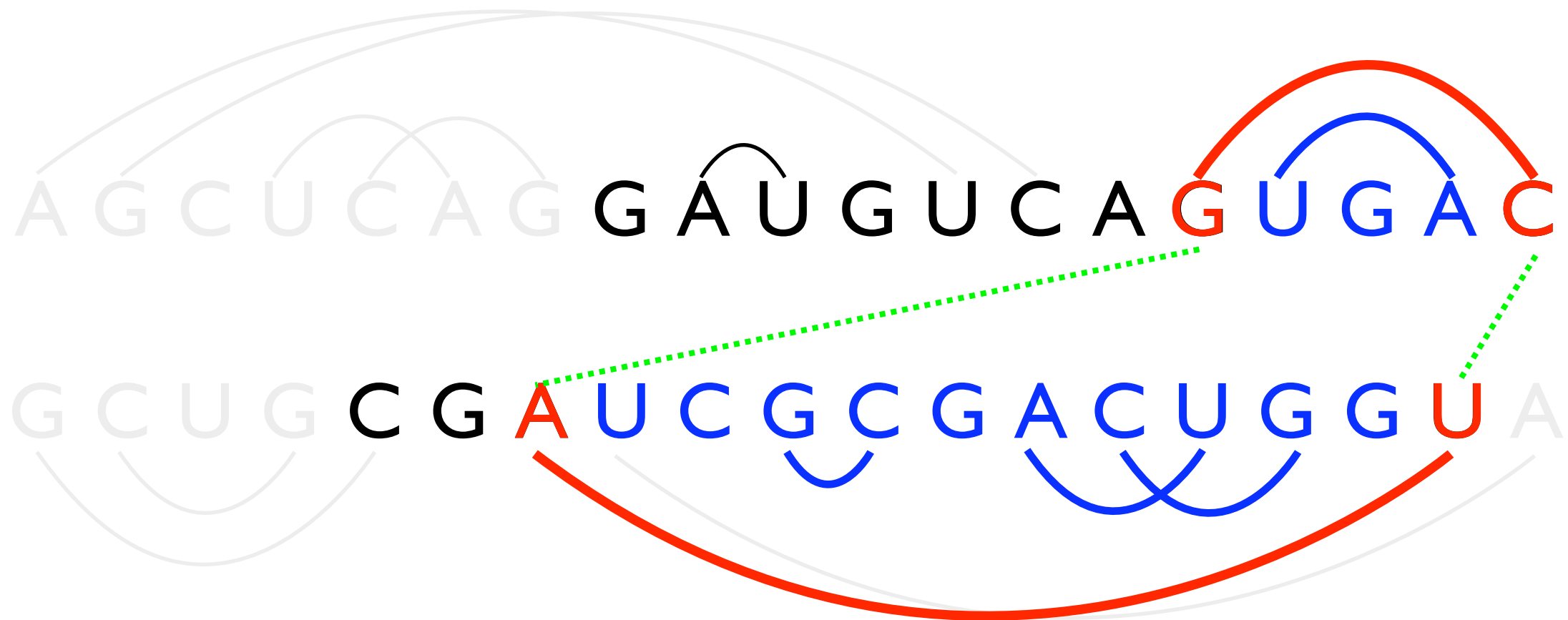
Arc Match / Relabel

Non-Crossing Consensus

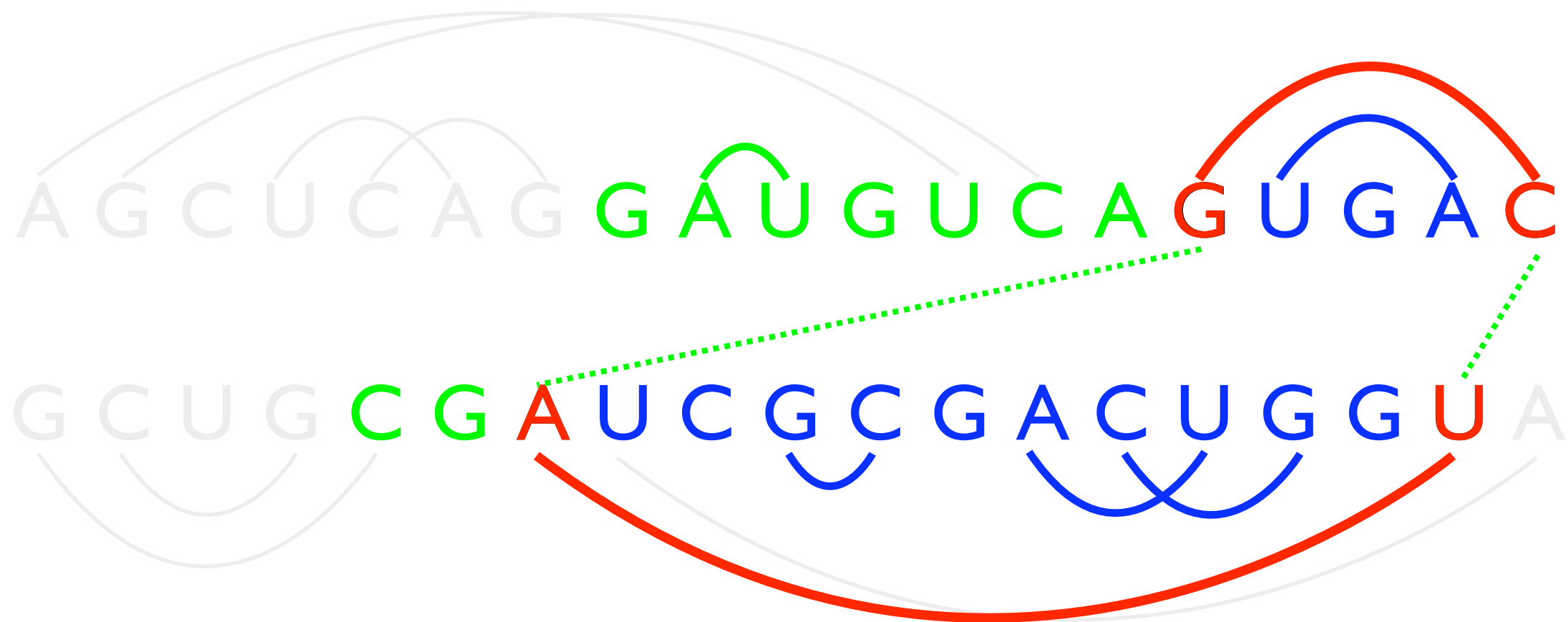


Arc Match / Relabel

Non-Crossing Consensus

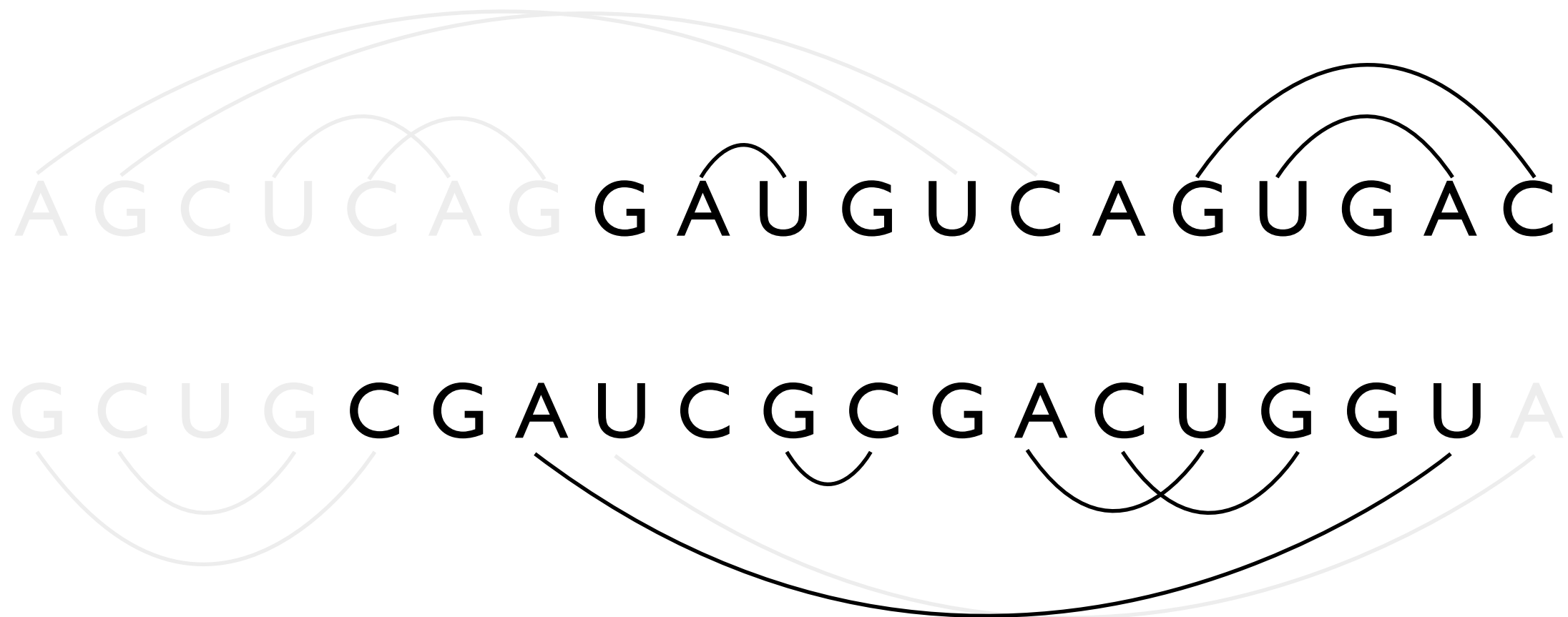


Non-Crossing Consensus



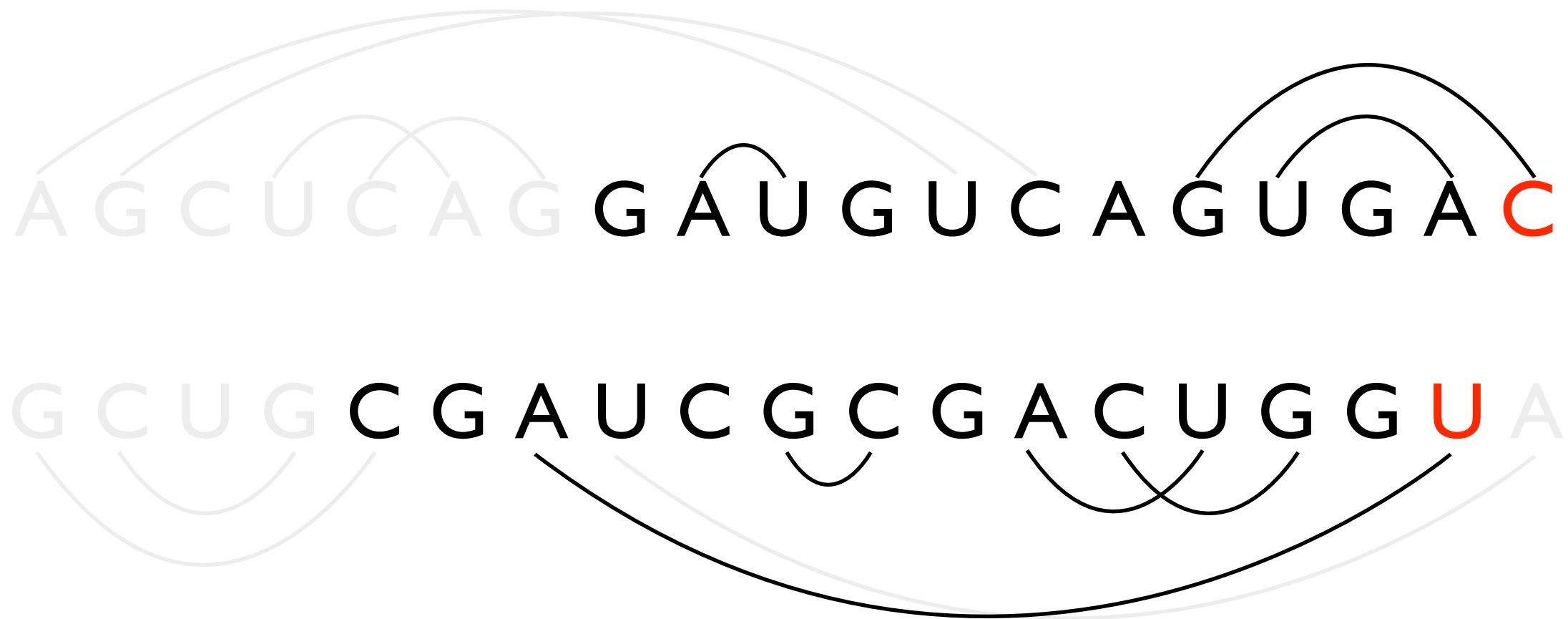
Arc Match / Relabel

Non-Crossing Consensus



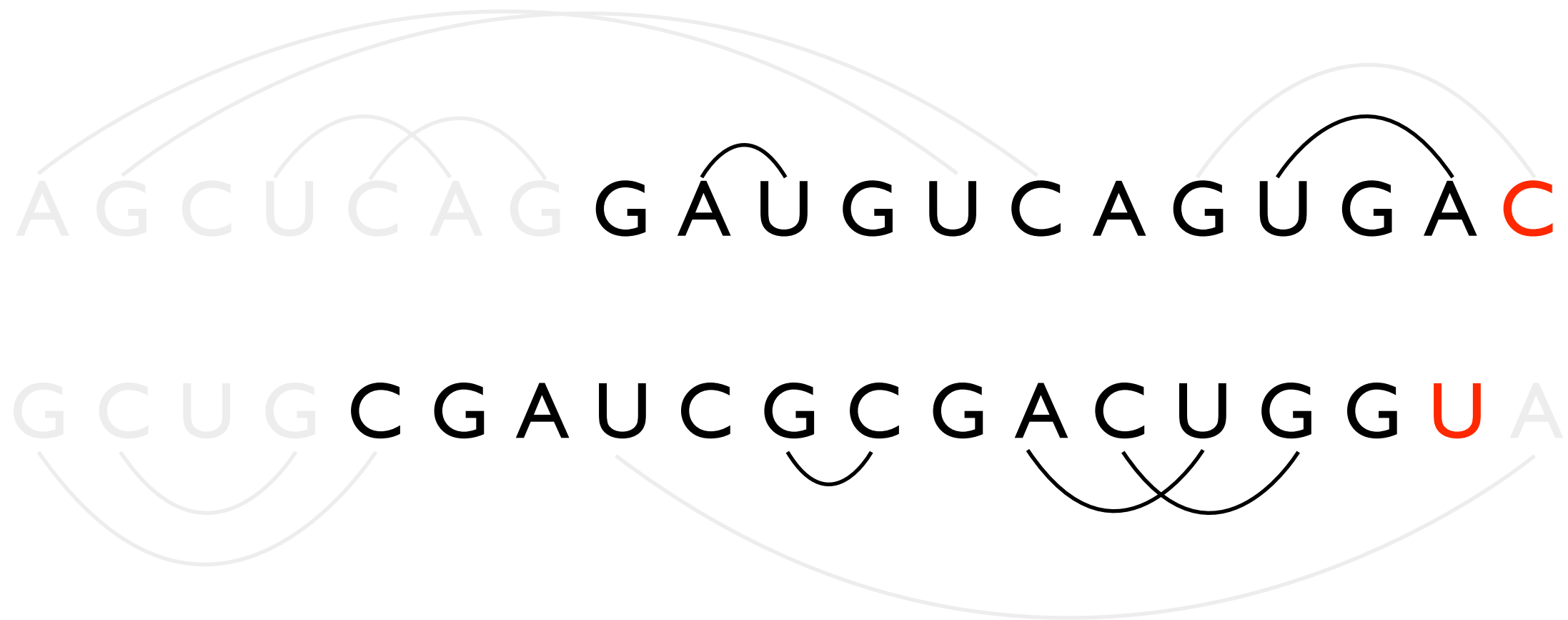
Character Match / Relabel

Non-Crossing Consensus



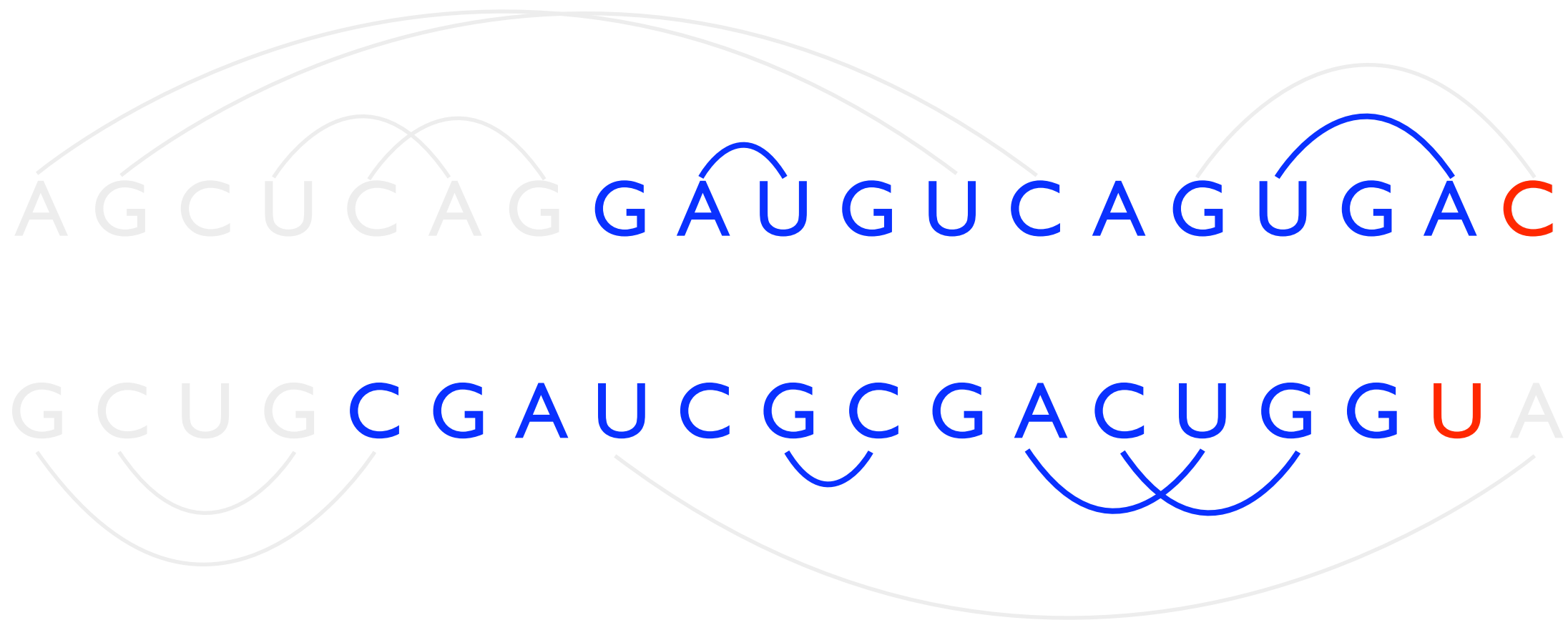
Character Match / Relabel

Non-Crossing Consensus



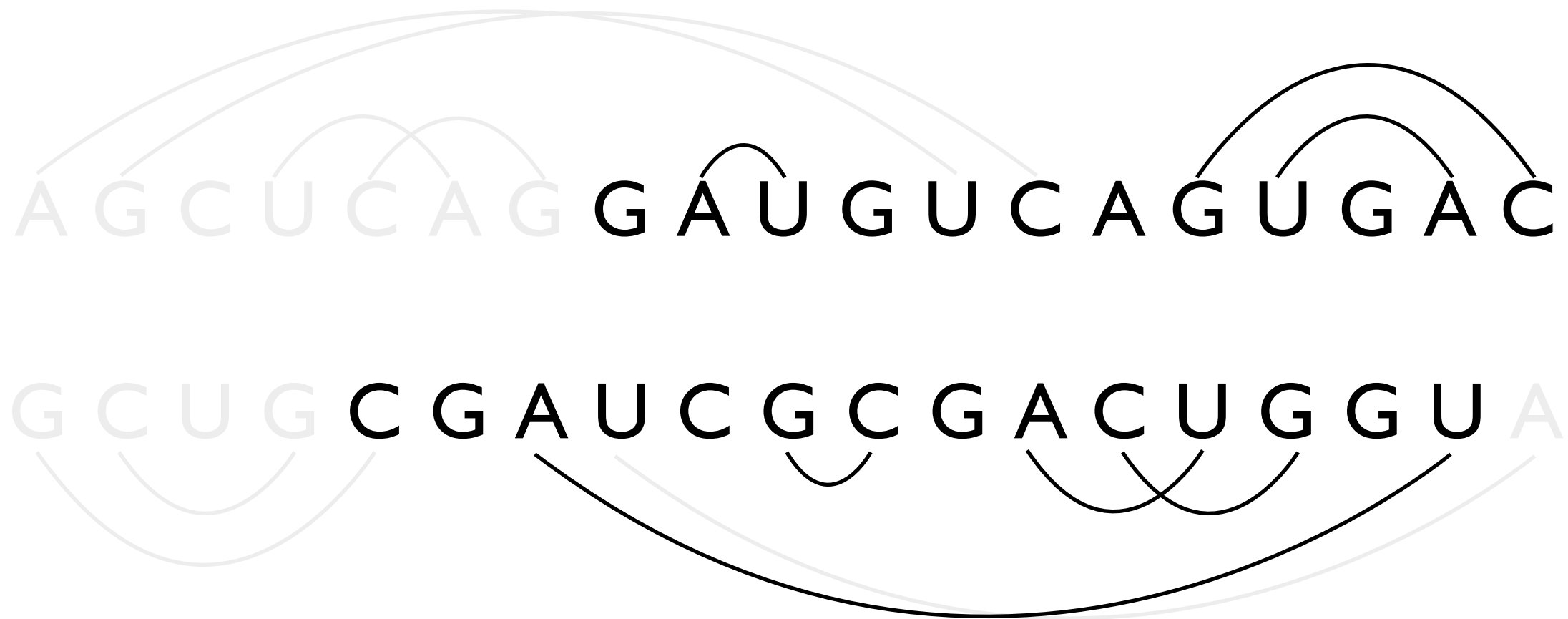
Character Match / Relabel

Non-Crossing Consensus



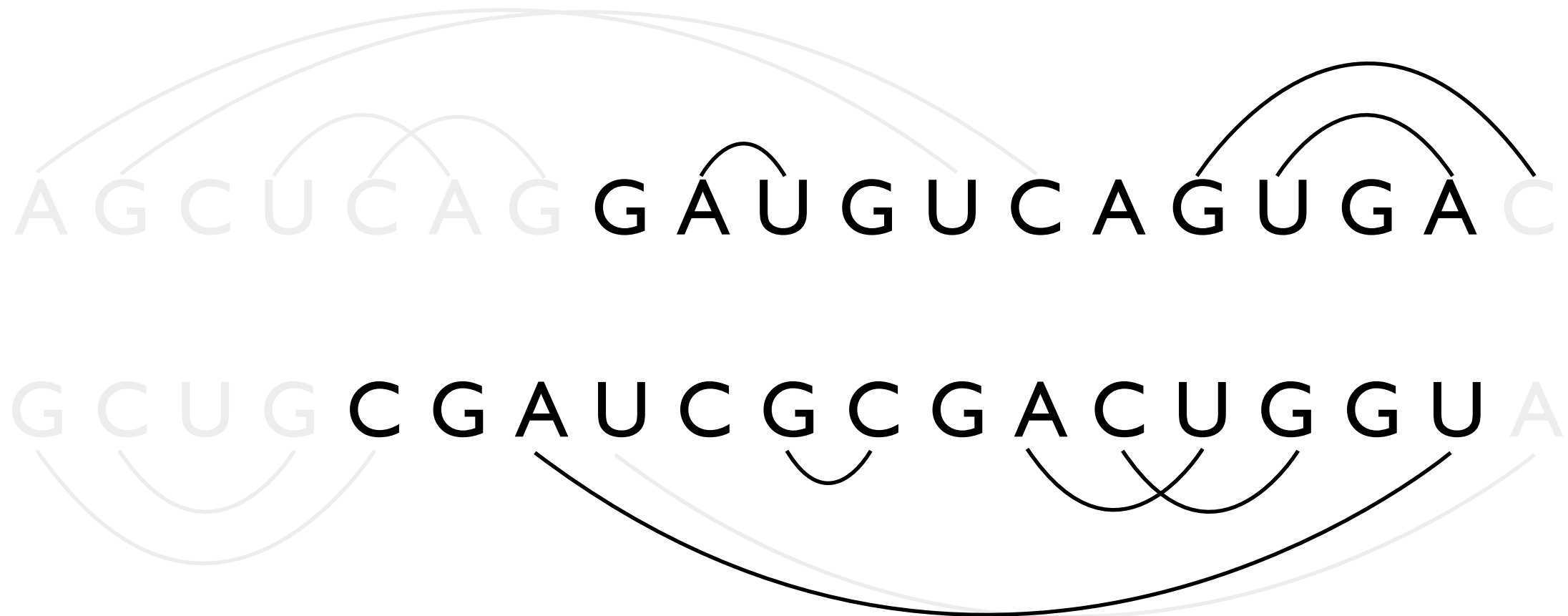
Character Match / Relabel

Non-Crossing Consensus



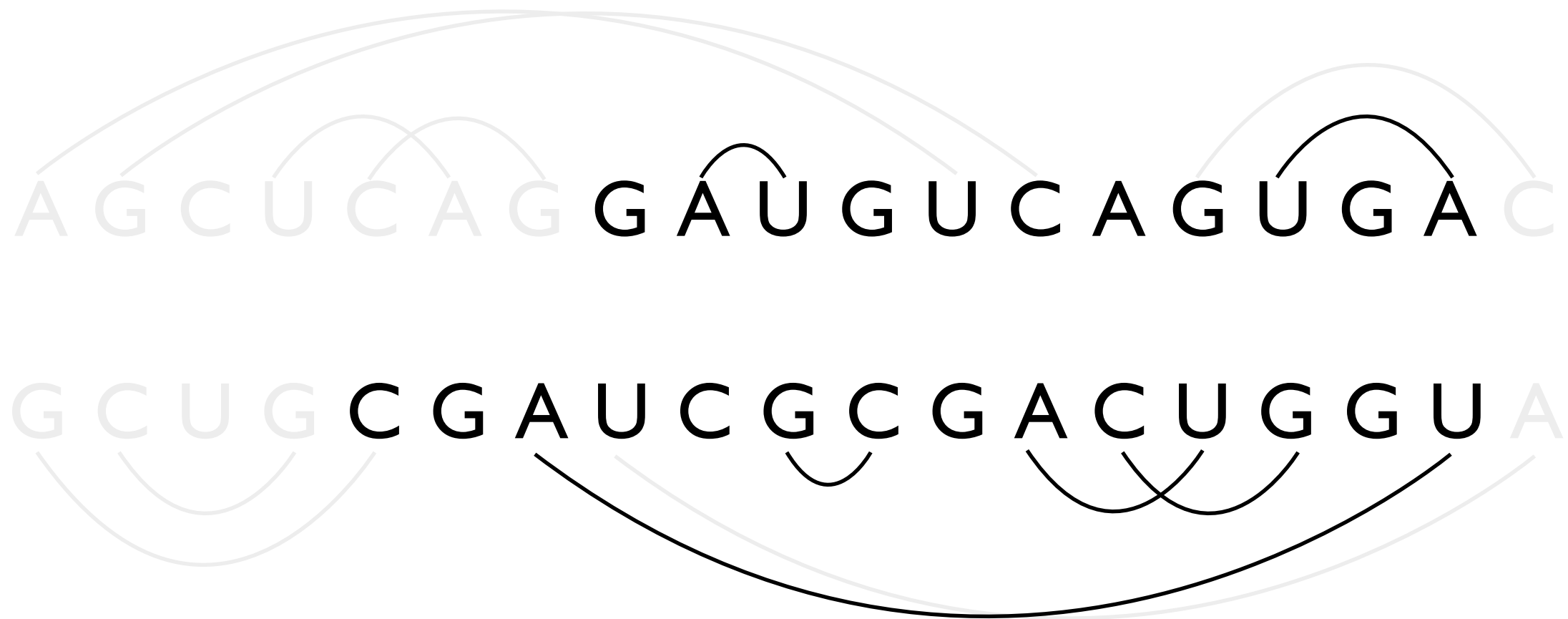
Character Deletion / Insertion

Non-Crossing Consensus



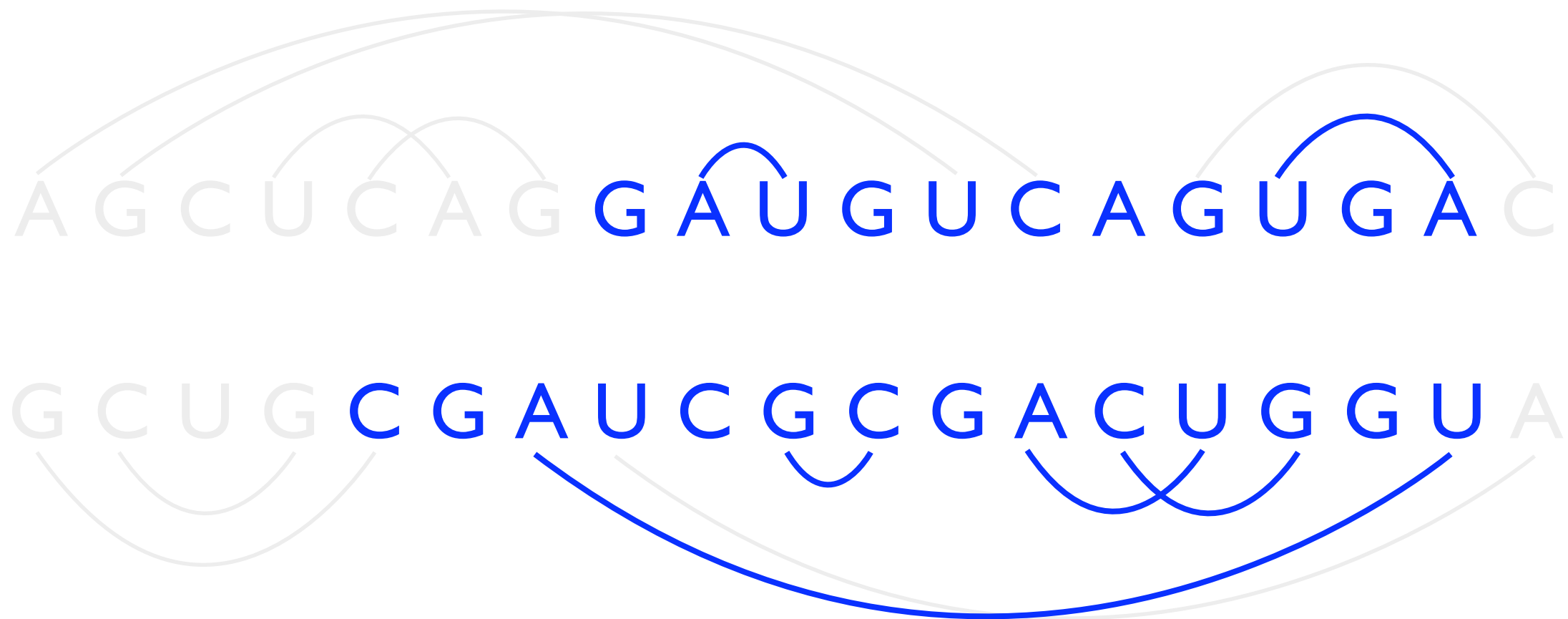
Character Deletion / Insertion

Non-Crossing Consensus



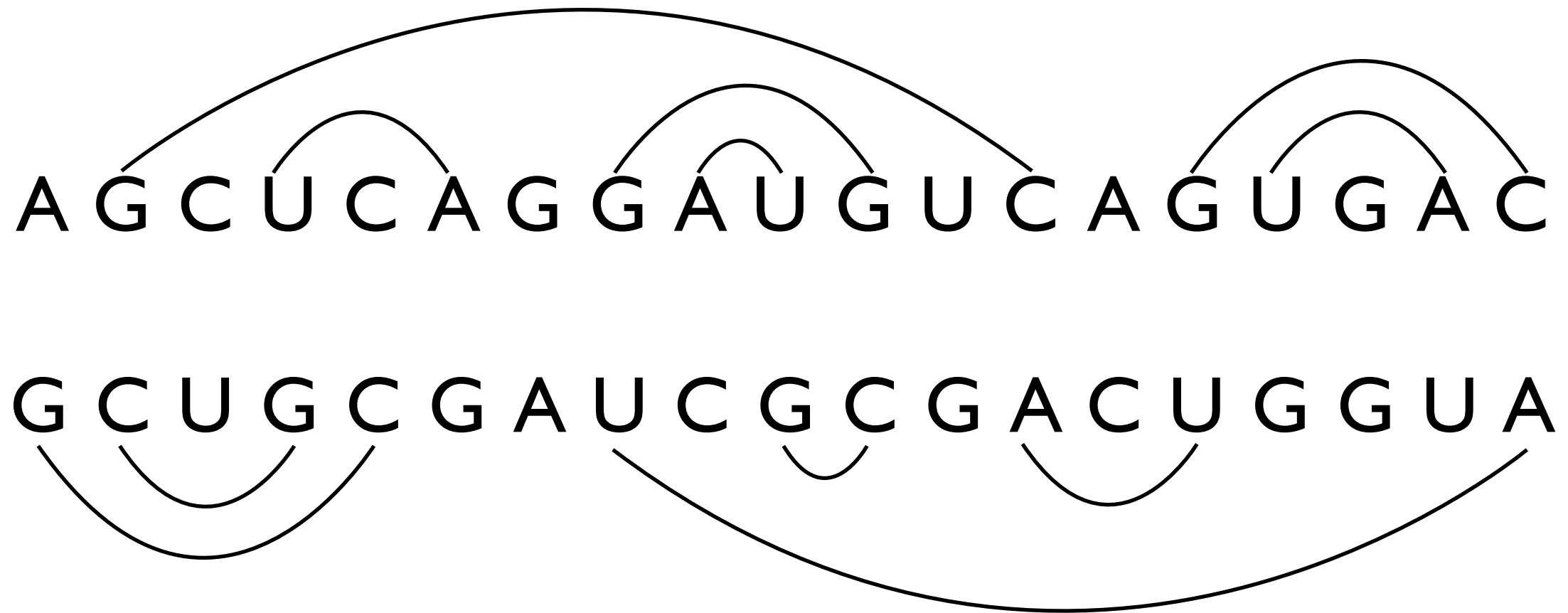
Character Deletion / Insertion

Non-Crossing Consensus

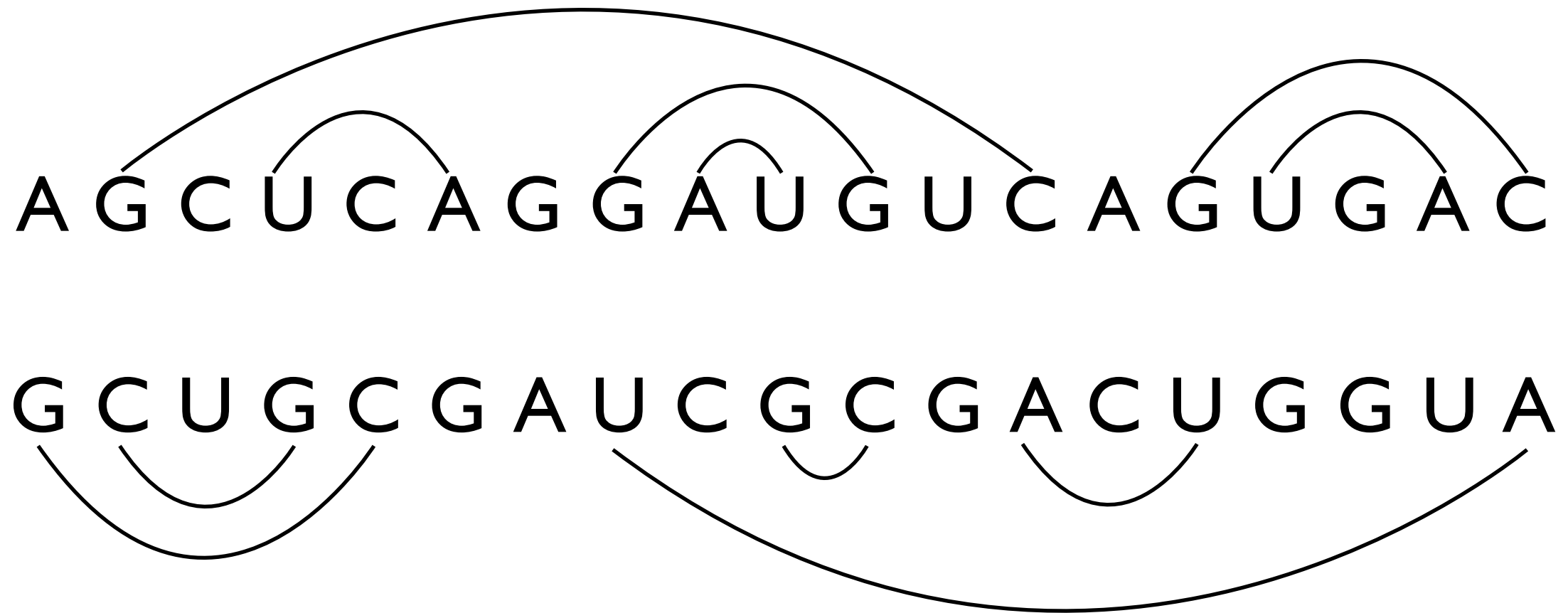


Character Deletion / Insertion

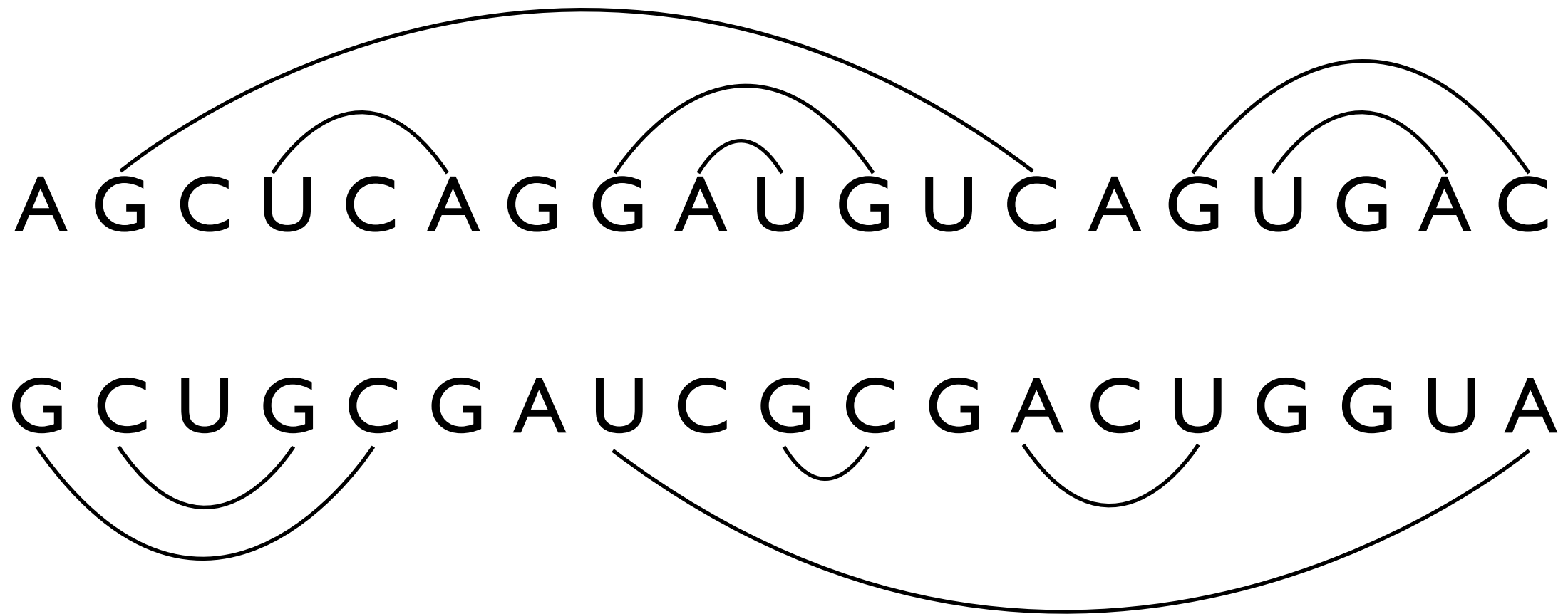
Non-Crossing Inputs



Non-Crossing Inputs = Tree Edit distance



Non-Crossing Inputs = Tree Edit distance



$O(n^6)$

[Tai 1979]

$O(n^4)$

[Shasha, Zhang 1989]

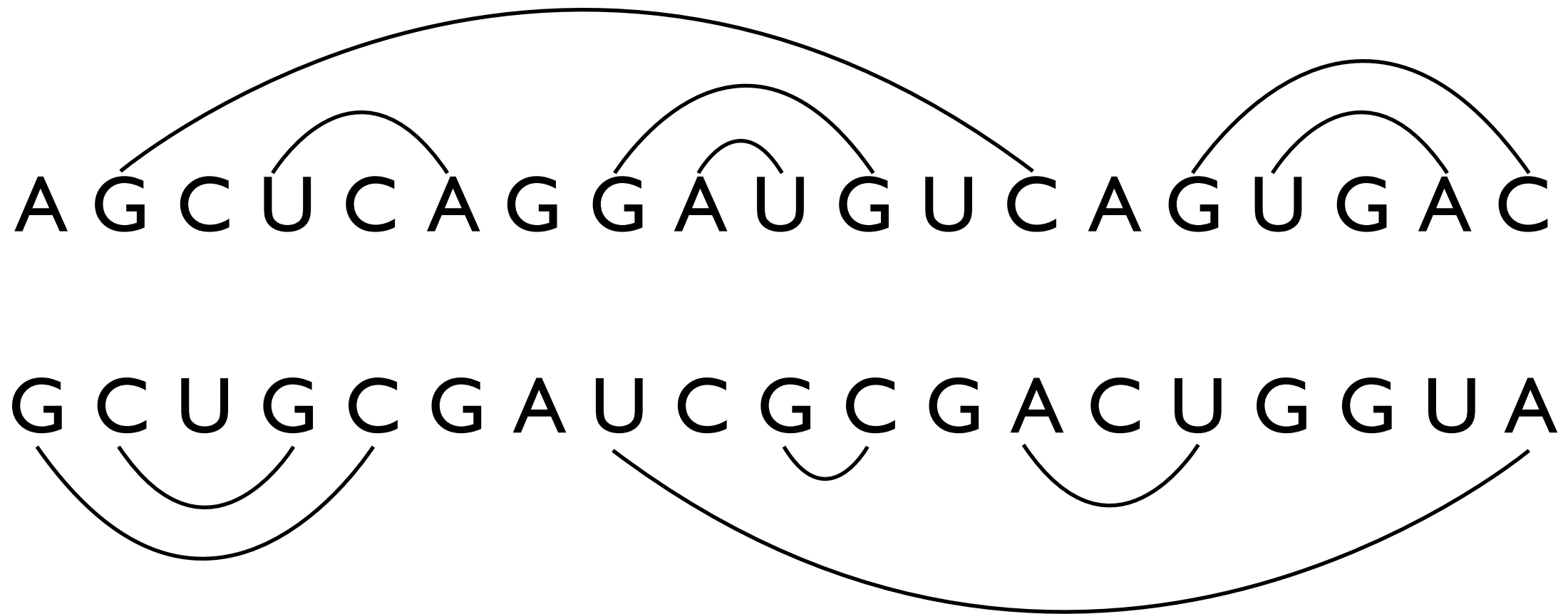
$O(n^3 \log n)$

[Klein 1998] [Dulucq, Touzet 2003] [Bille 2005]

$O(n^3)$

[Demaine, Mozes, Rossman, W, 2007]

Non-Crossing Inputs = Tree Edit distance



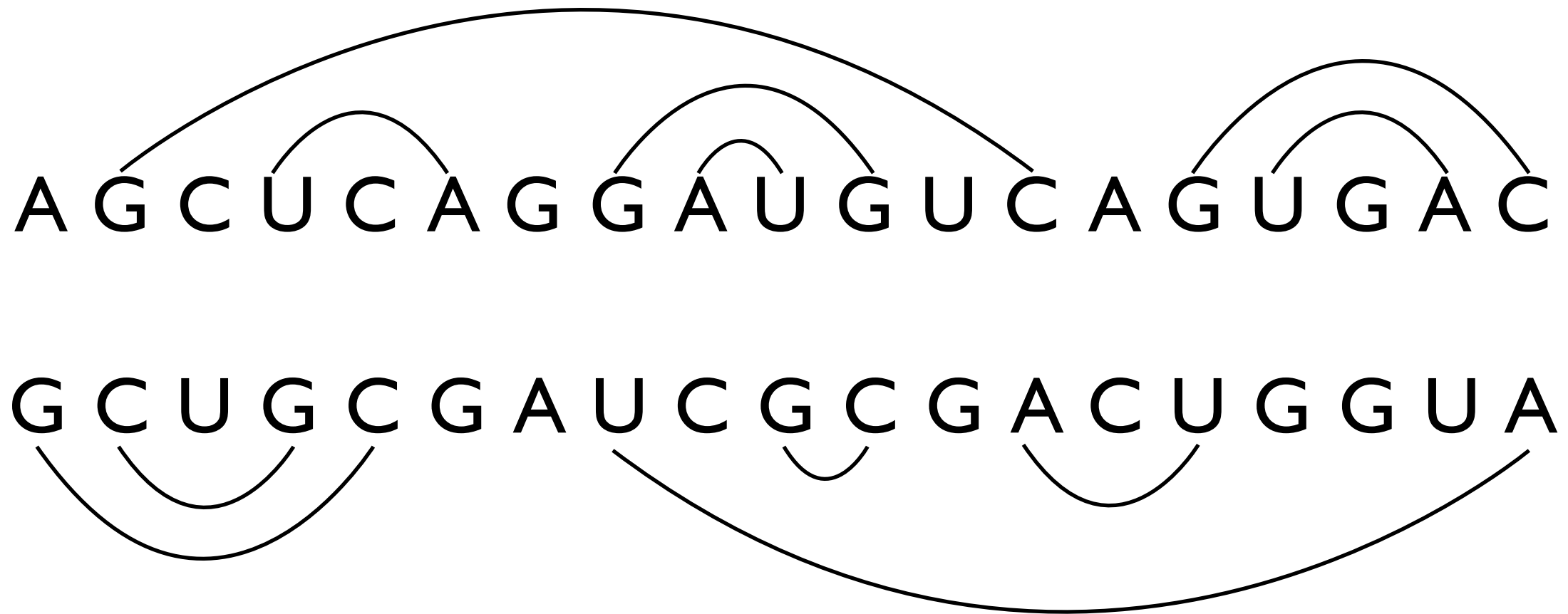
$O(n^3 \log n)$ [Klein 1998]

Non-Crossing Inputs = Tree Edit distance

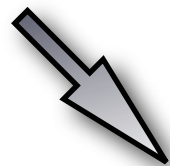


$O(n^3 \log n)$ [Klein 1998]

Non-Crossing Inputs = Tree Edit distance

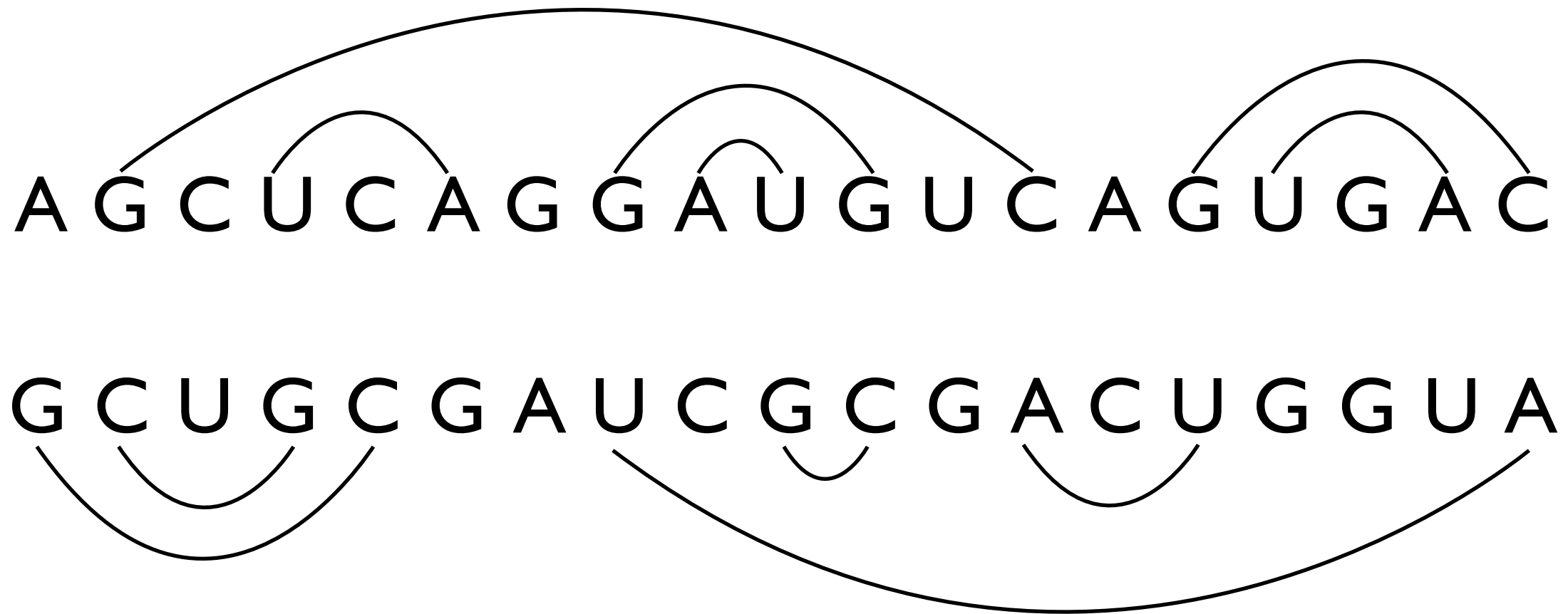


n^2



$O(n^3 \log n)$ [Klein 1998]

Non-Crossing Inputs = Tree Edit distance

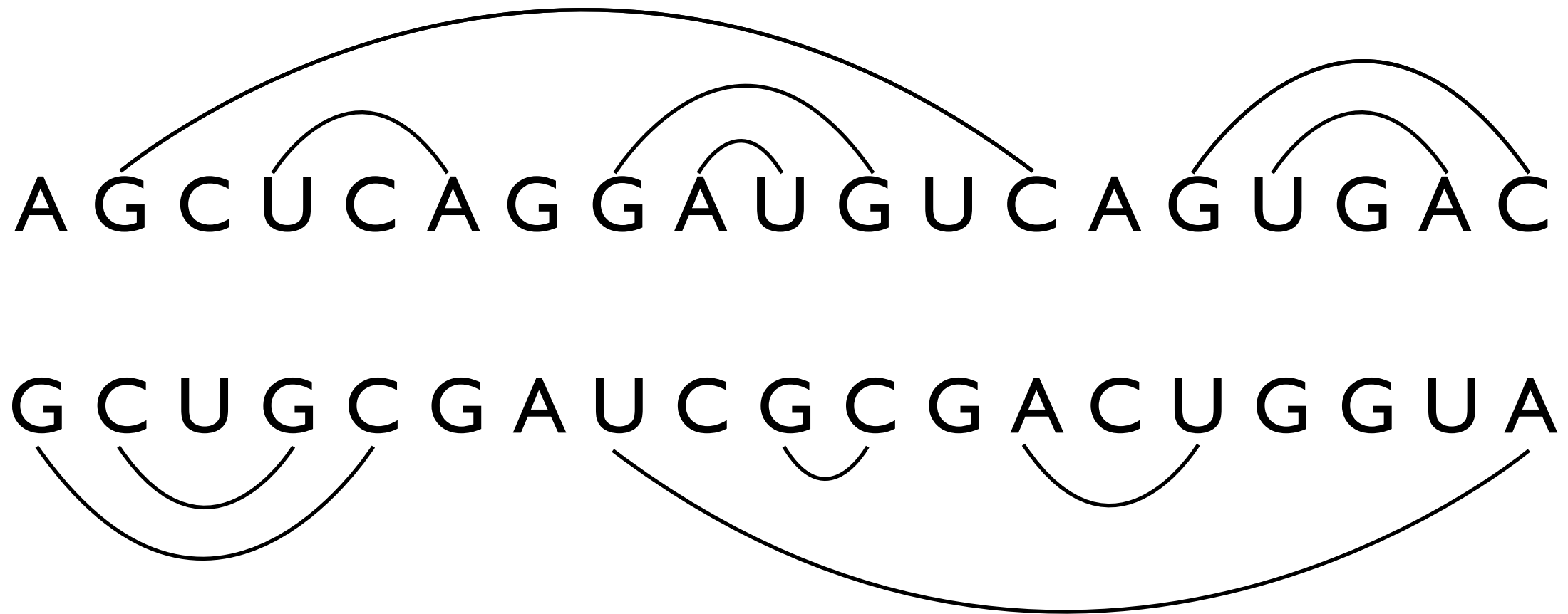


$$n^2 \times n \log n$$

$\swarrow \searrow$

$O(n^3 \log n)$ [Klein 1998]

Non-Crossing Inputs = Tree Edit distance

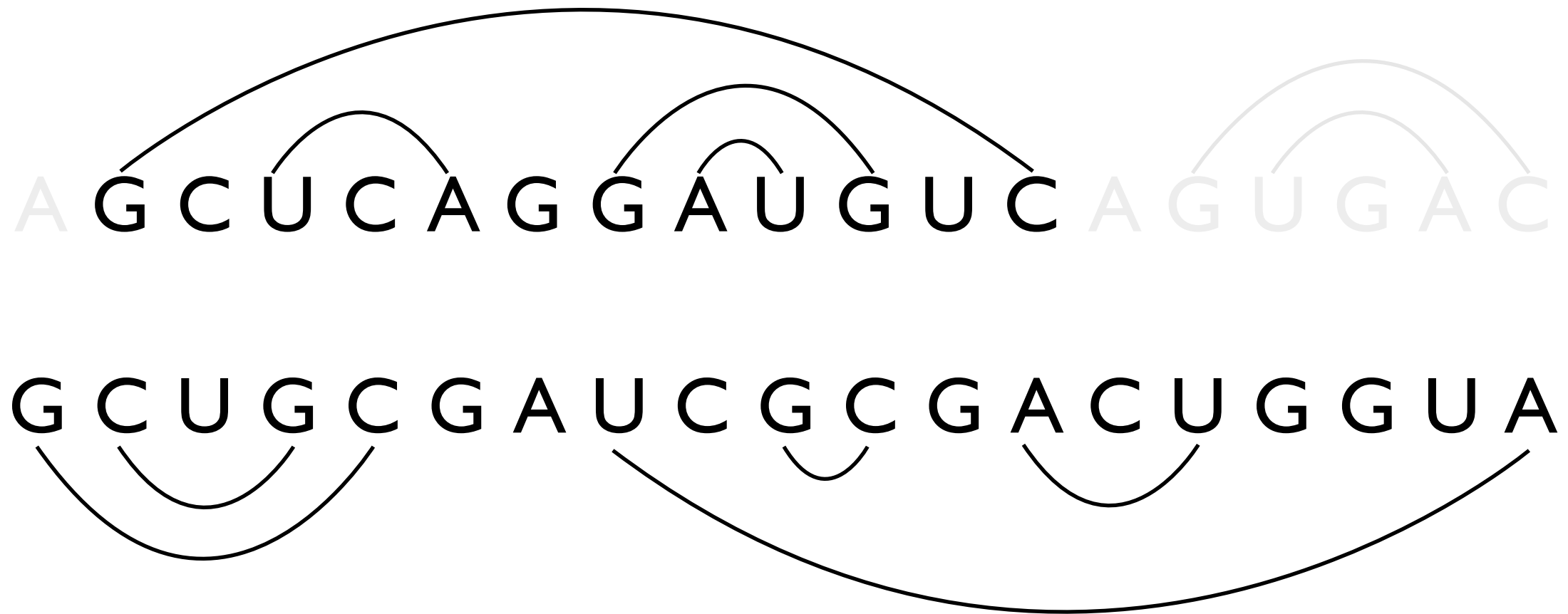


$$n^2 \times n \log n$$

$\swarrow \quad \searrow$

$O(n^3 \log n)$ [Klein 1998]

Non-Crossing Inputs = Tree Edit distance

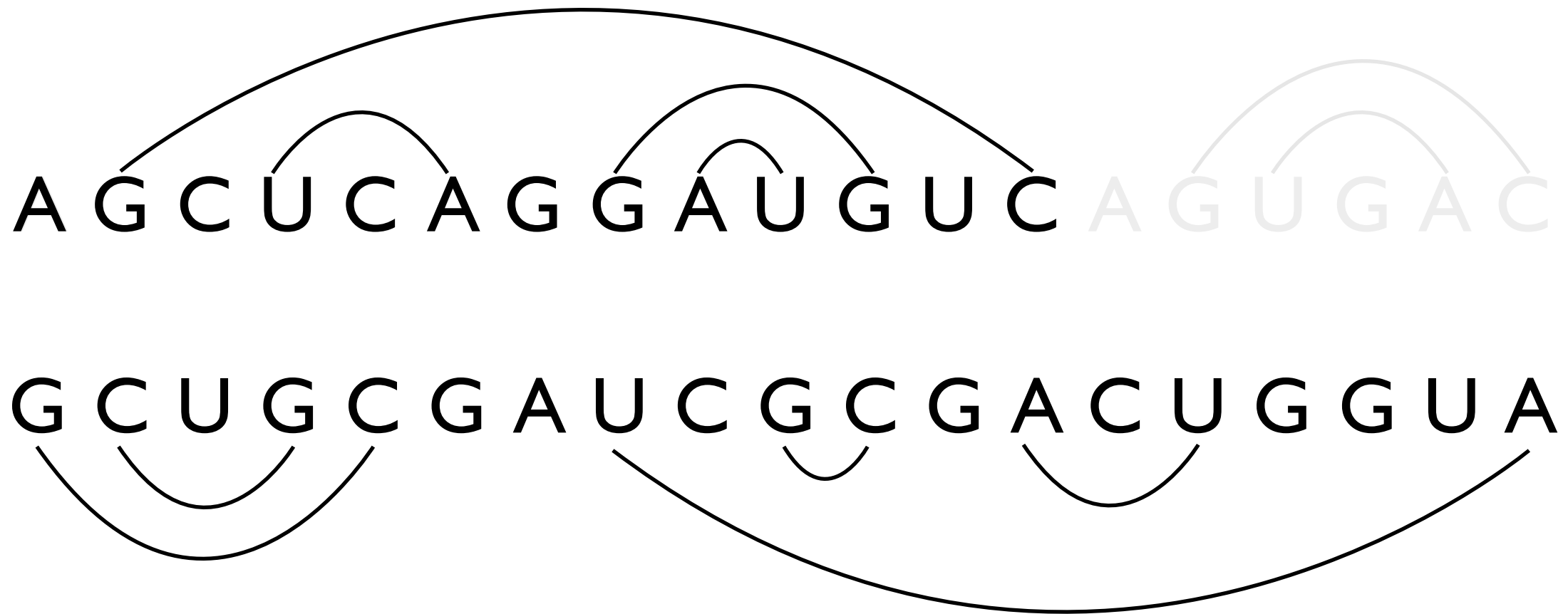


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$O(n^3 \log n)$ [Klein 1998]

Non-Crossing Inputs = Tree Edit distance

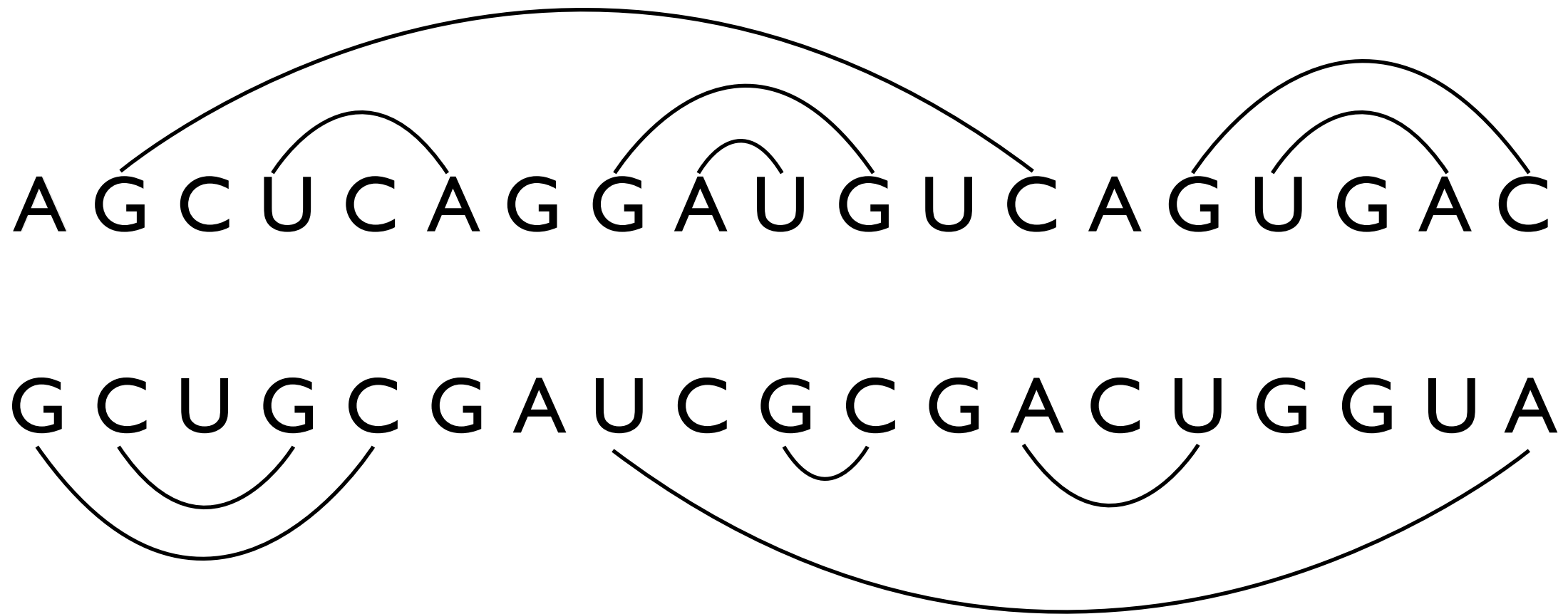


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$\swarrow \quad \searrow$

$O(n^3 \log n)$ [Klein 1998]

Non-Crossing Inputs = Tree Edit distance

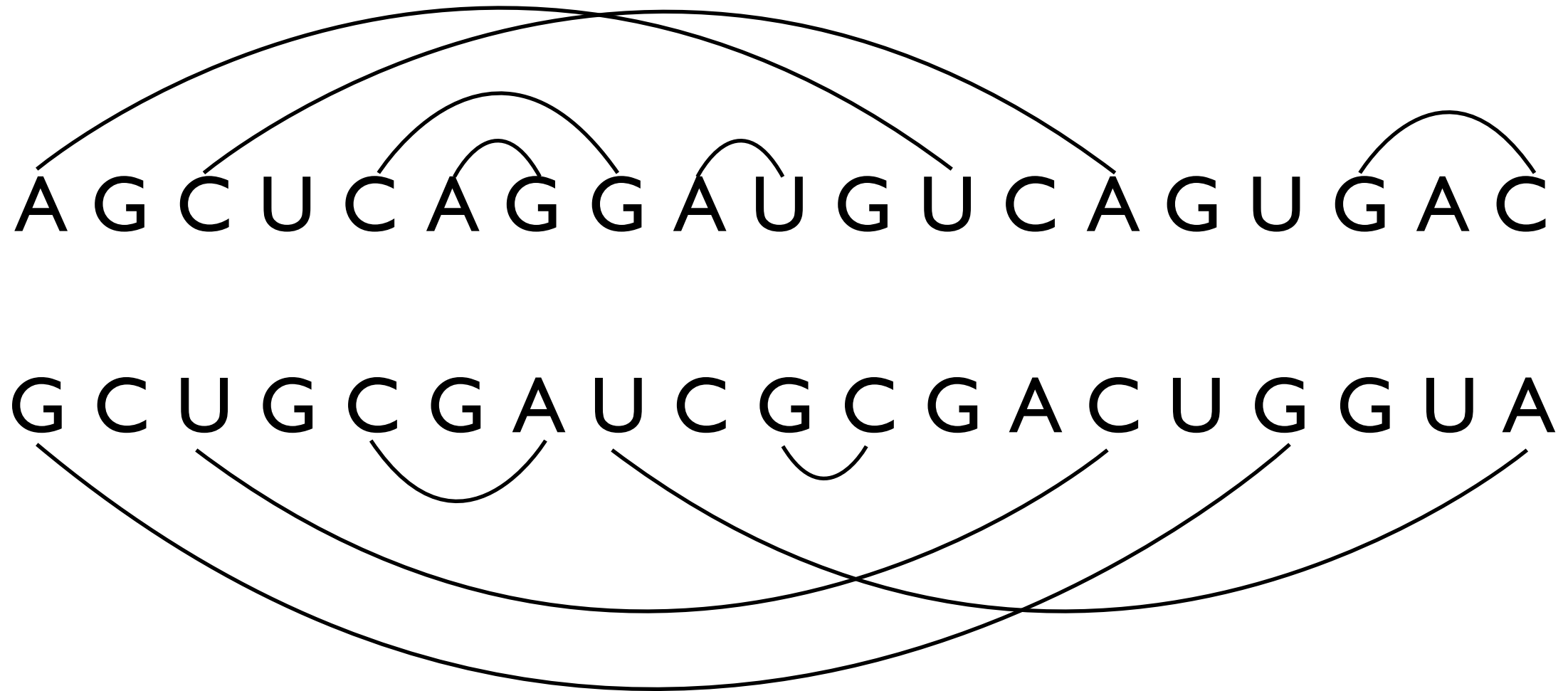


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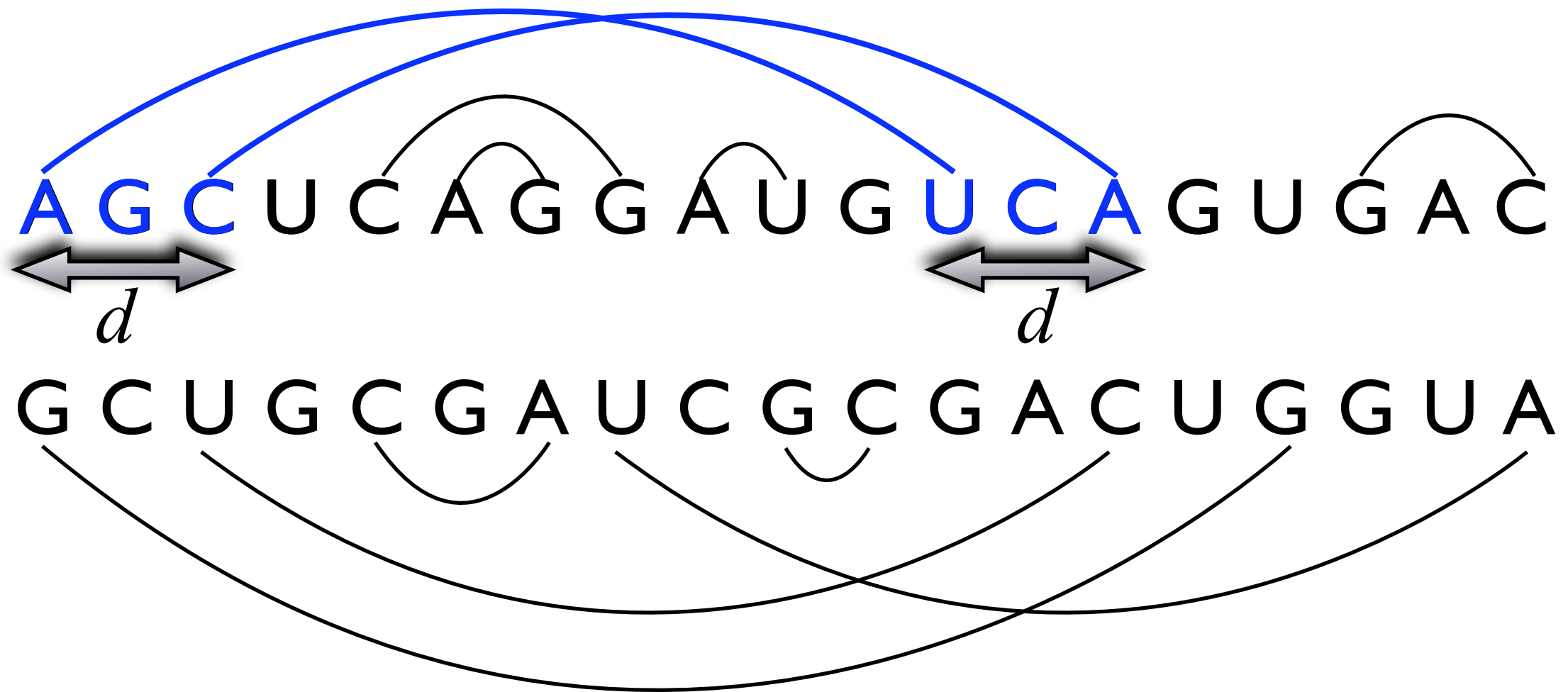
$\swarrow \quad \searrow$

$O(n^3 \log n)$ [Klein 1998]

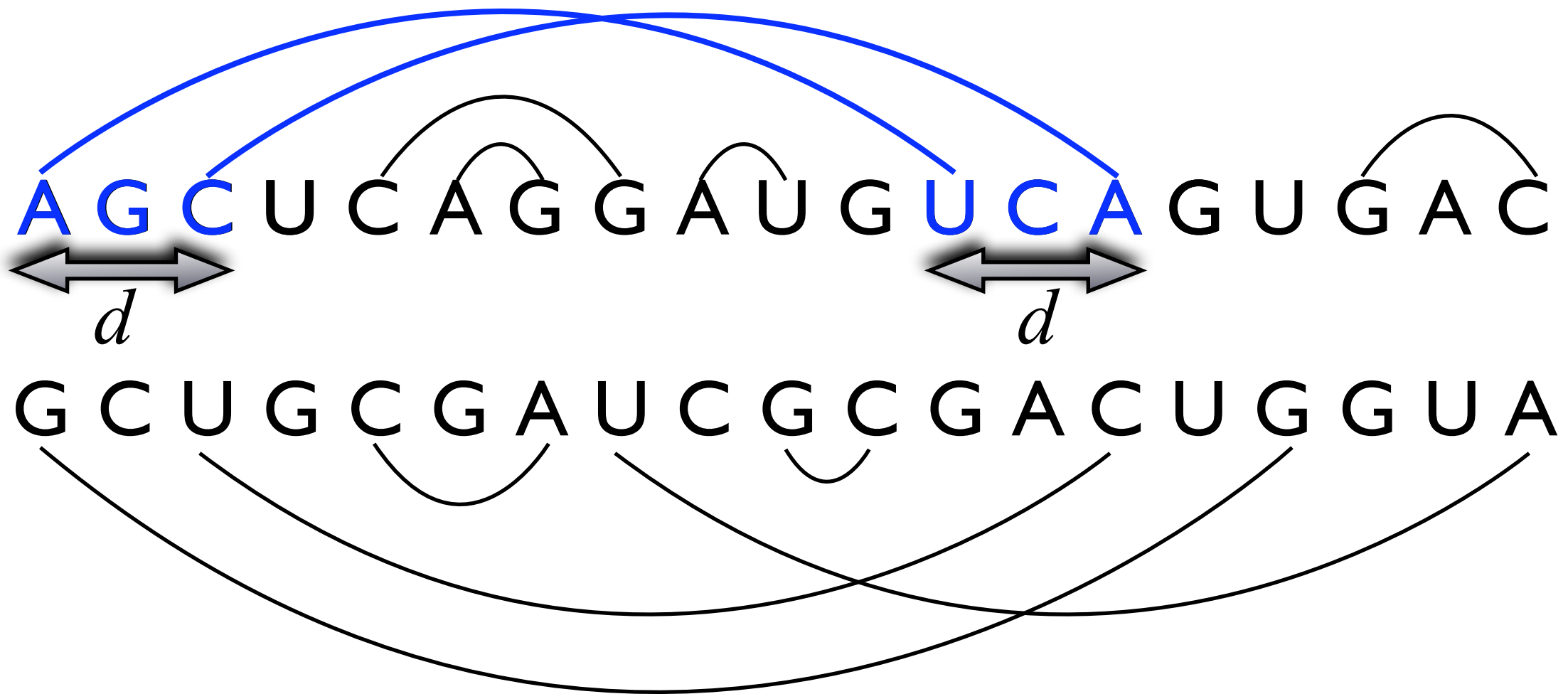
Our Result: *d*-crossing inputs



Our Result: d -crossing inputs

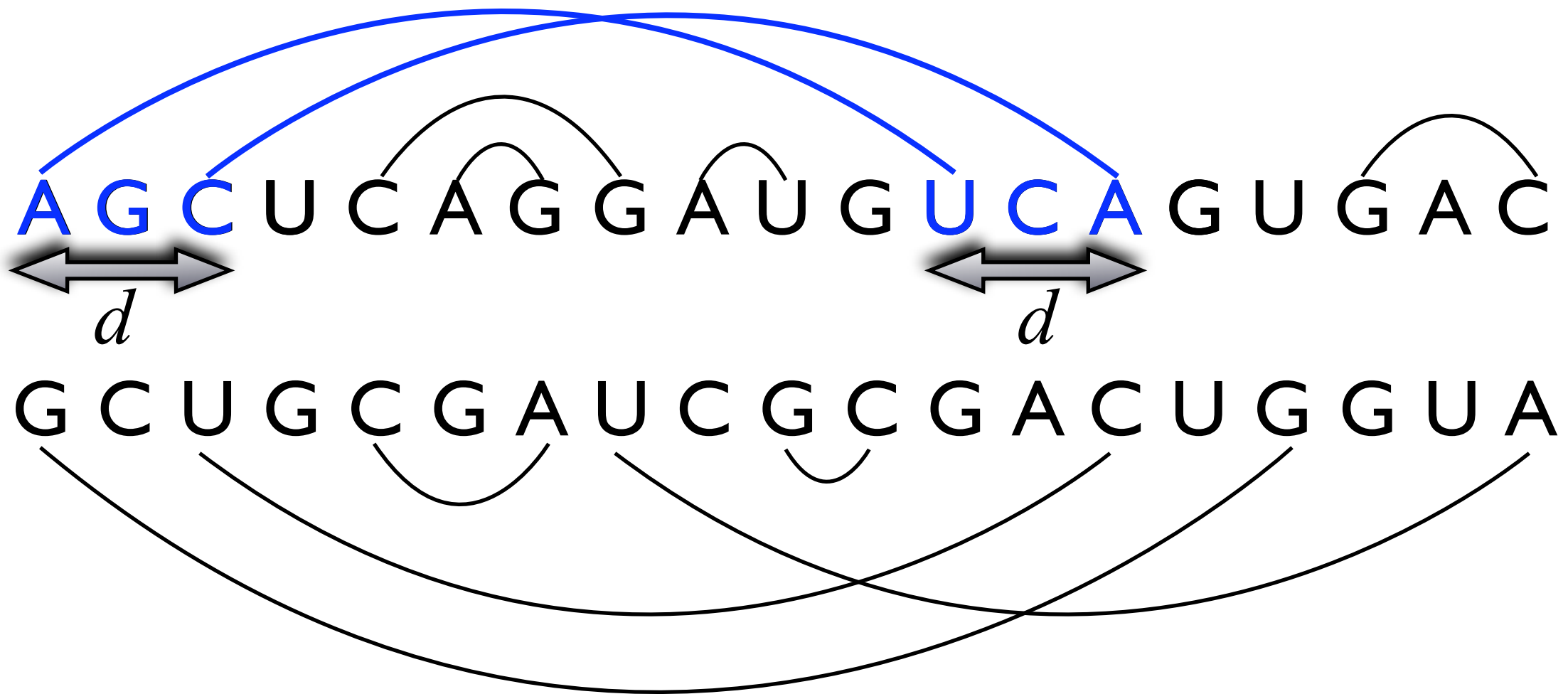


Our Result: d -crossing inputs



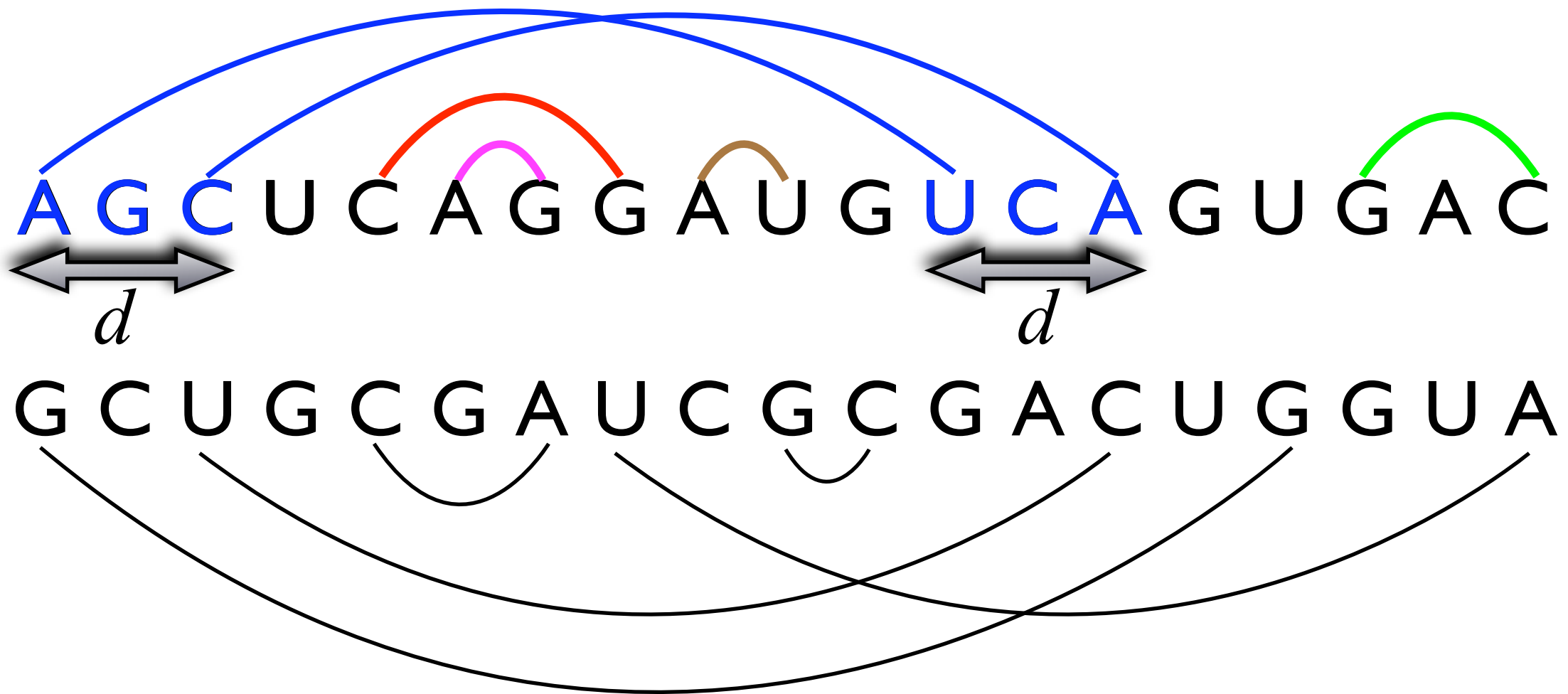
$$O(dn^3 \log n)$$

Our Result: d -crossing inputs



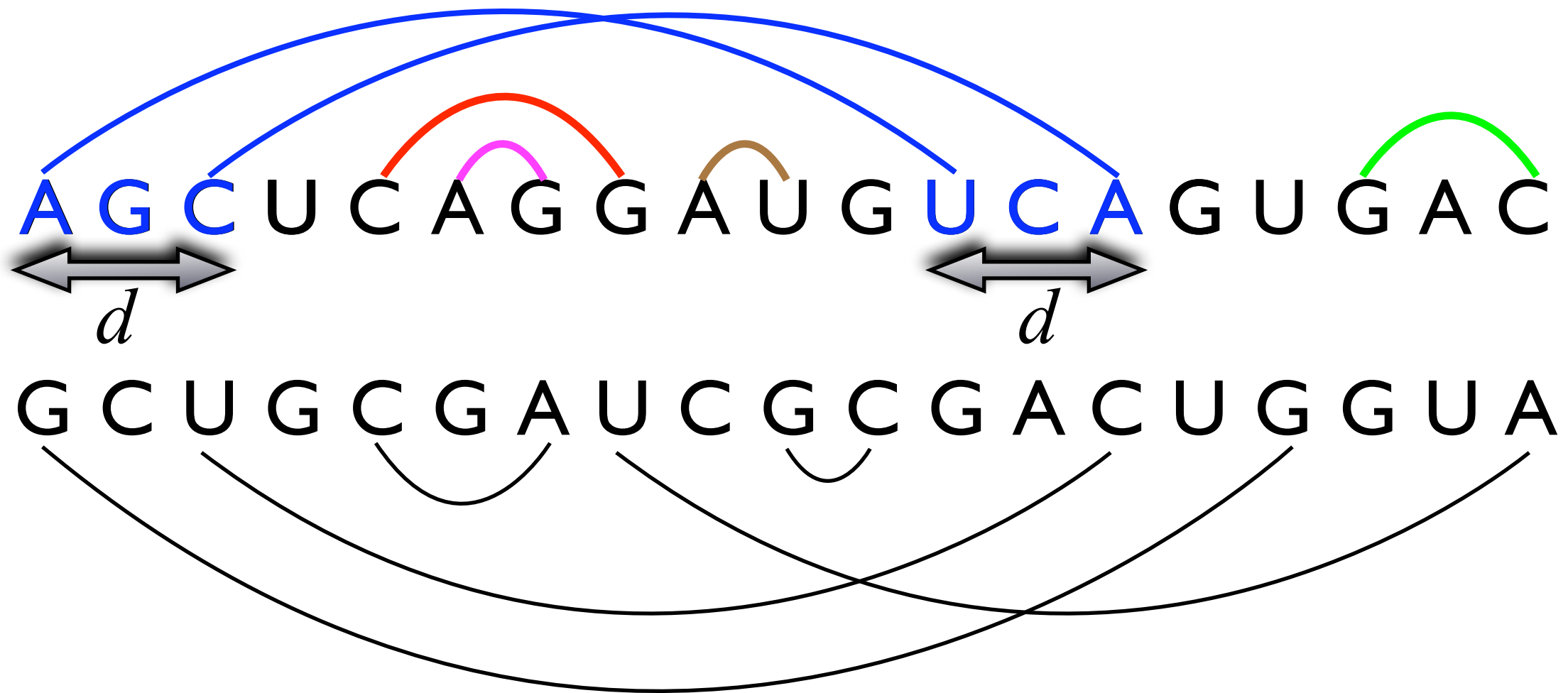
$$O(dn^3 \log n) \quad \text{Vs.} \quad O(n^4)$$

Our Result: d -crossing inputs



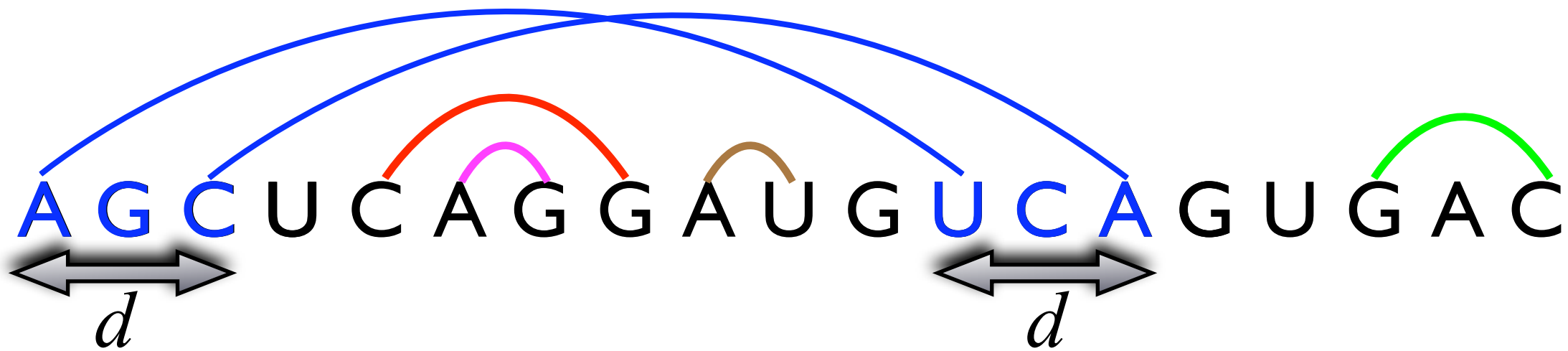
$$O(dn^3 \log n)$$

Our Result: d -crossing inputs



$$O(dn^3 \log n) = n^2 \times dn \log n$$

Our Result: d -crossing inputs



$$O(dn^3 \log n) = n^2 \times dn \log n$$

The Algorithm

The Algorithm



The Algorithm



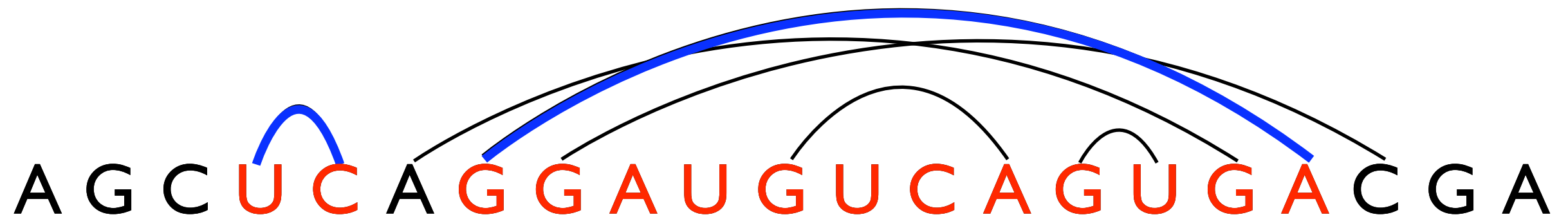
I. Compute every **infix** of length $< 2d$ in $O(dn^3)$

The Algorithm



1. Compute every **infix** of length $< 2d$ in $O(dn^3)$
2. Pick largest **arc** in every cluster

The Algorithm



1. Compute every **infix** of length $< 2d$ in $O(dn^3)$
2. Pick largest **arc** in every cluster
3. Recursively compute **infix** below these arcs

The Algorithm



1. Compute every **infix** of length $< 2d$ in $O(dn^3)$
2. Pick largest **arc** in every cluster
3. Recursively compute **infix** below these arcs
+ its extension by d to both left and right

The Algorithm



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3. Recursively compute **infix** below these arcs
+ its extension by d to both left and right
4. Extend largest **infix** to the left and then to the right

The Algorithm



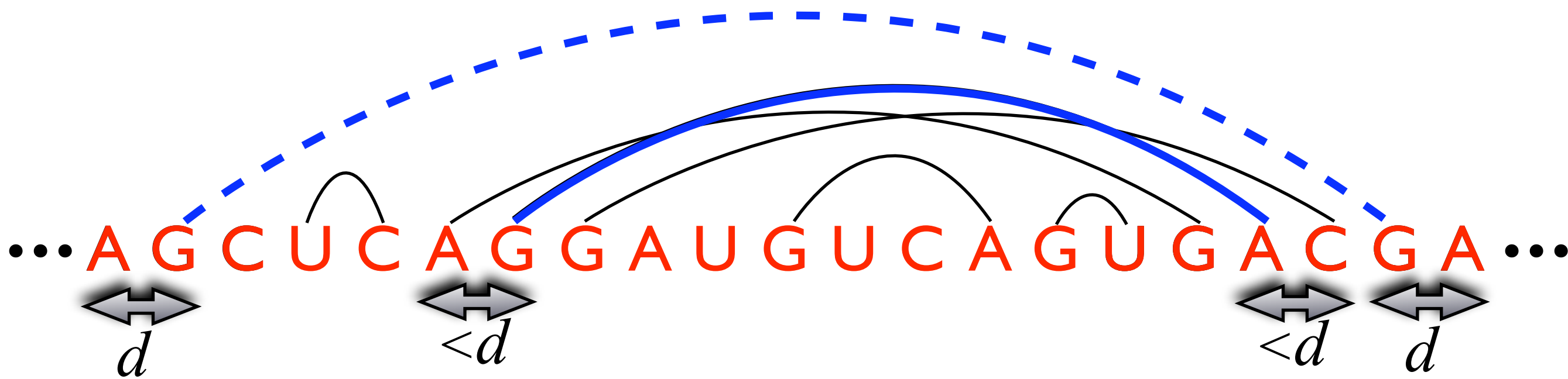
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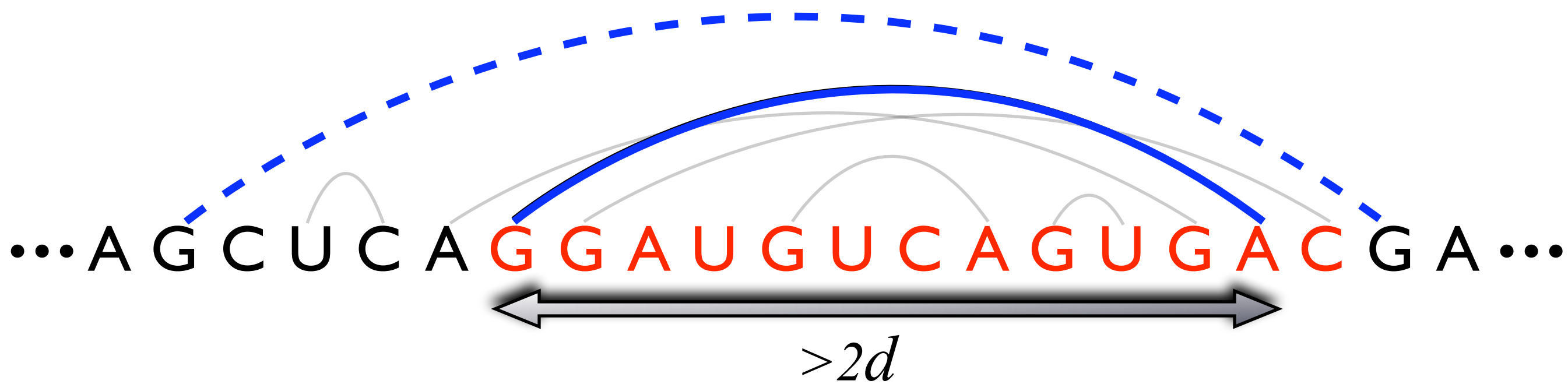
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Why does this work?



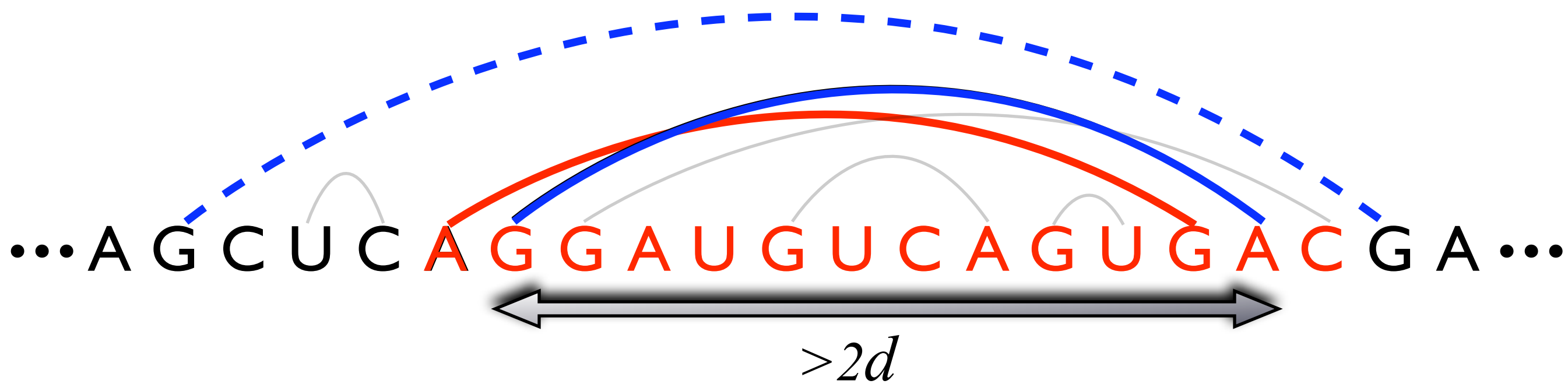
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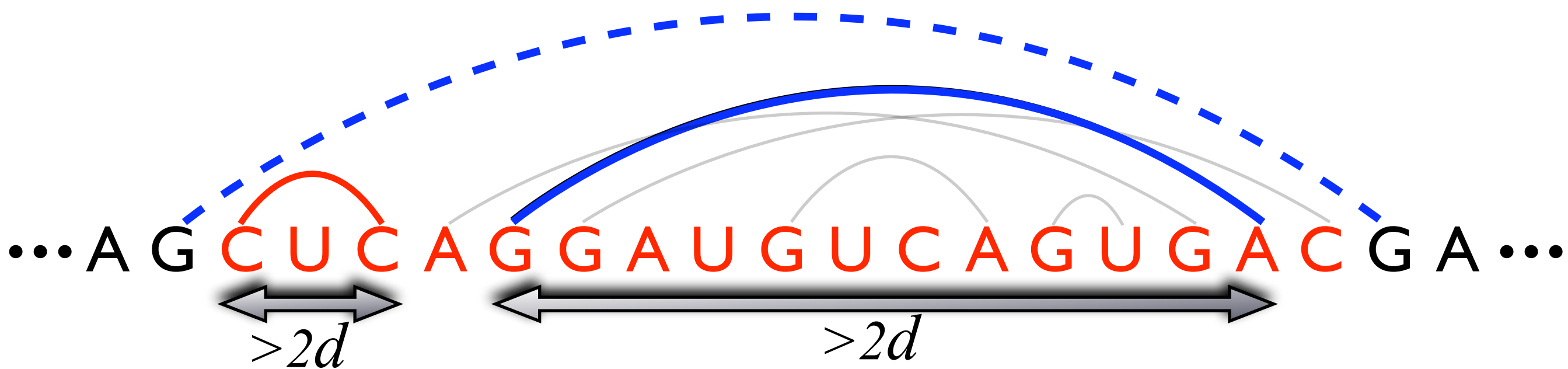
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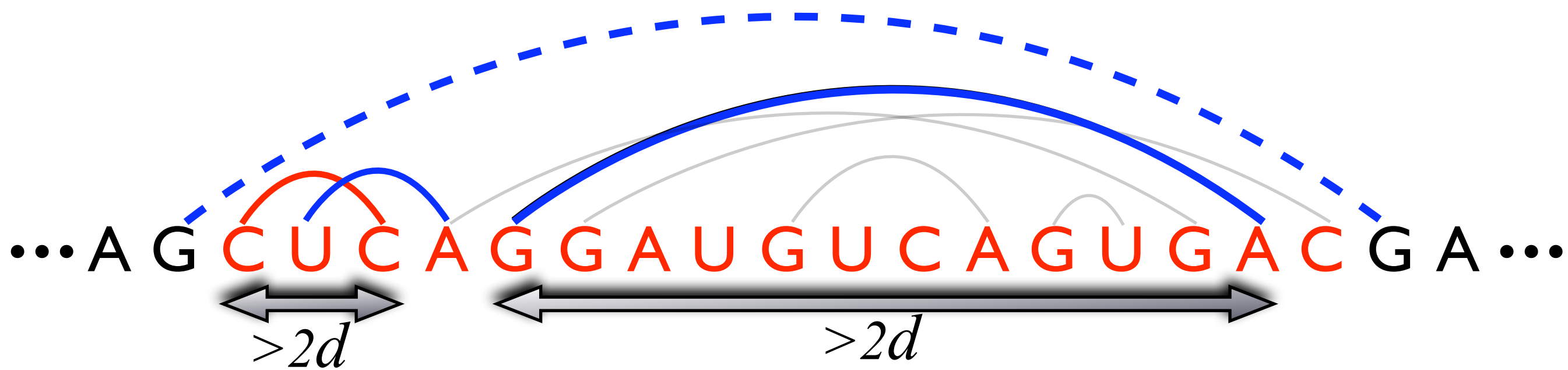
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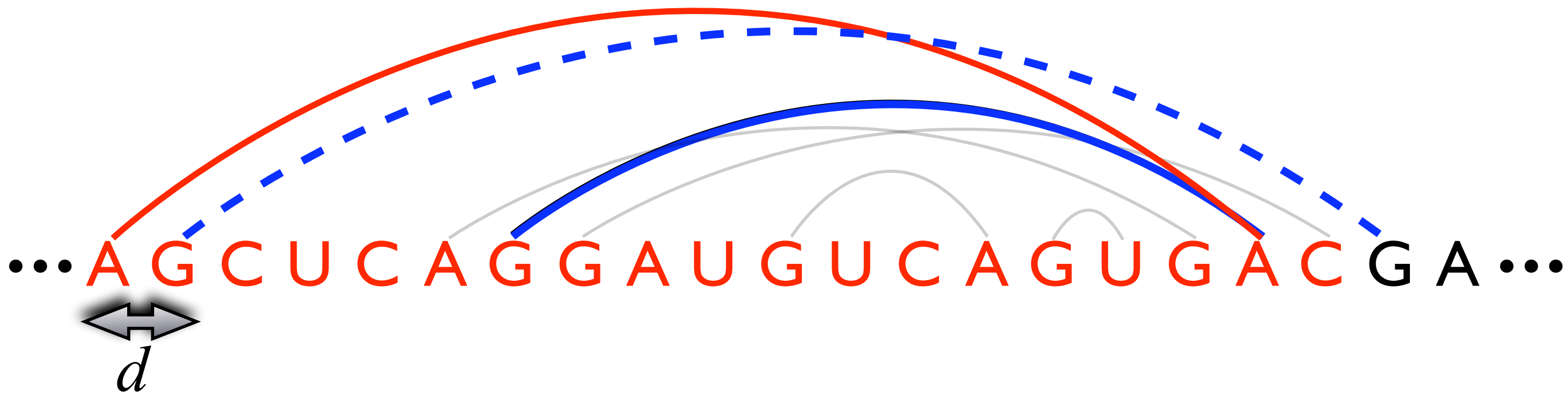
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Why does this work?



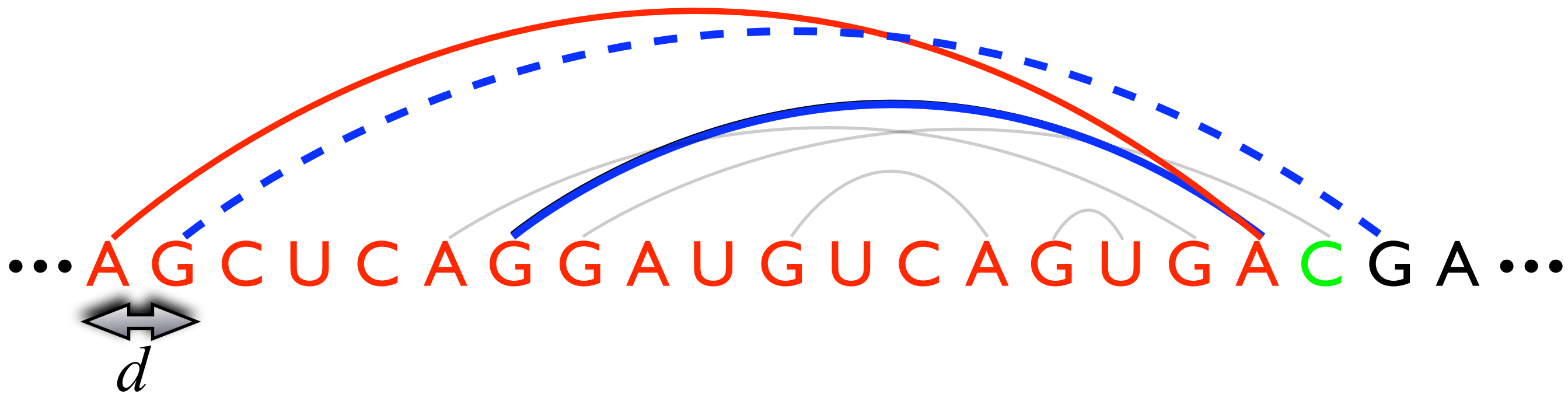
1. Compute every **infix** of length $< 2d$ in $O(dn^3)$
2. Pick largest **arc** in every cluster
3. Recursively compute **infix** below these arcs
+ its extension by d to both left and right
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Thank You!
and happy birthday CPM