Fault-Tolerant Distance Labeling for Planar Graphs

Aviv Bar-Natan,
Panagiotis Charalampopoulos,
Paweł Gawrychowski,
Shay Mozes,
Oren Weimann

Slides by Aviv Bar-Natan

Distance Labeling

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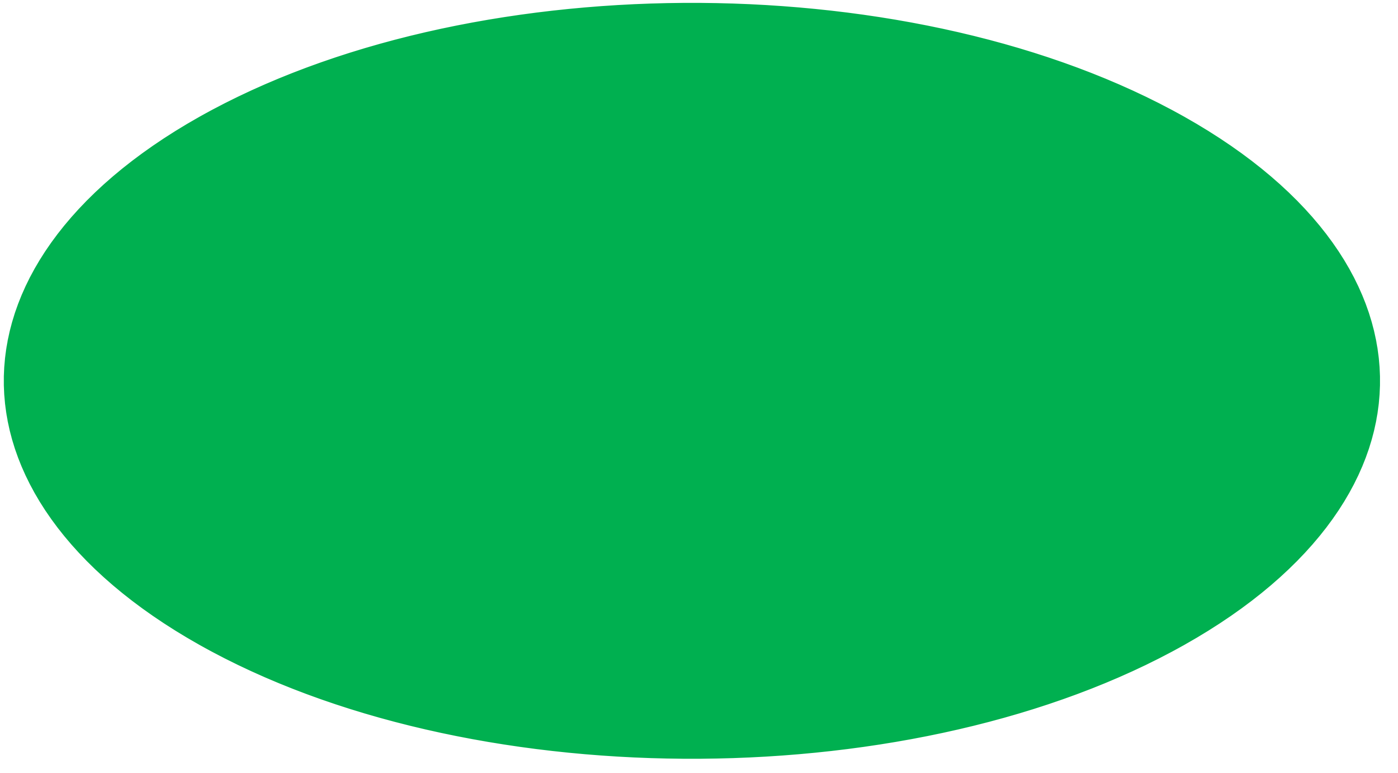

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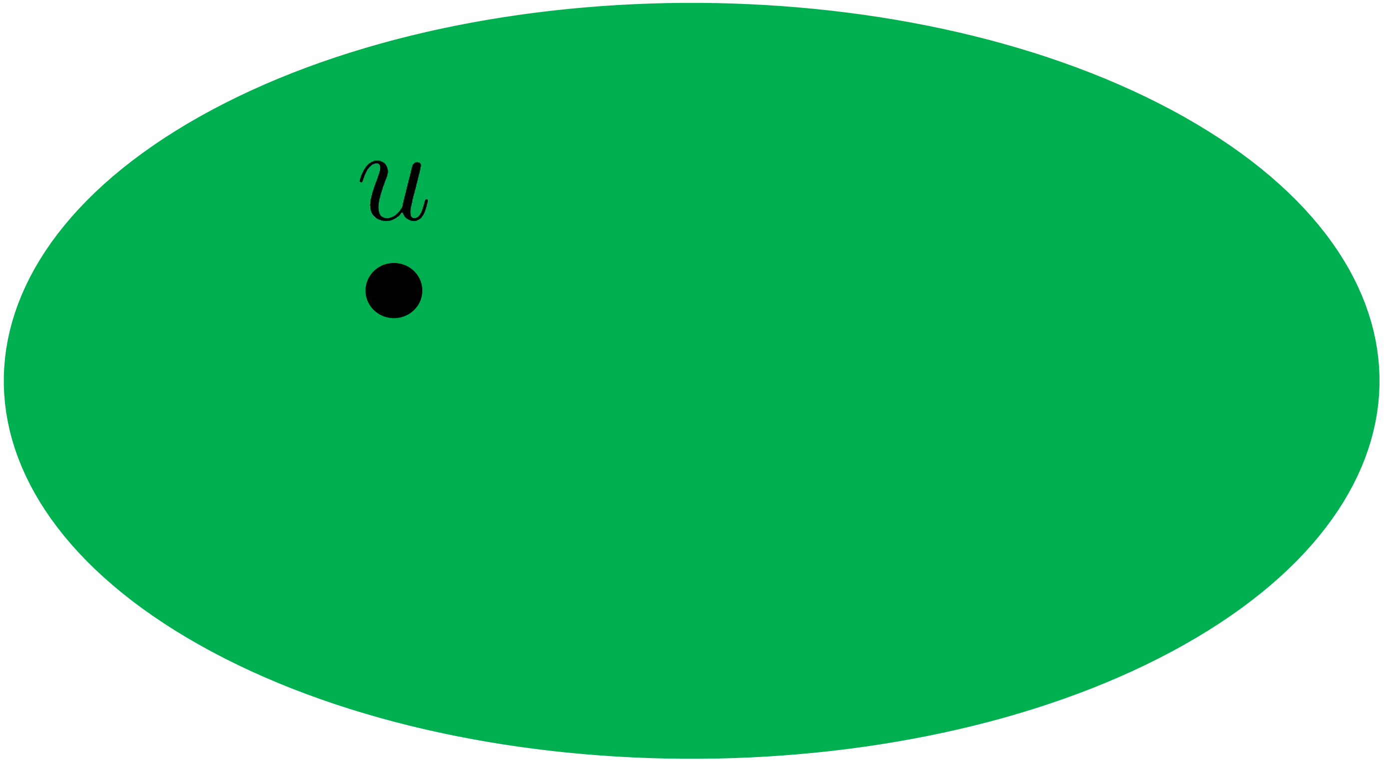

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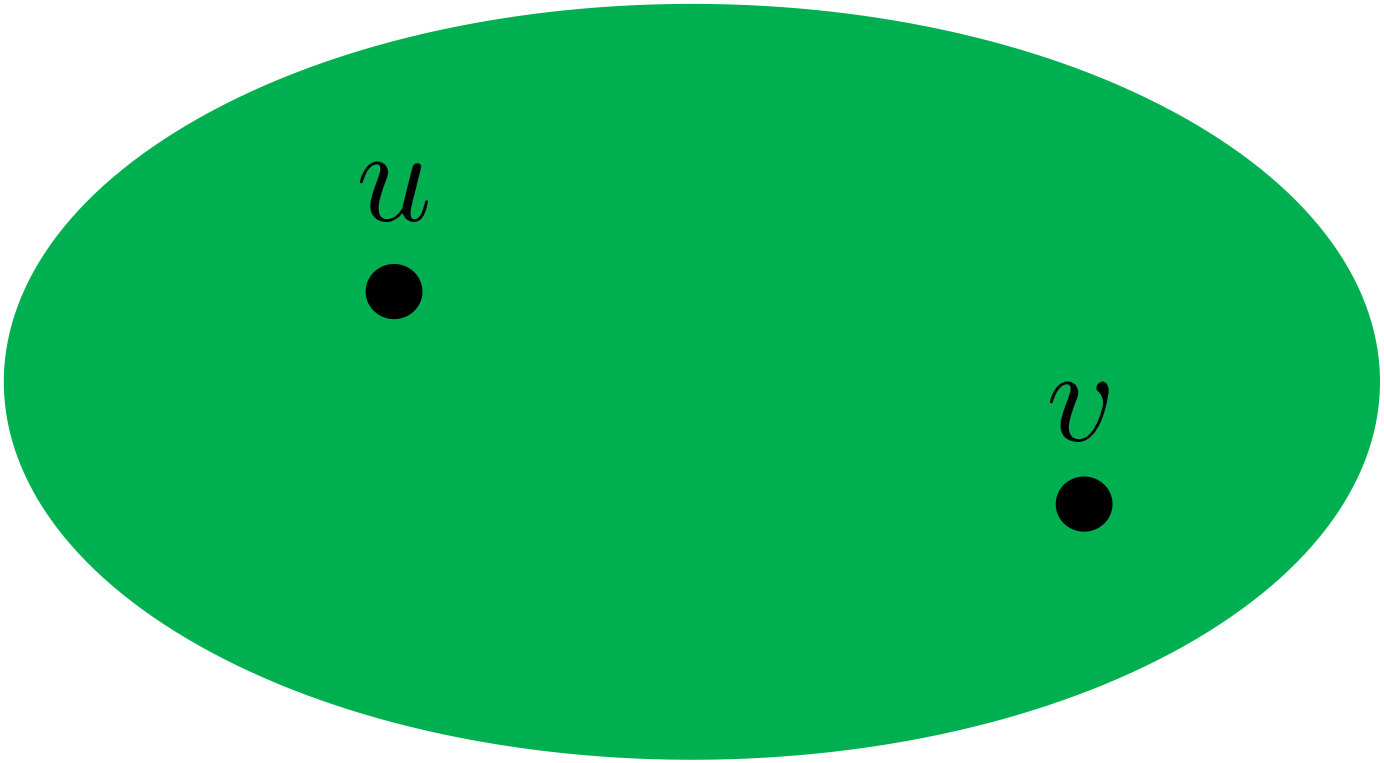

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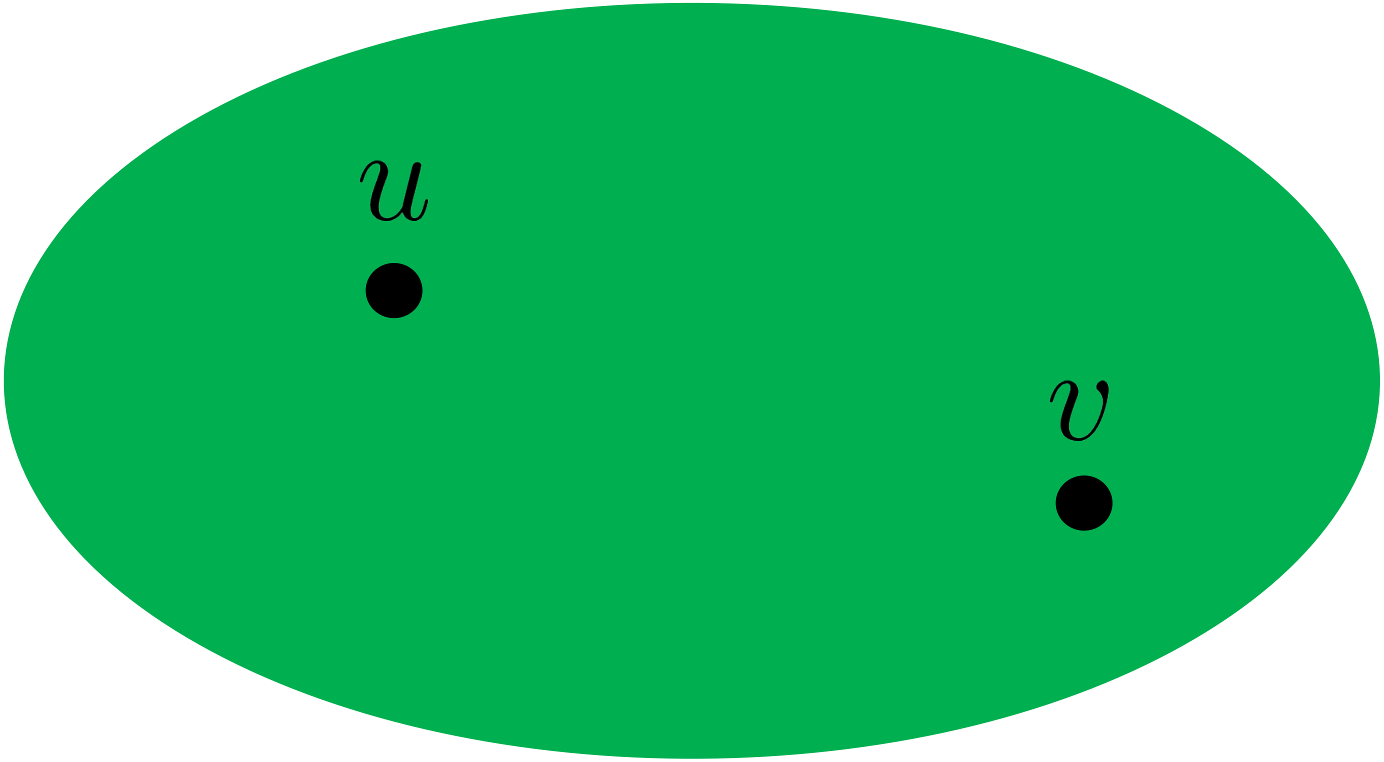

label of u

label of v

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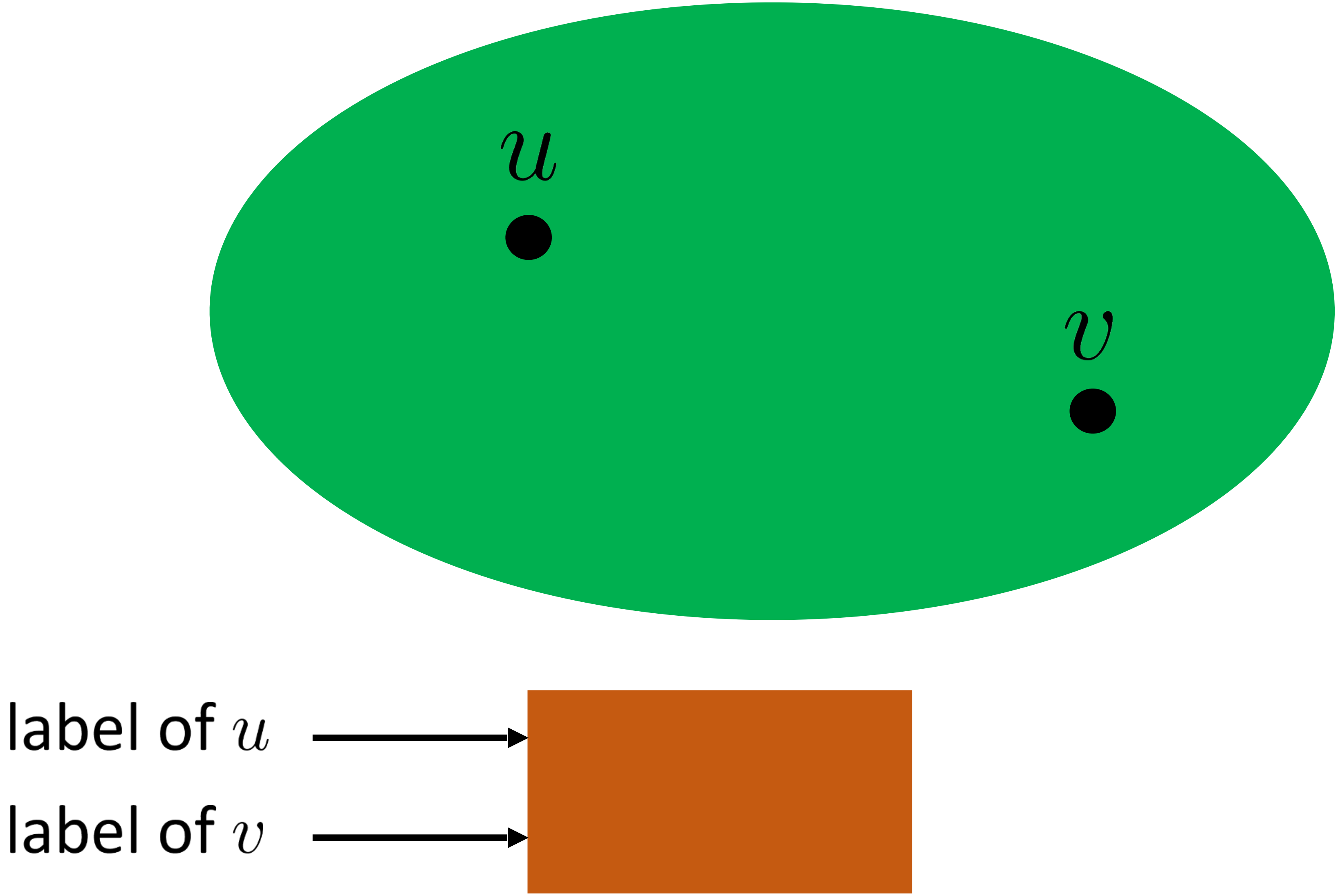

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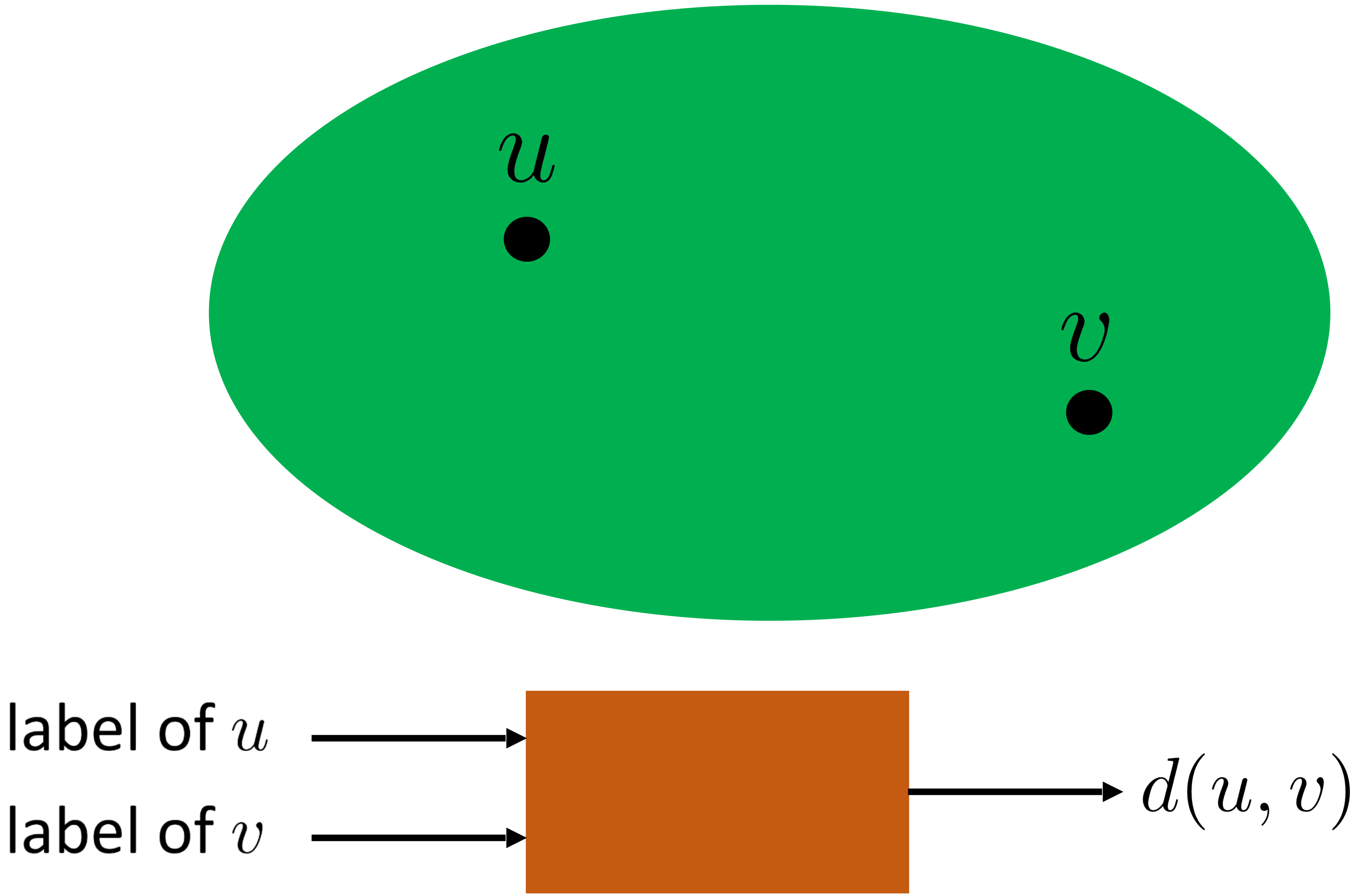

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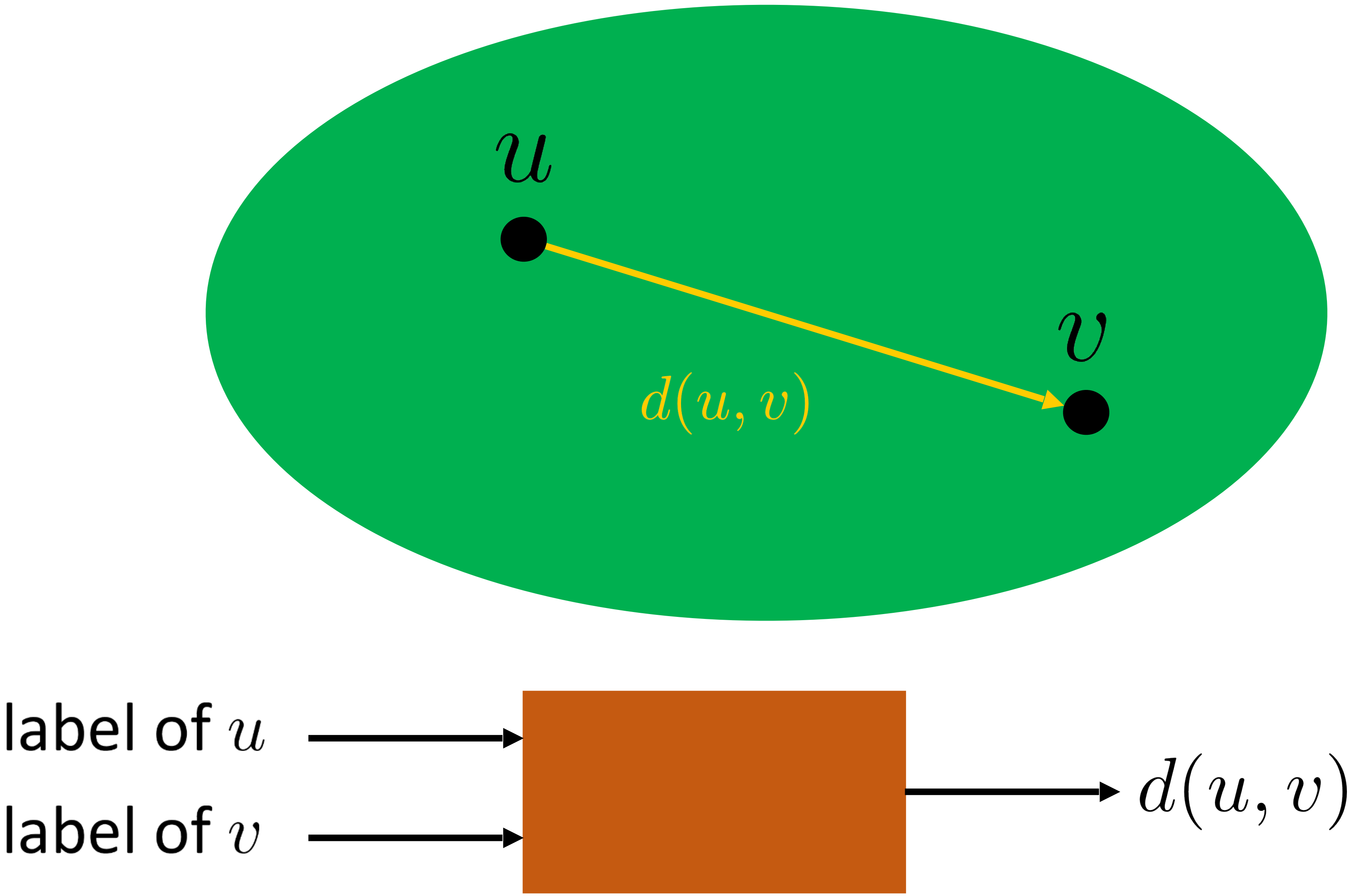

Some Results

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- General graphs: $\tilde{\Theta}(n)$

  [Gavoille et al. SODA '01]

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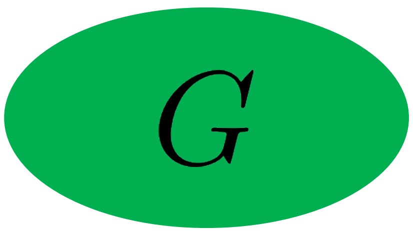

$$L(v_1), ..., L(v_n)$$

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$2^{\binom{n}{2}}$ different strings

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string size is $\log(2^{\binom{n}{2}}) = \Omega(n^2)$

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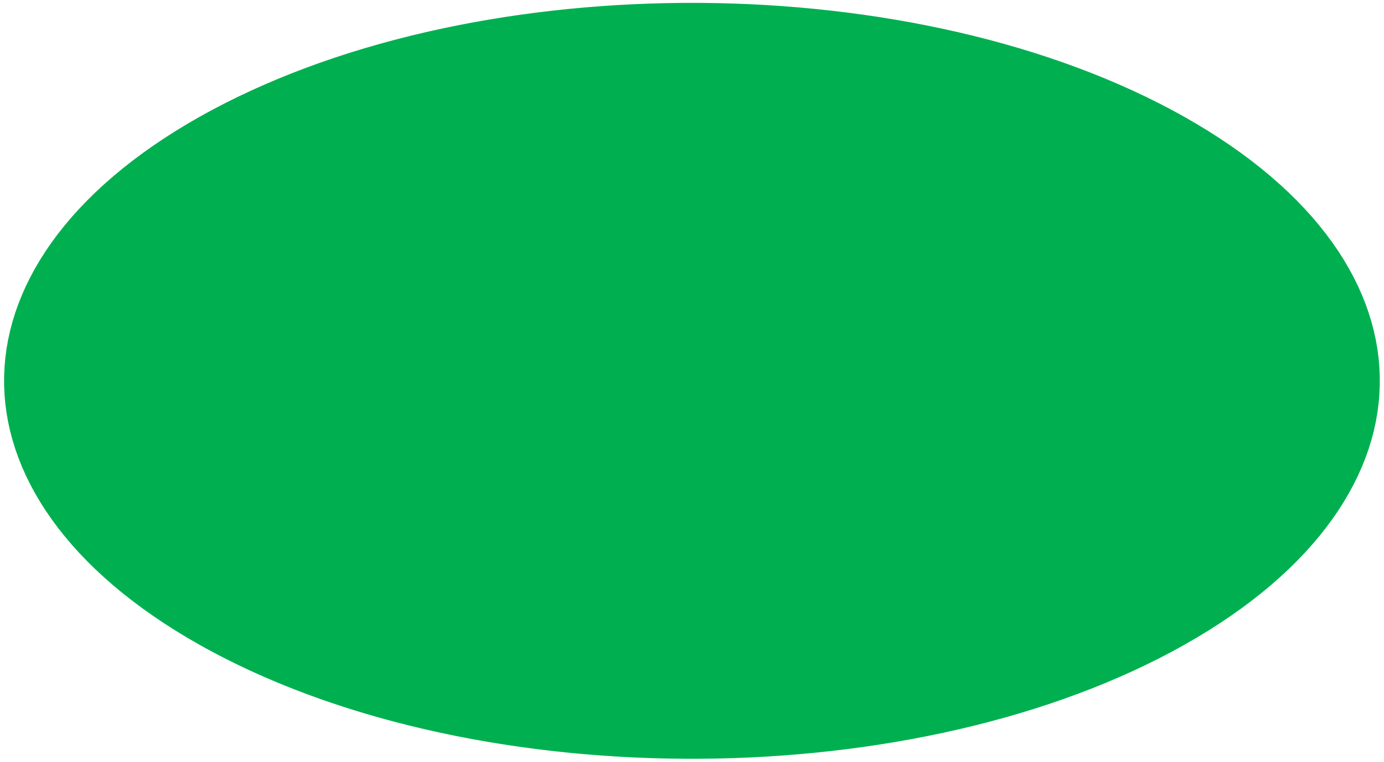
- Planar graphs $(1+\epsilon)$-approximation: $O(\log n/\epsilon)$
  [Thorup 2004]

Small Separator



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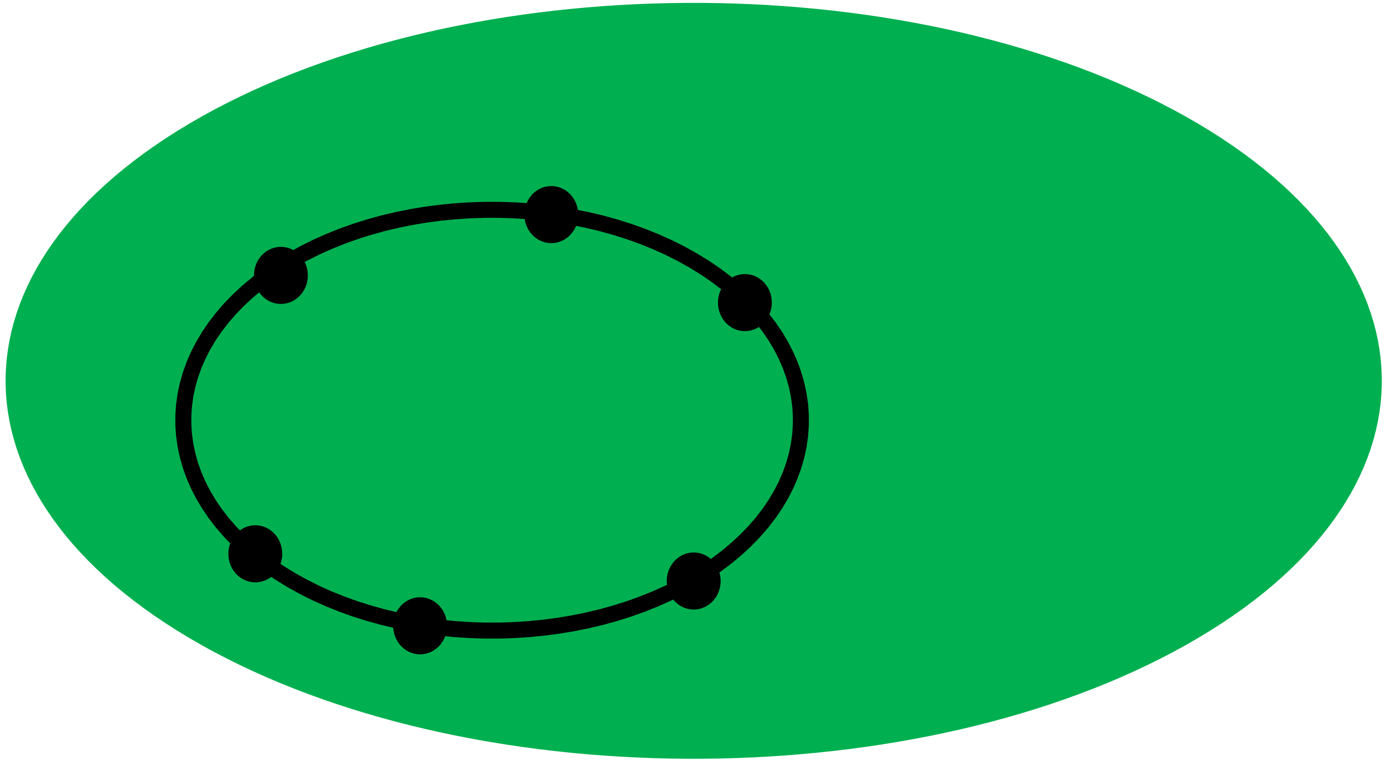

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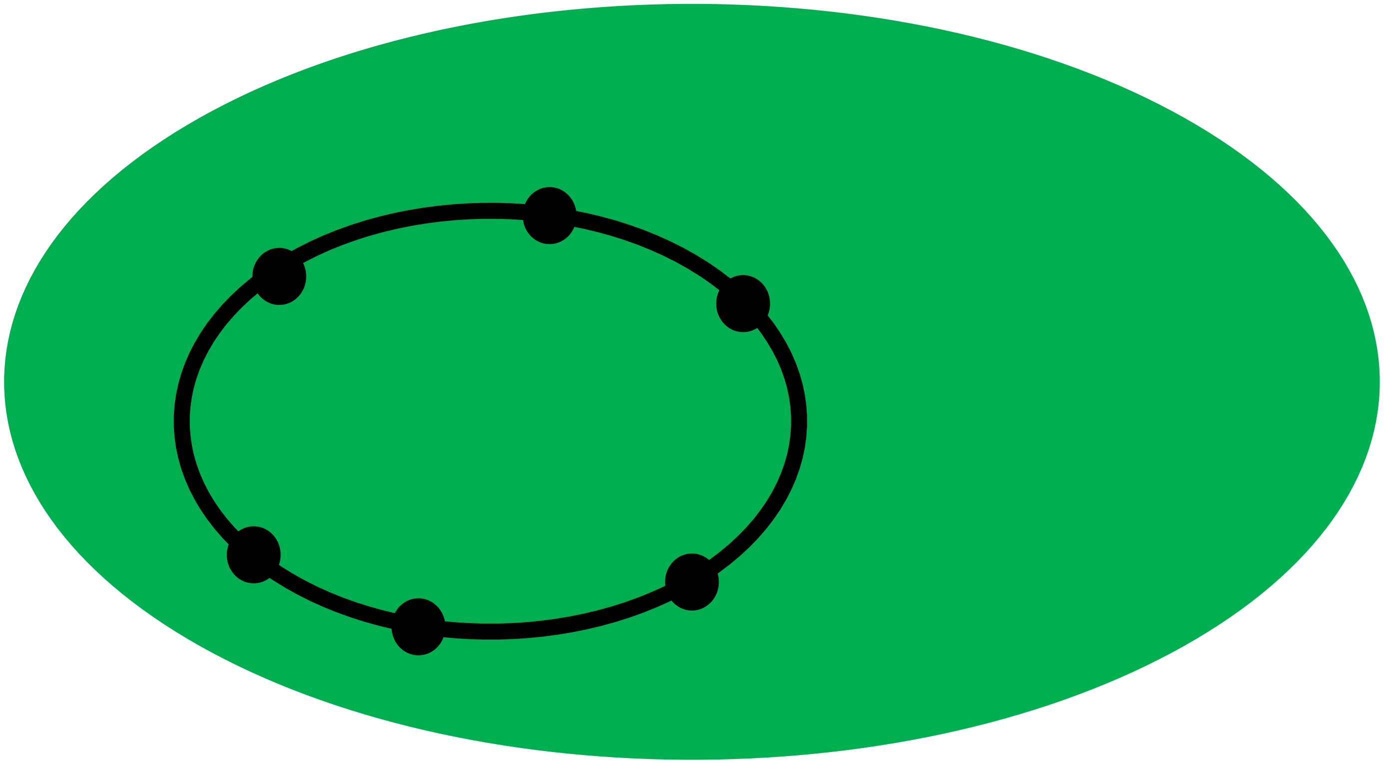

Seperator size $O(\sqrt{n})$

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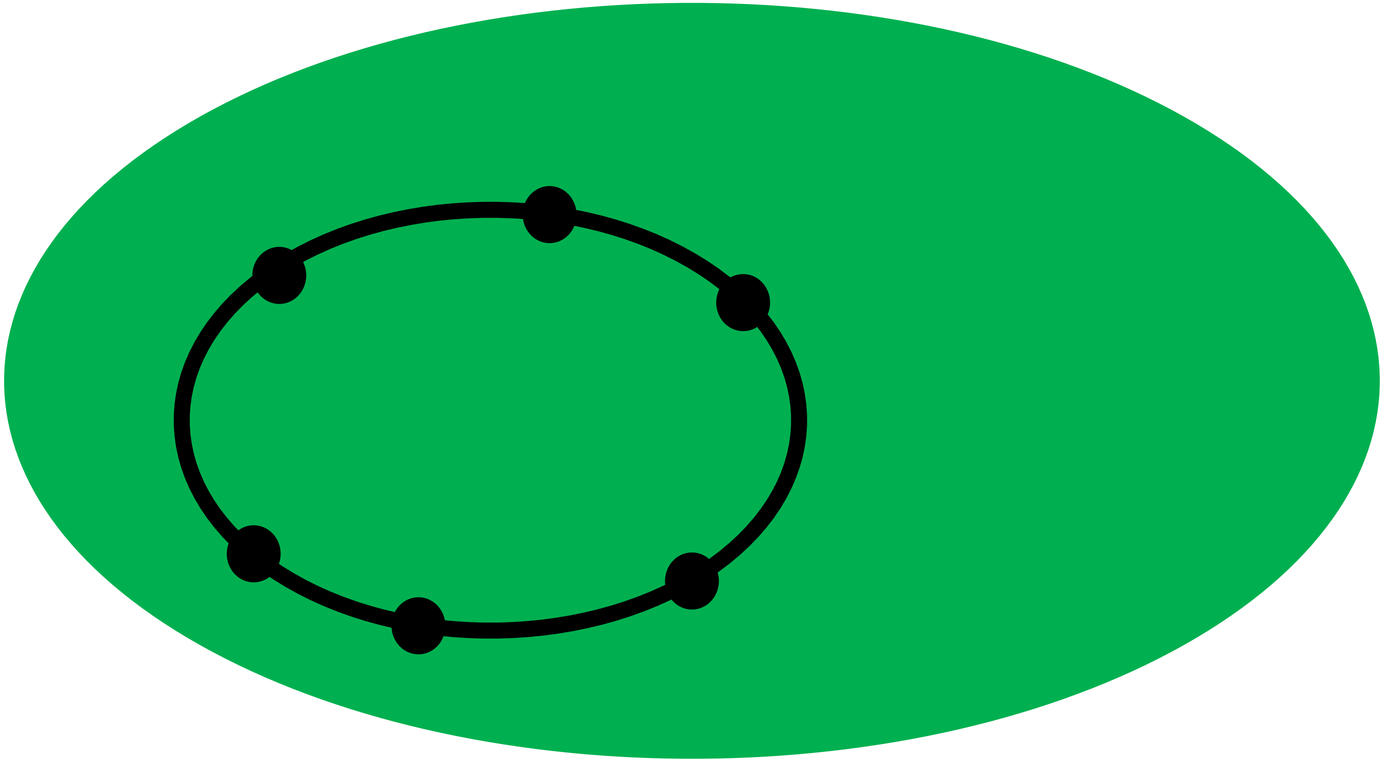
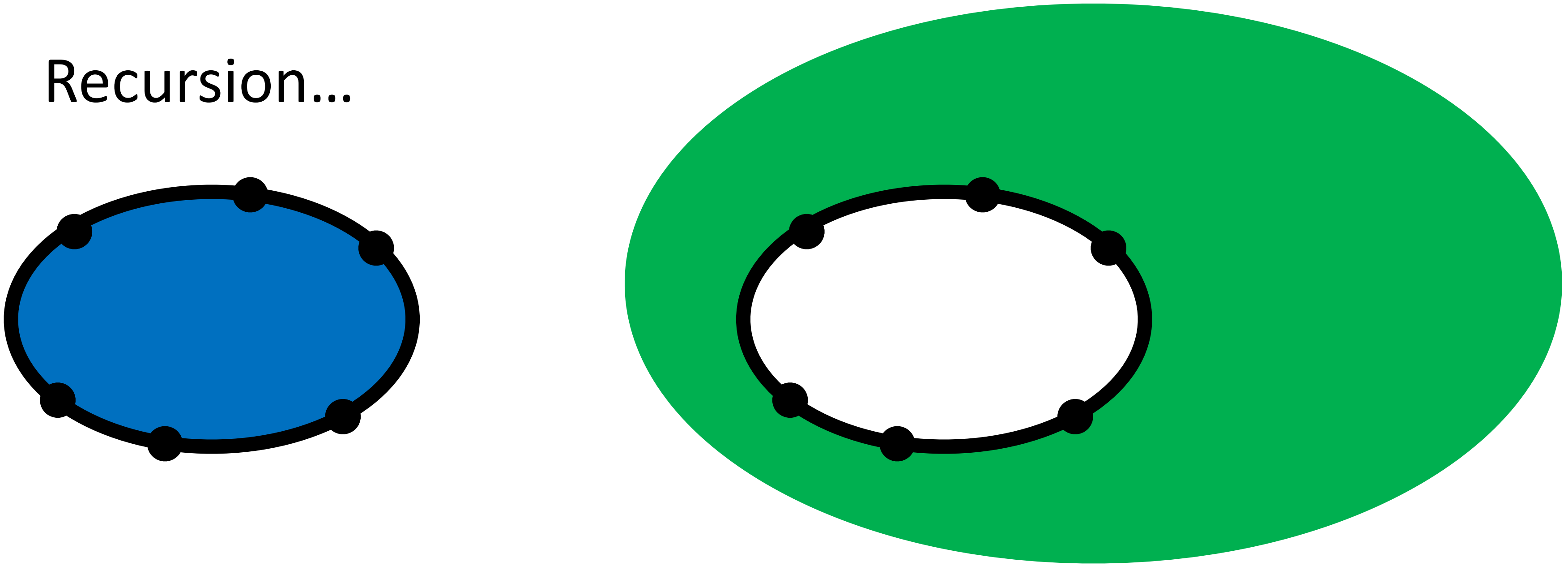


Seperator size $O(\sqrt{n})$

$$|green\ piece|, |blue\ piece| \leq 2n/3$$

Small Separator

Recursion…



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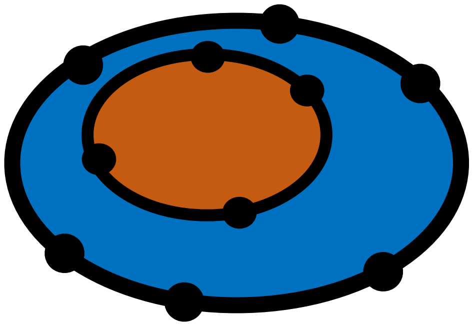

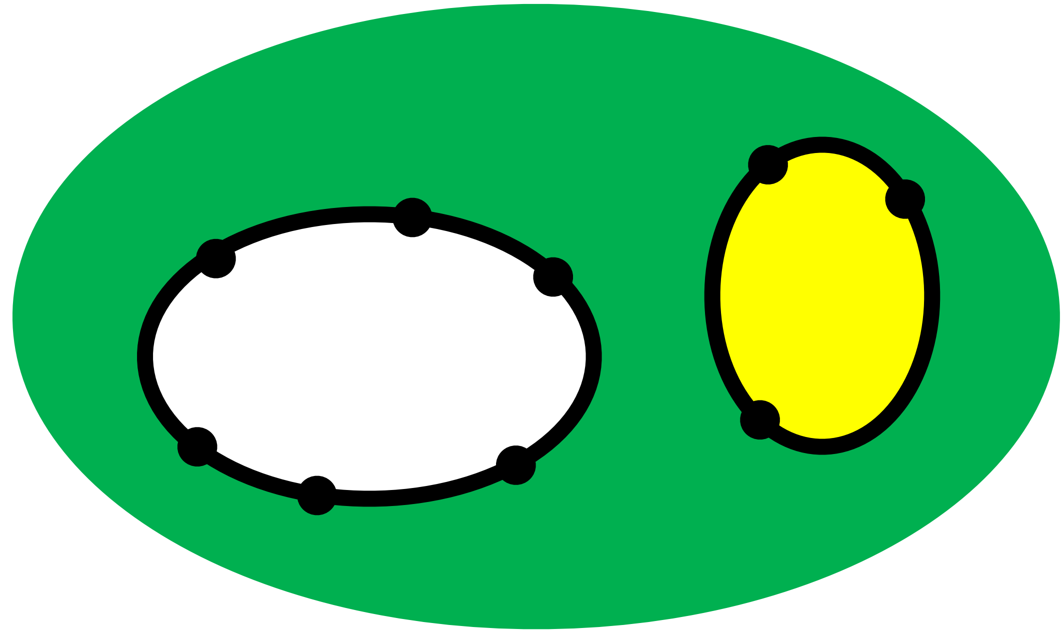

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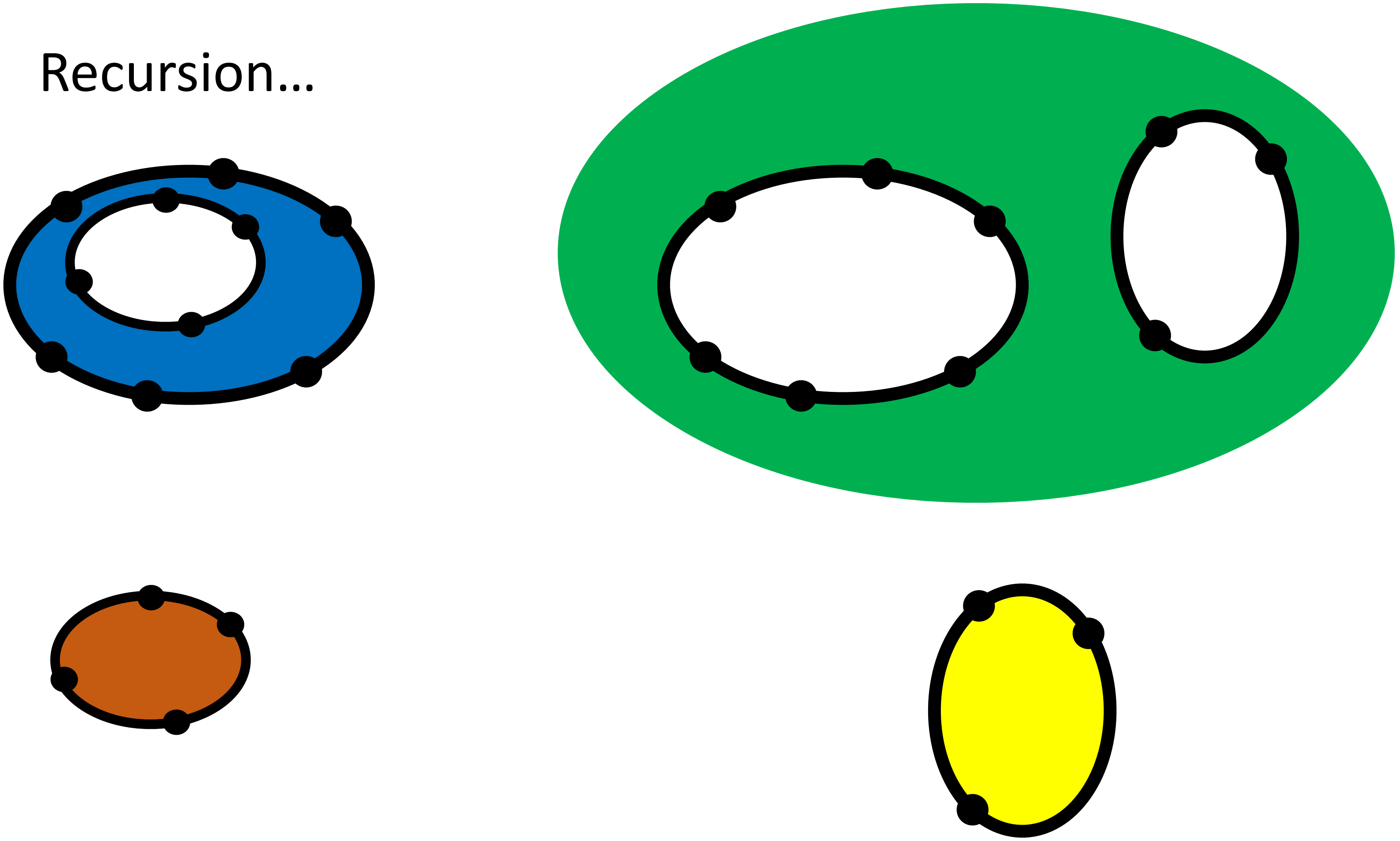
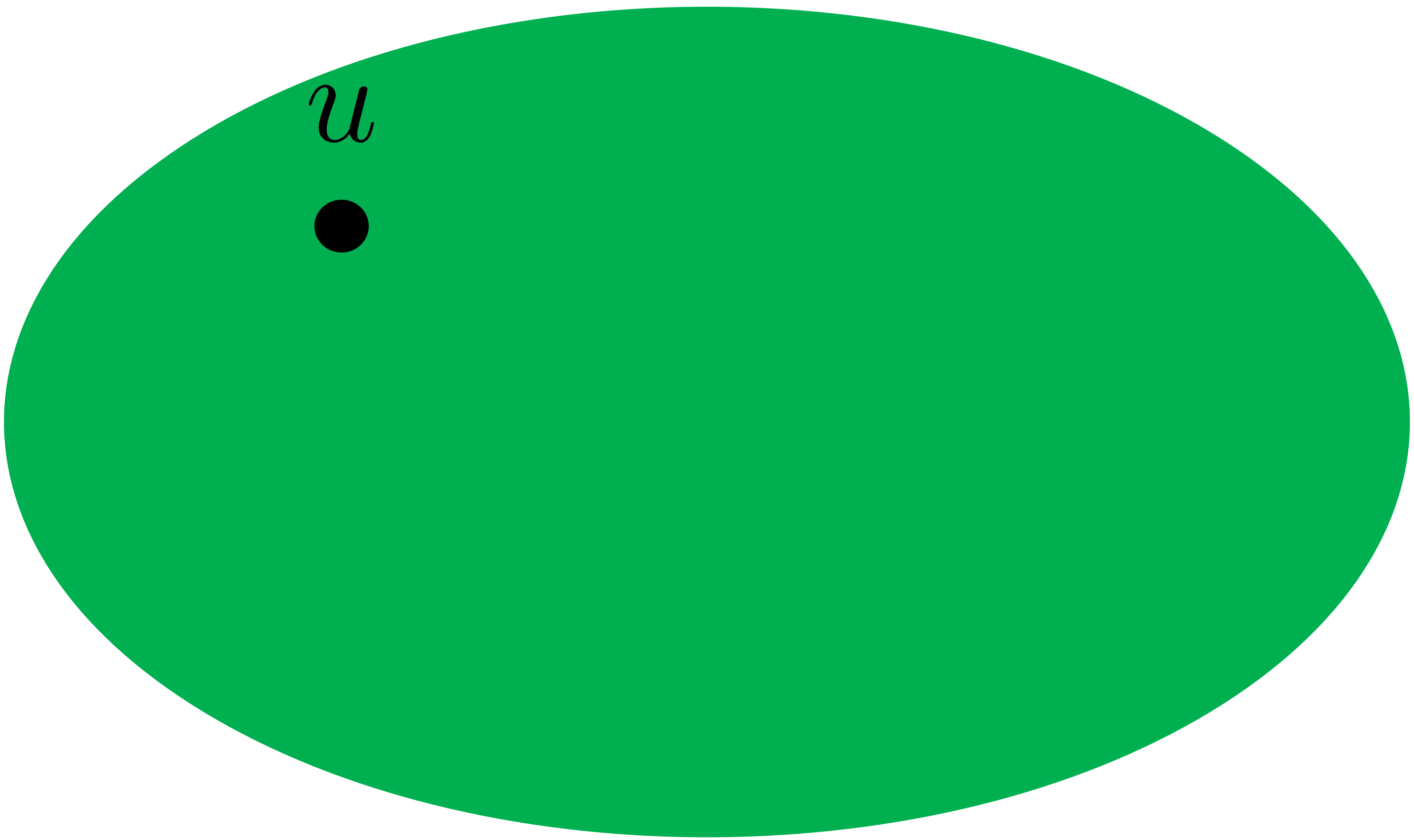


Example: $O(\sqrt{n} \cdot \log n)$ distance labeling

u's label:



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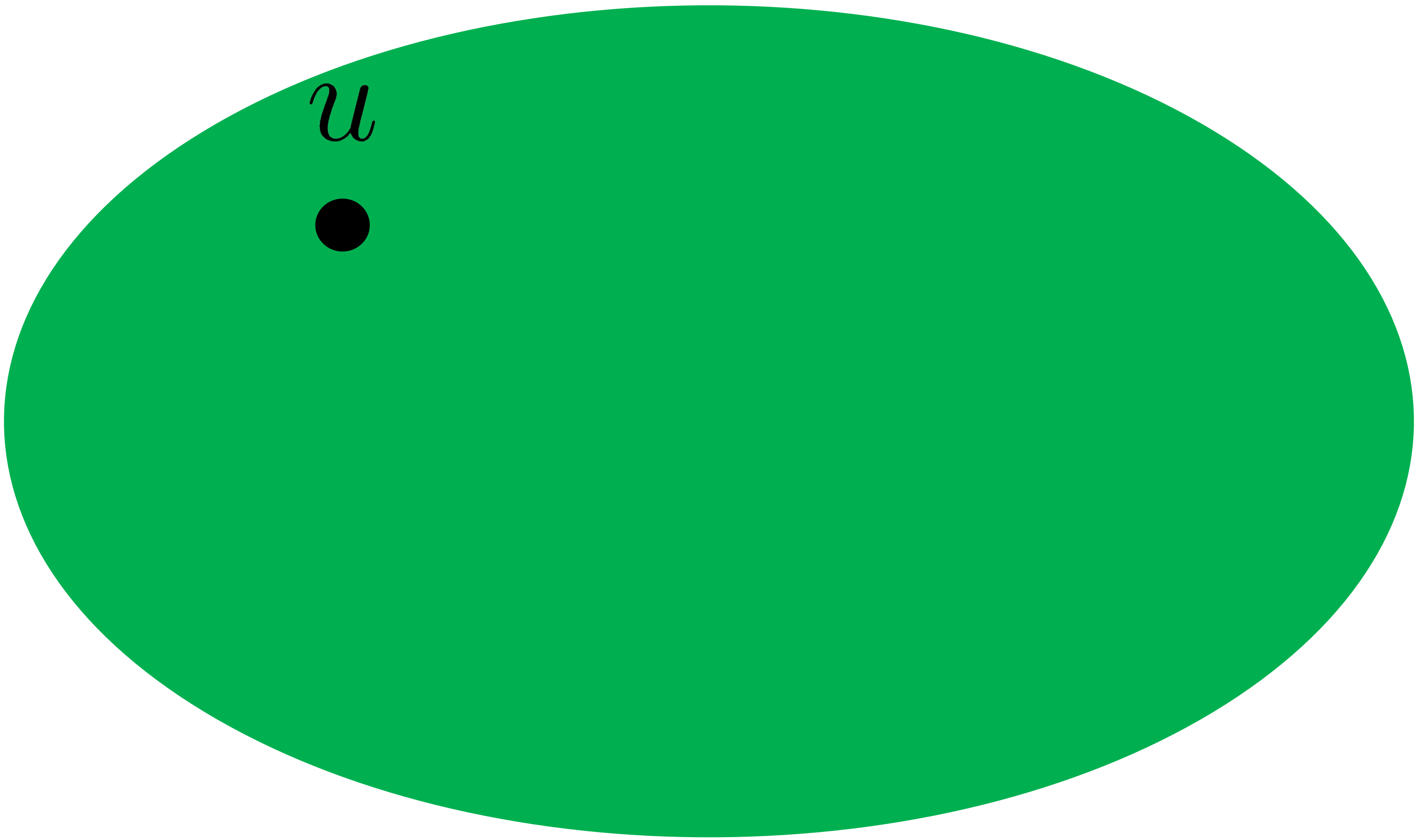

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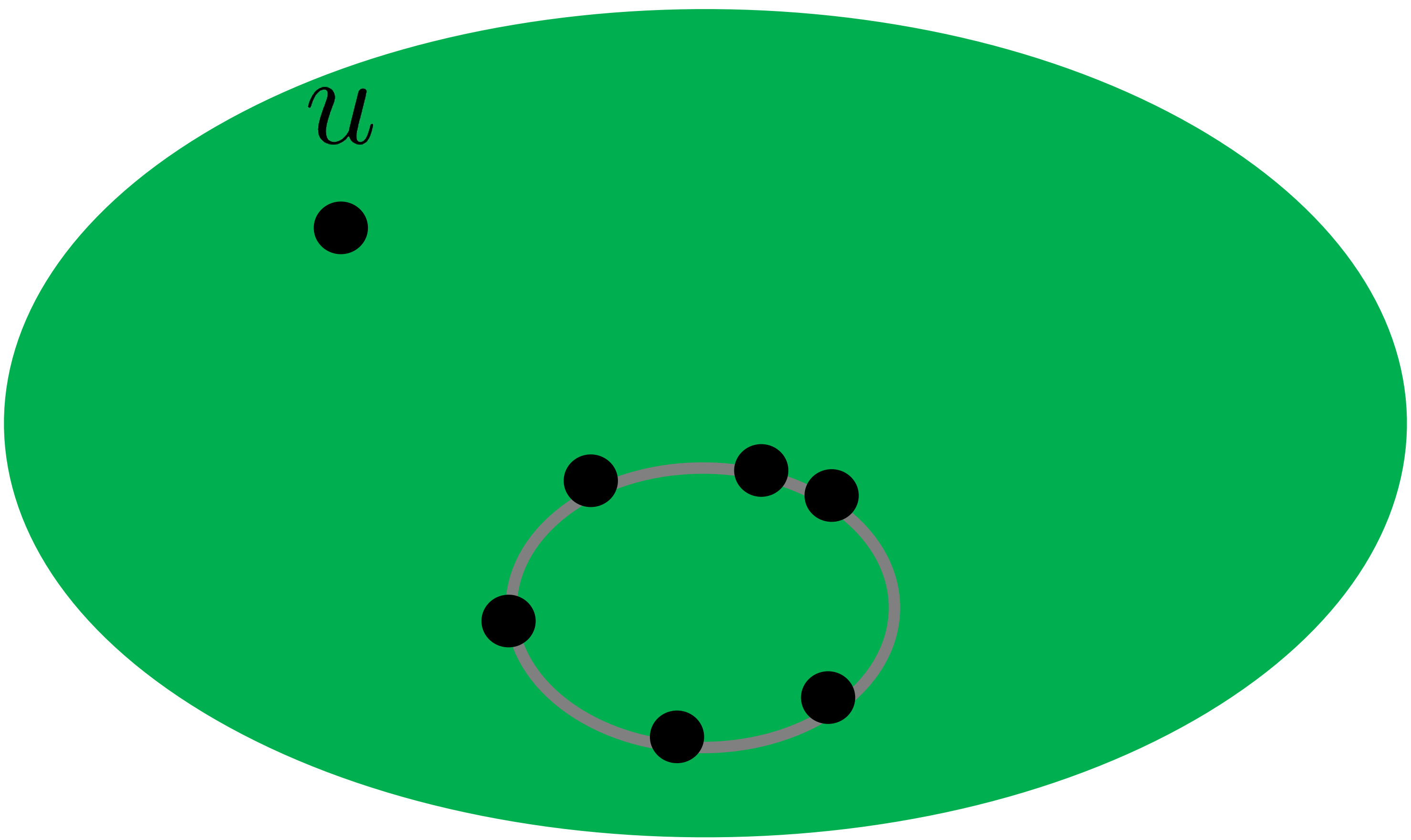

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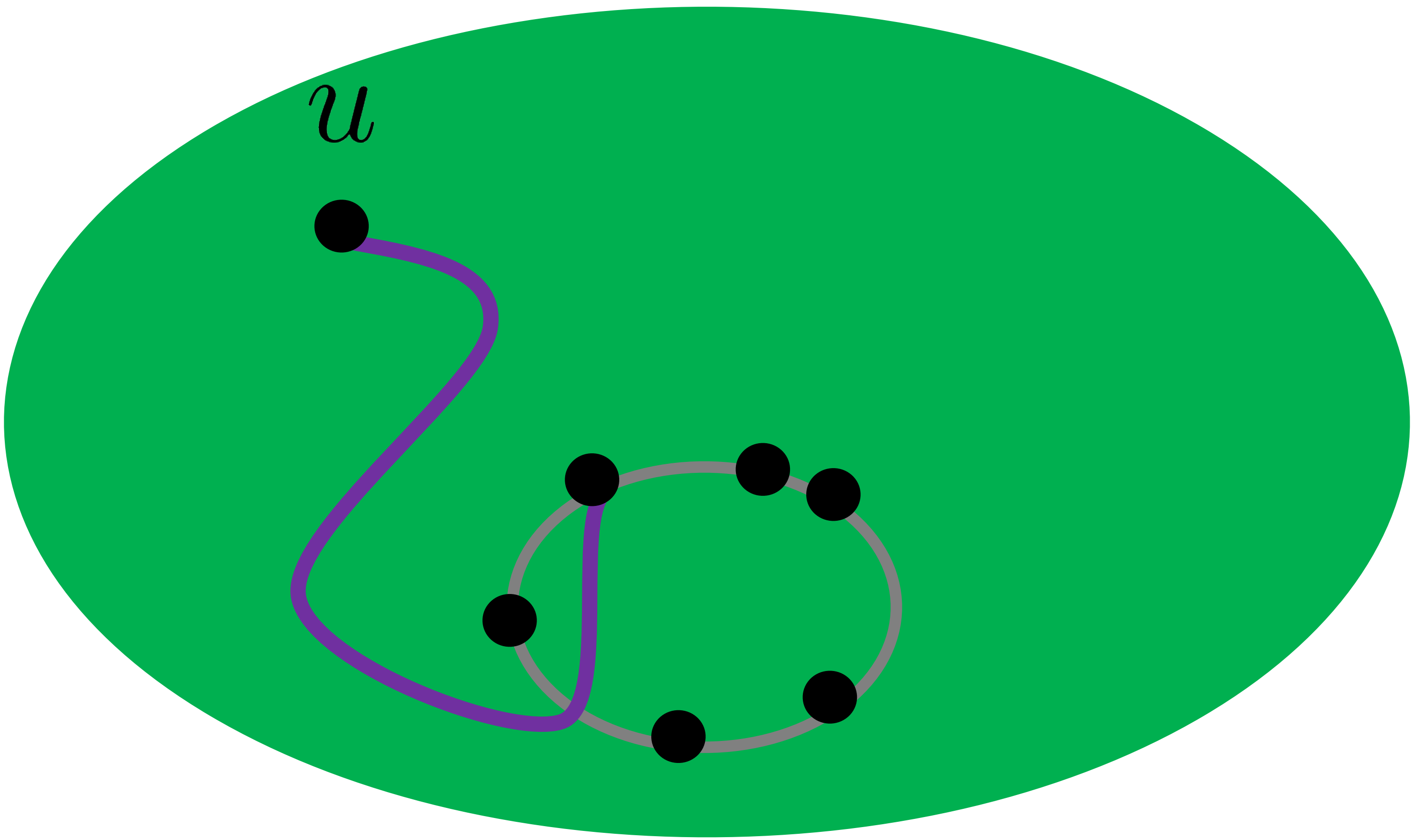

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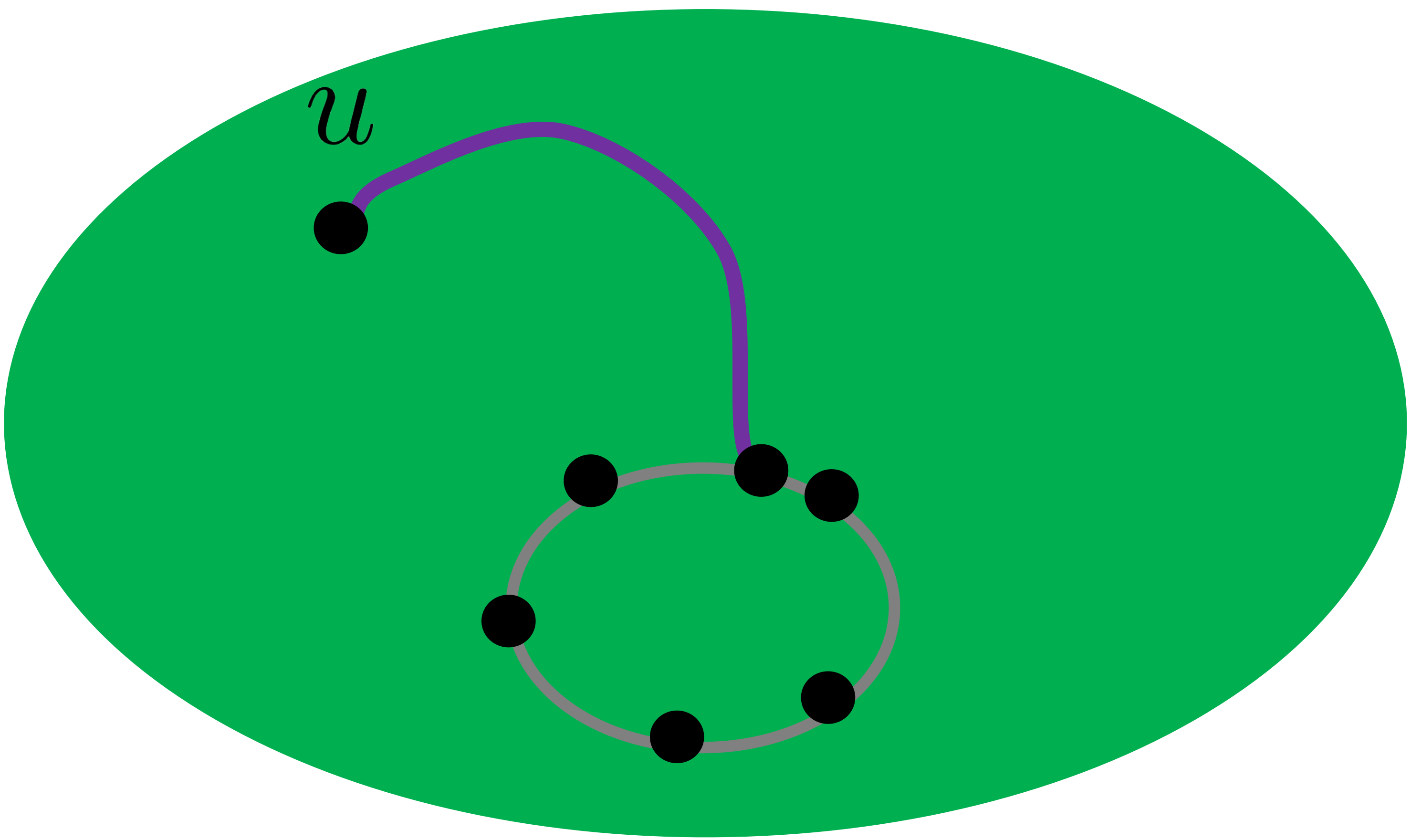

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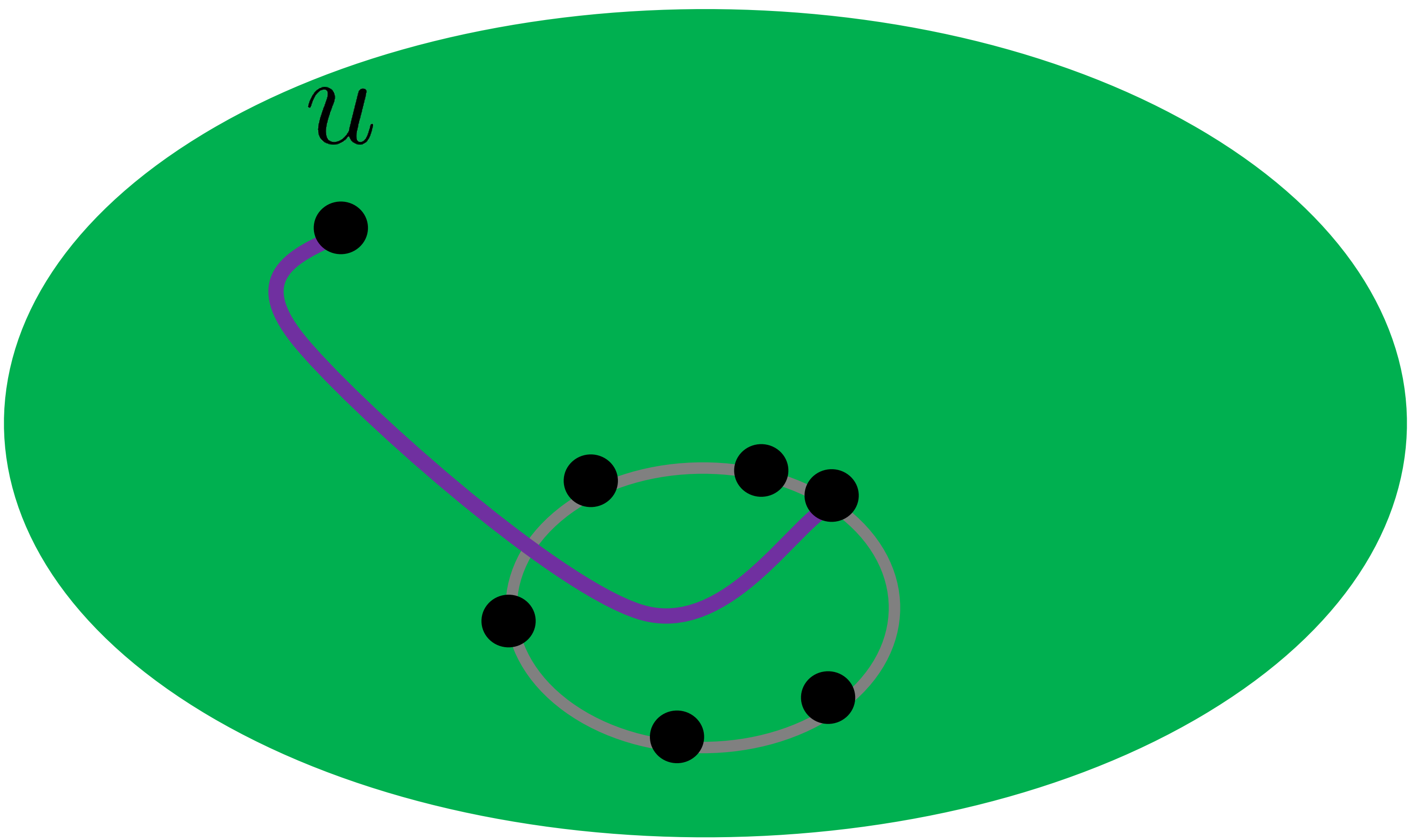

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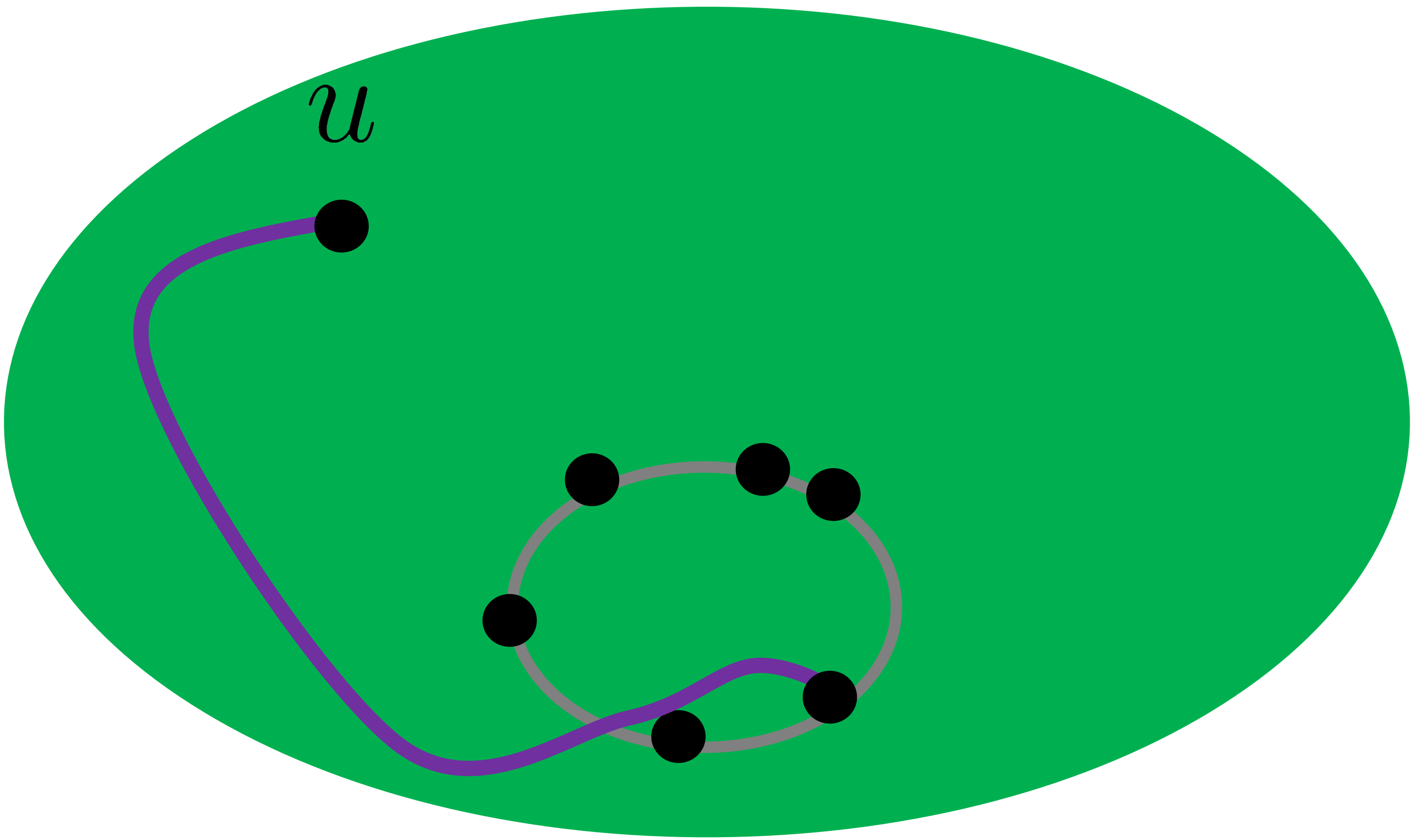

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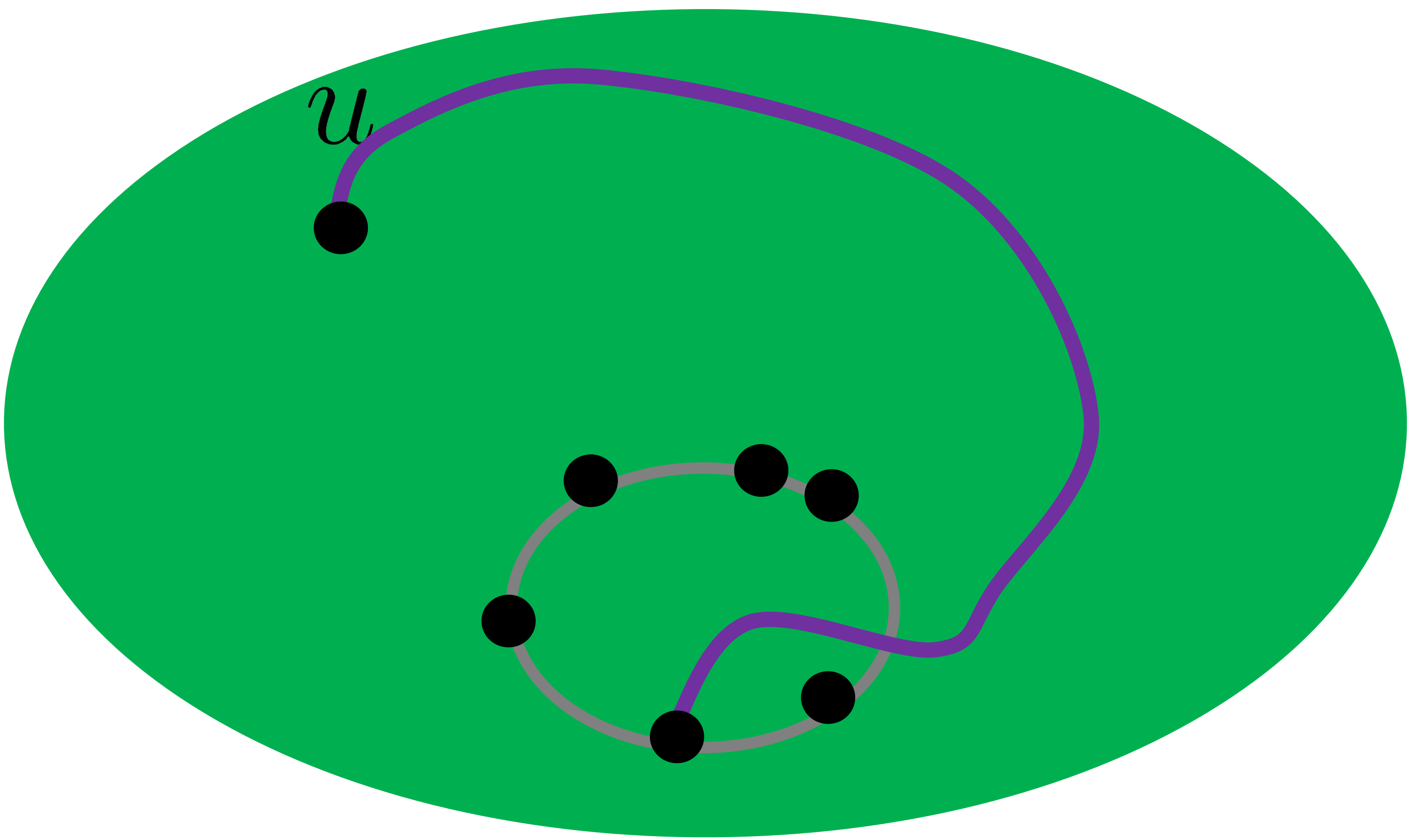

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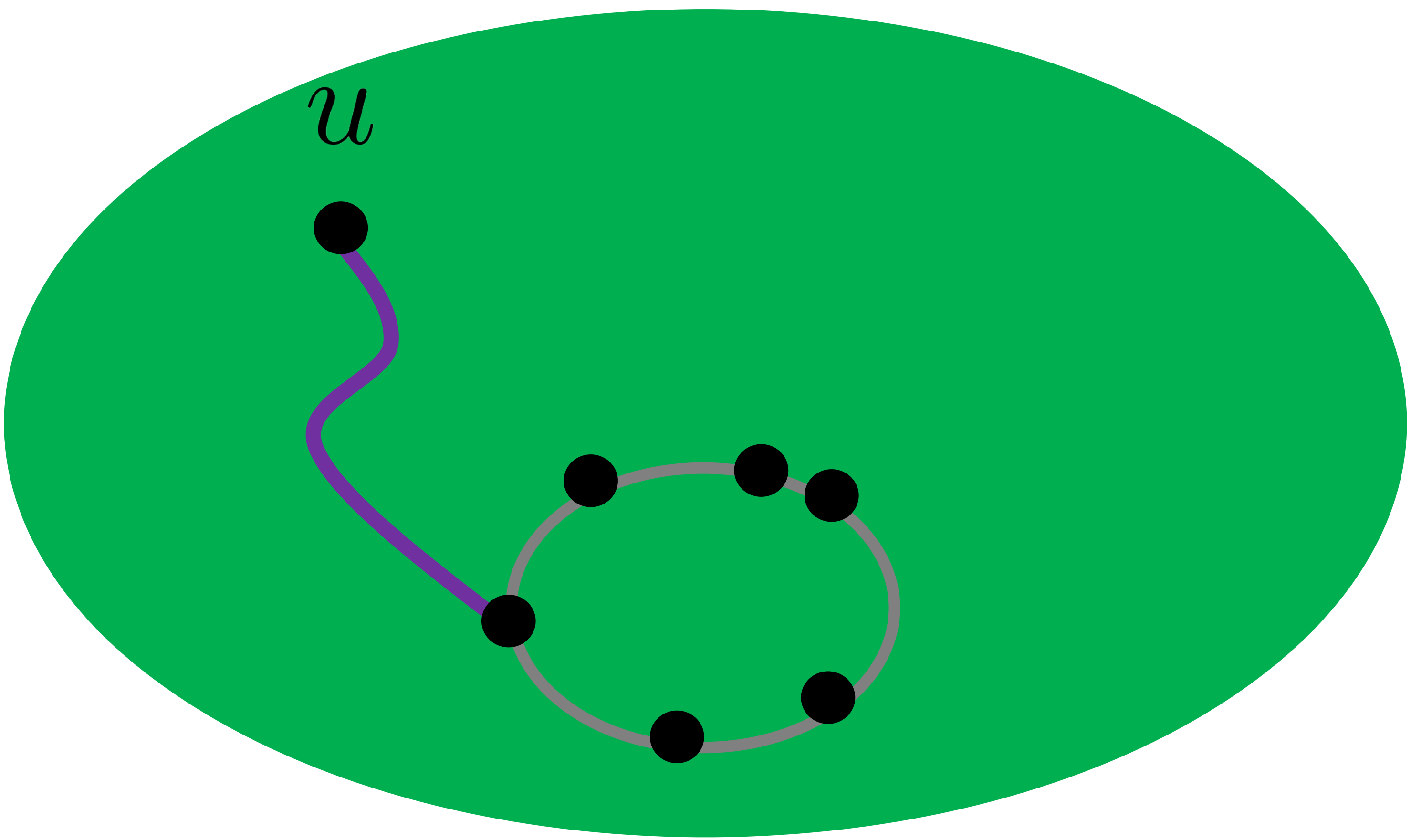

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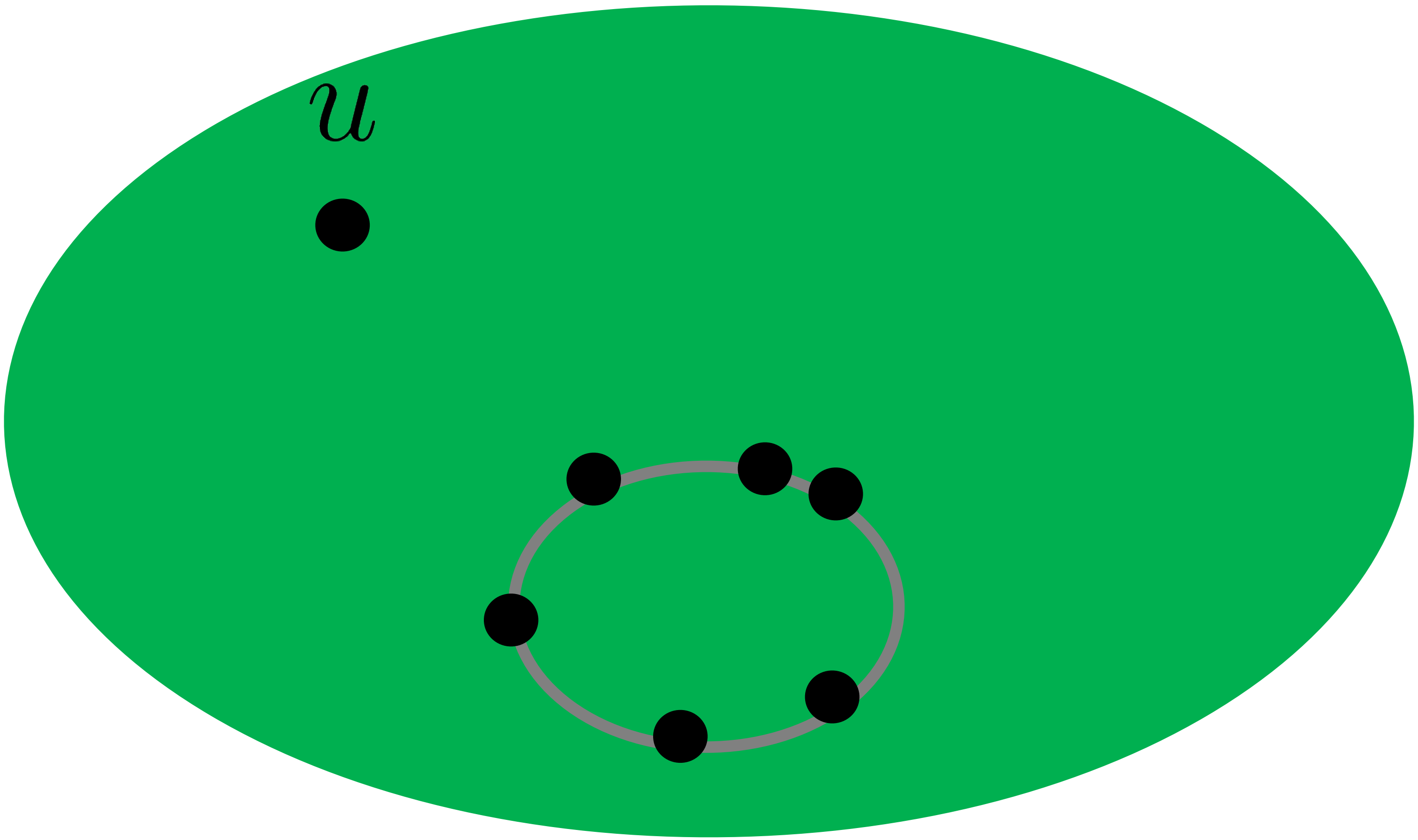

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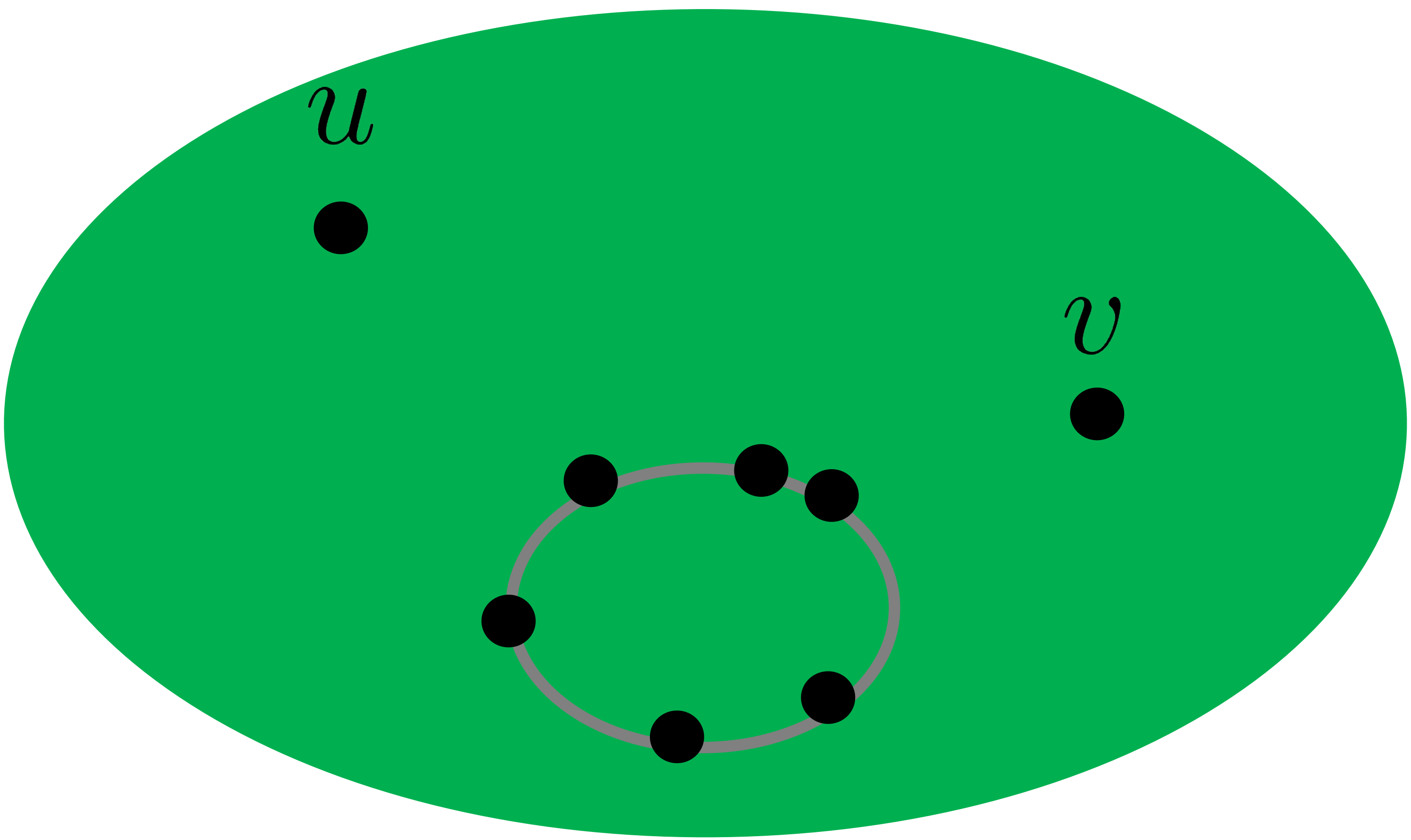
u, v query:



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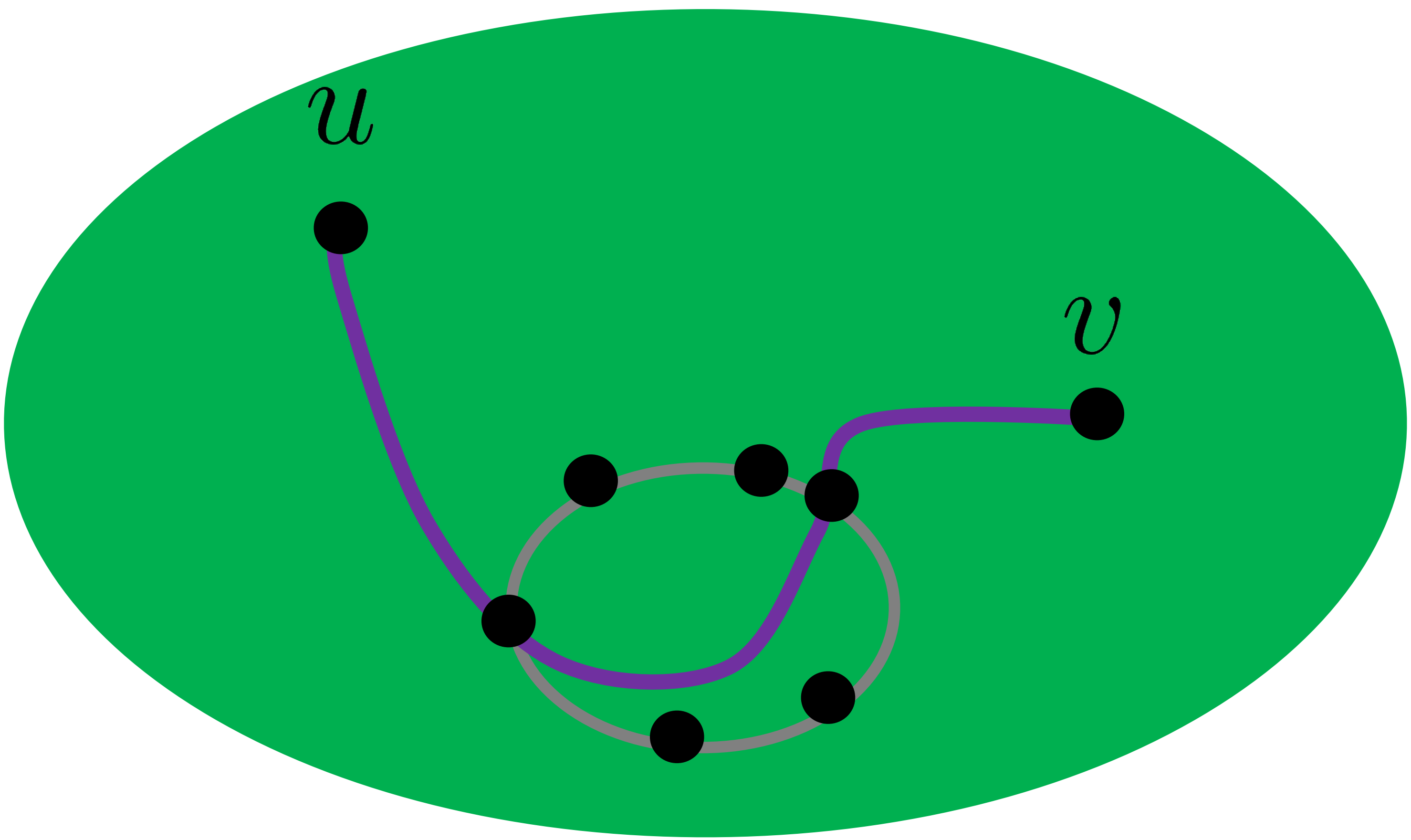

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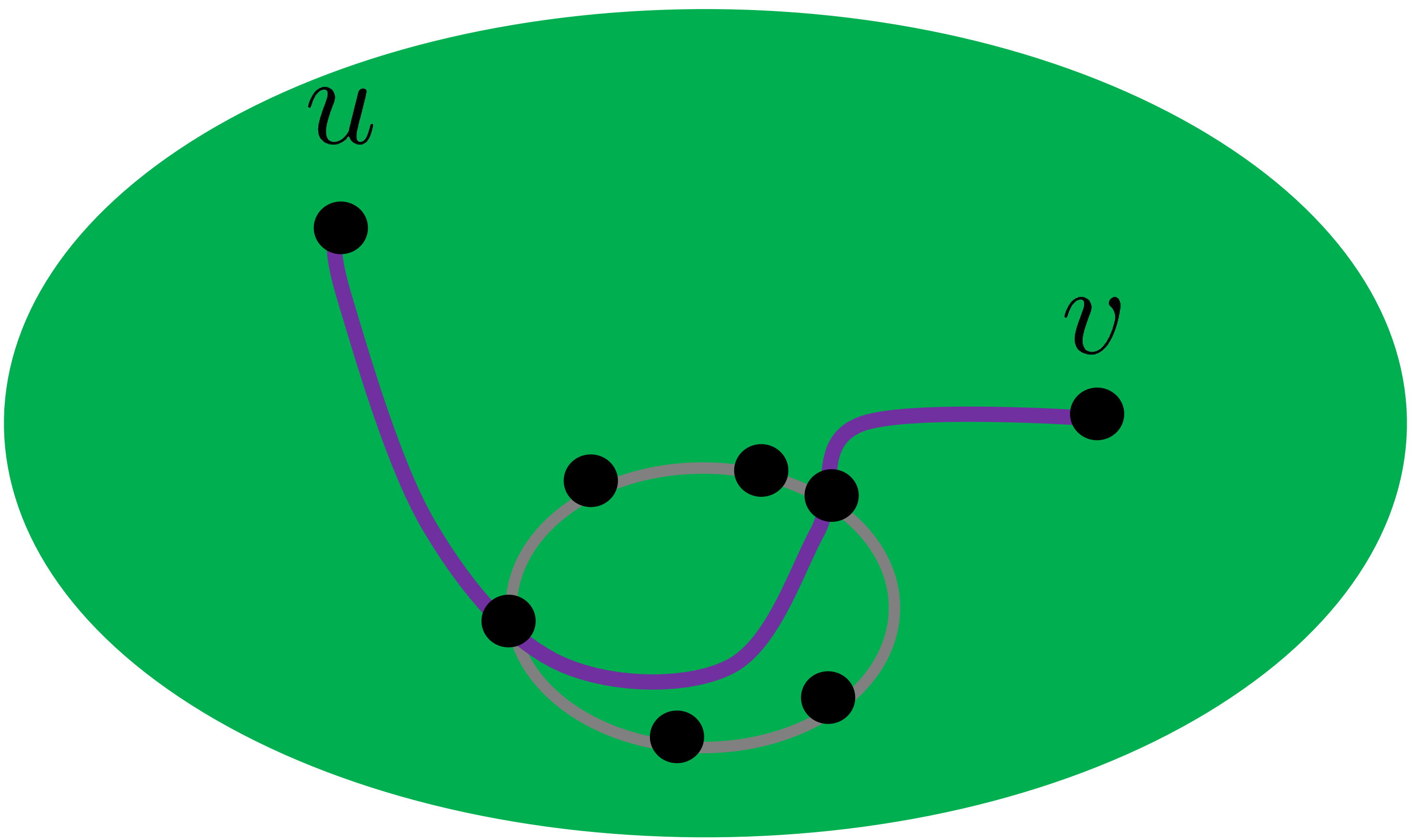

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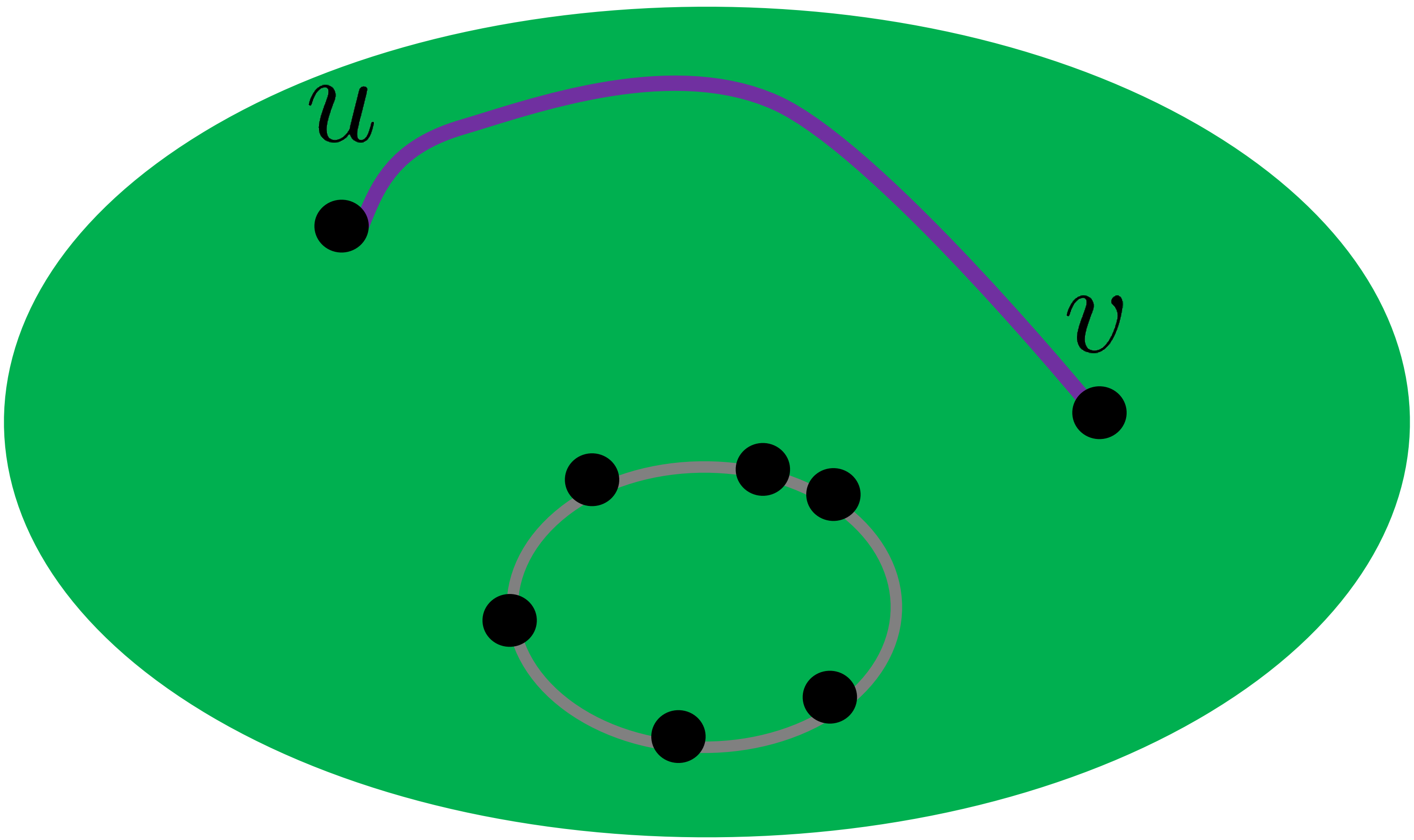
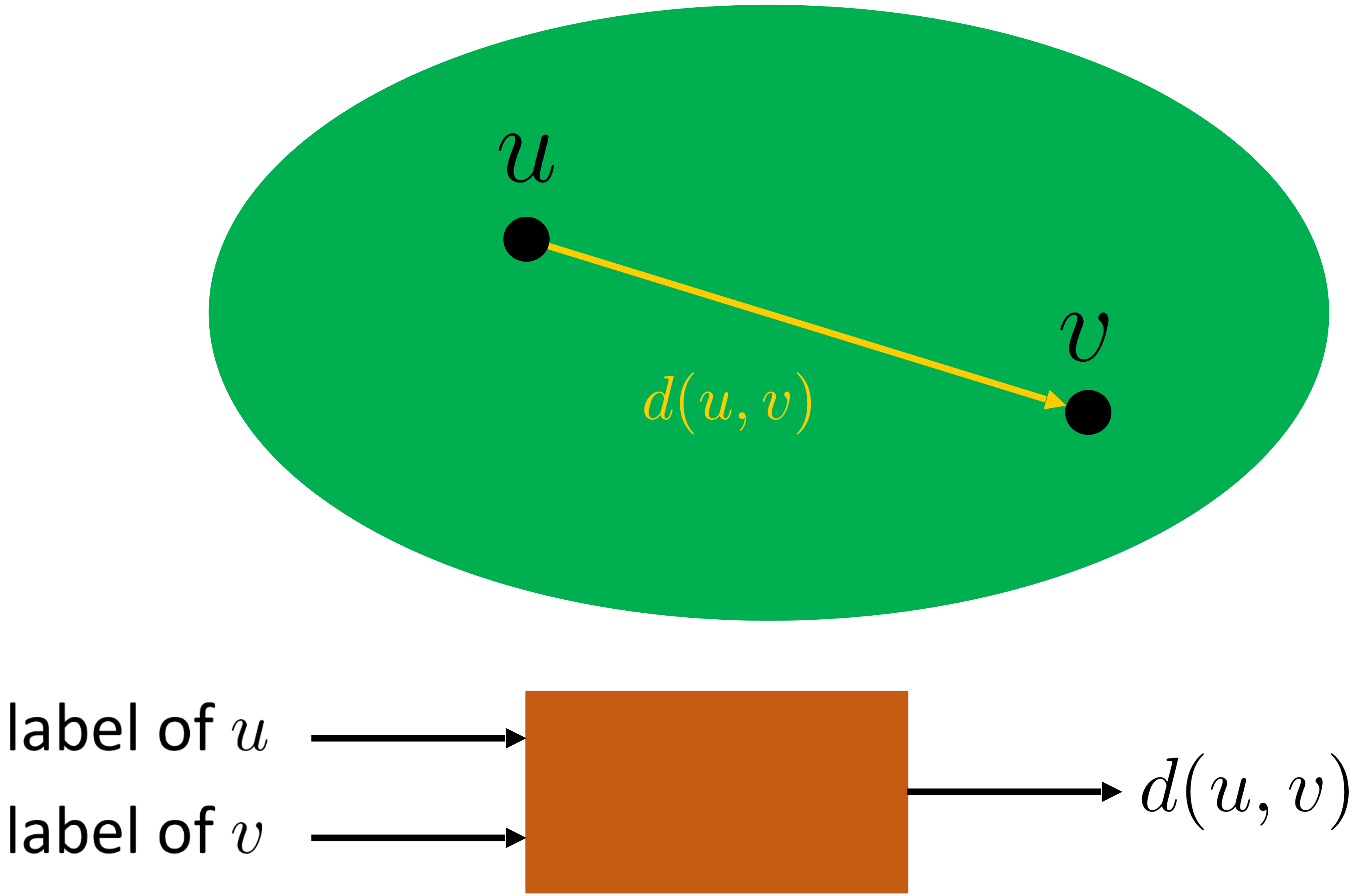


Fault-Tolerant Setting



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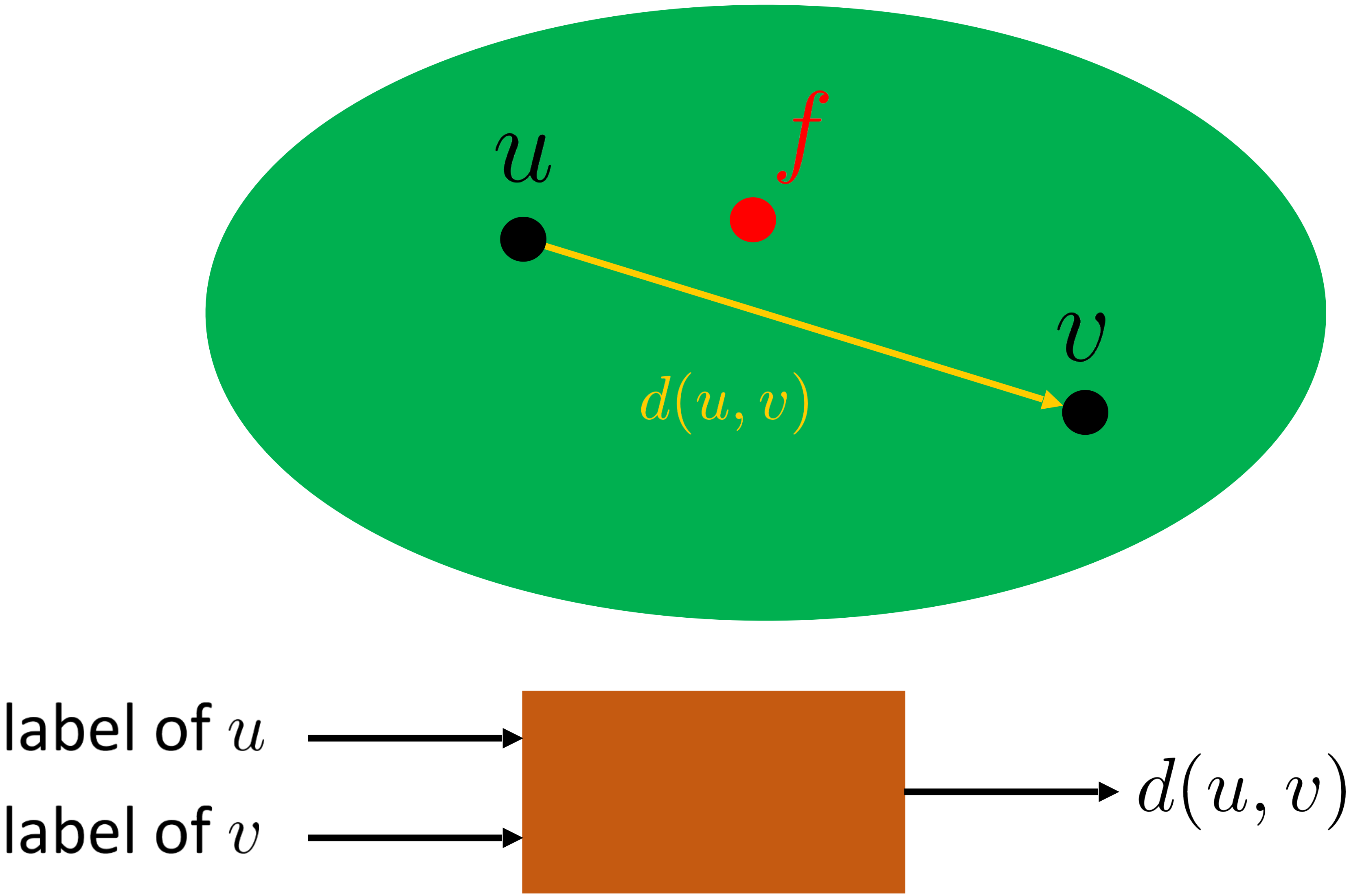

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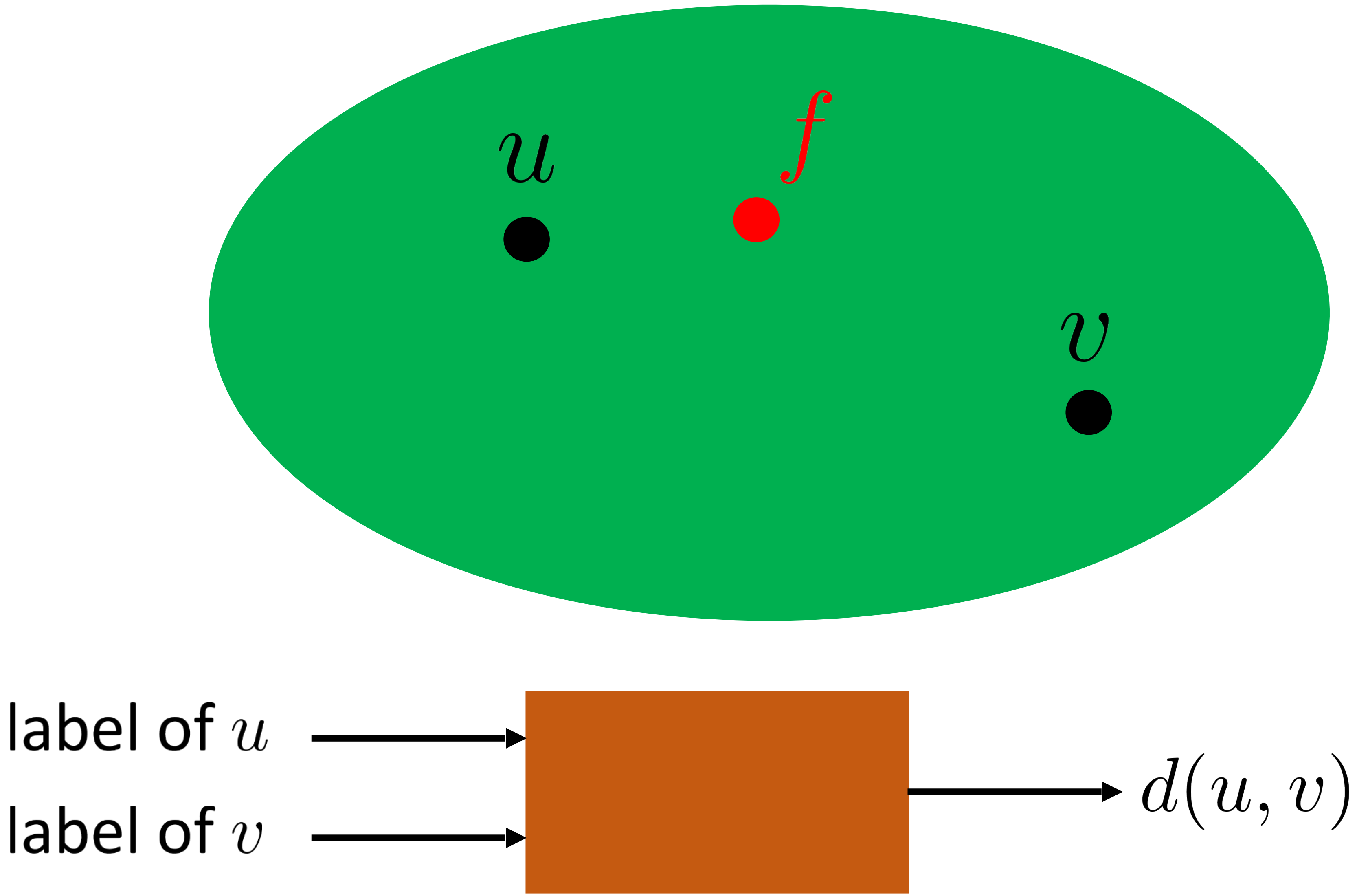

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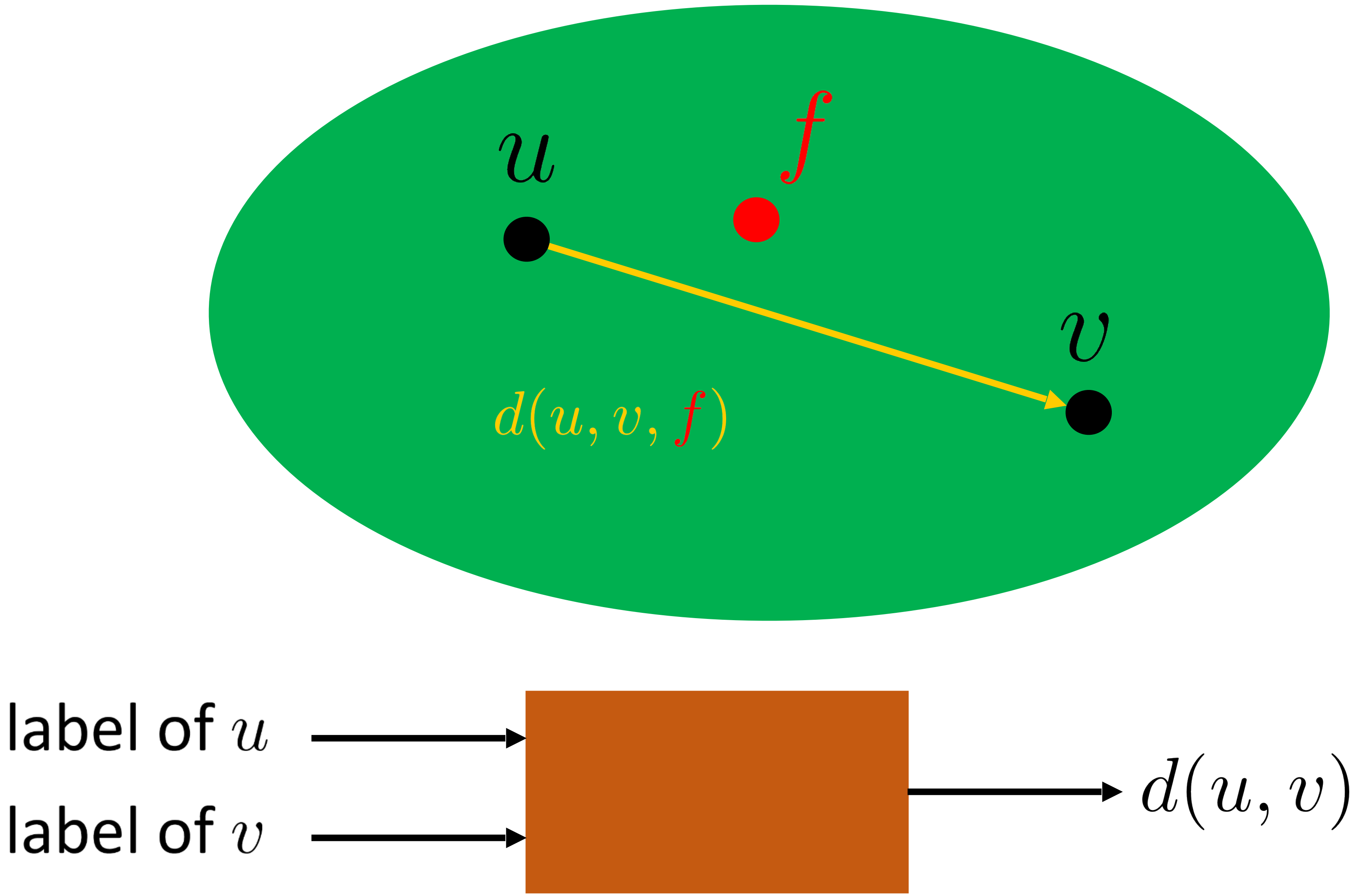

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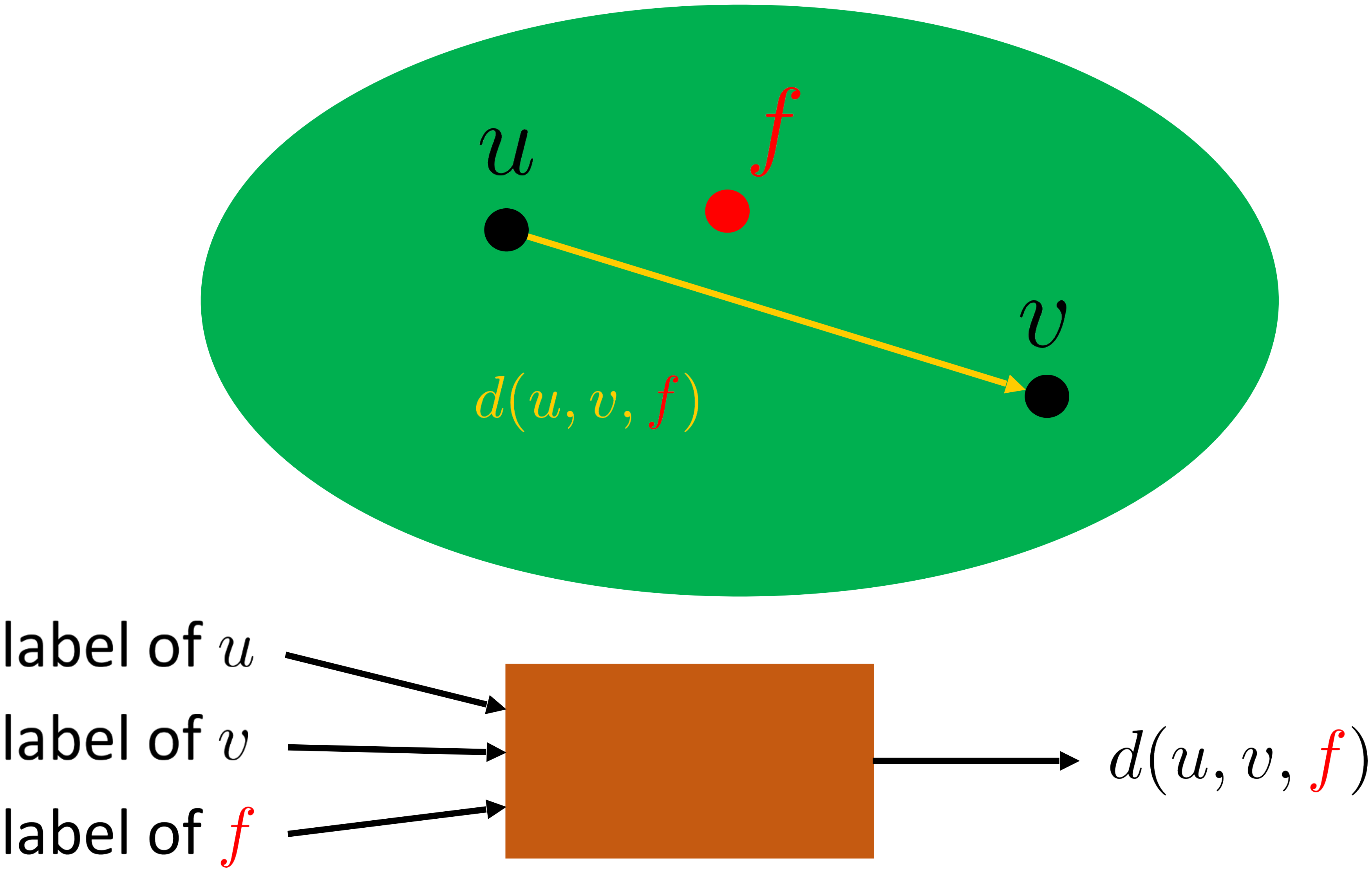

Our result

Labels of size $\tilde{O}(n^{2/3})$ for fault-tolerant distance labeling in planar graphs

Recursive decomposition tree

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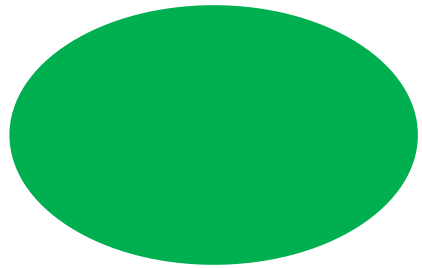

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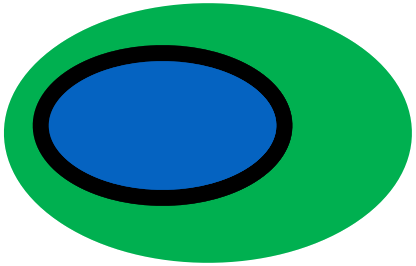

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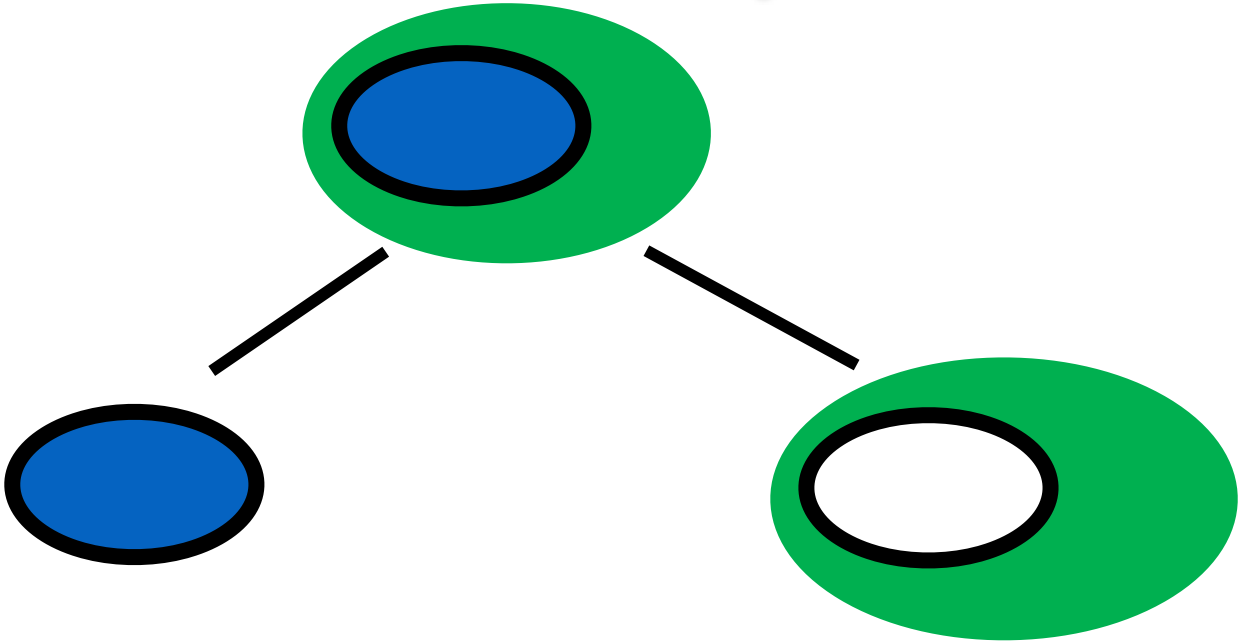

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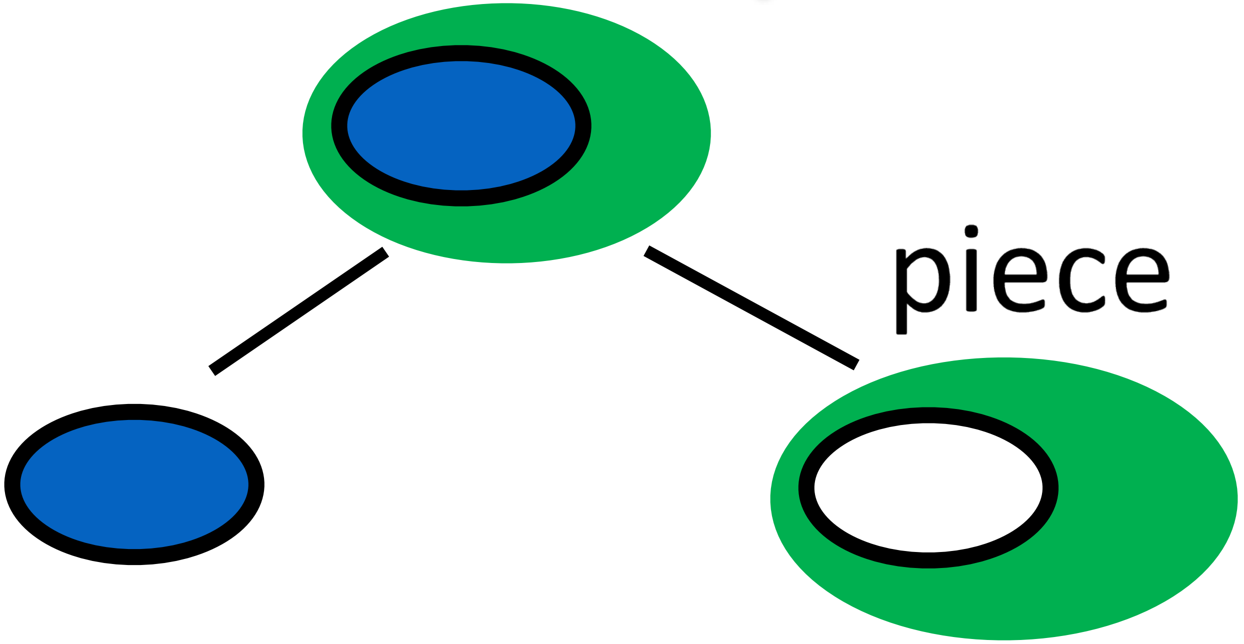

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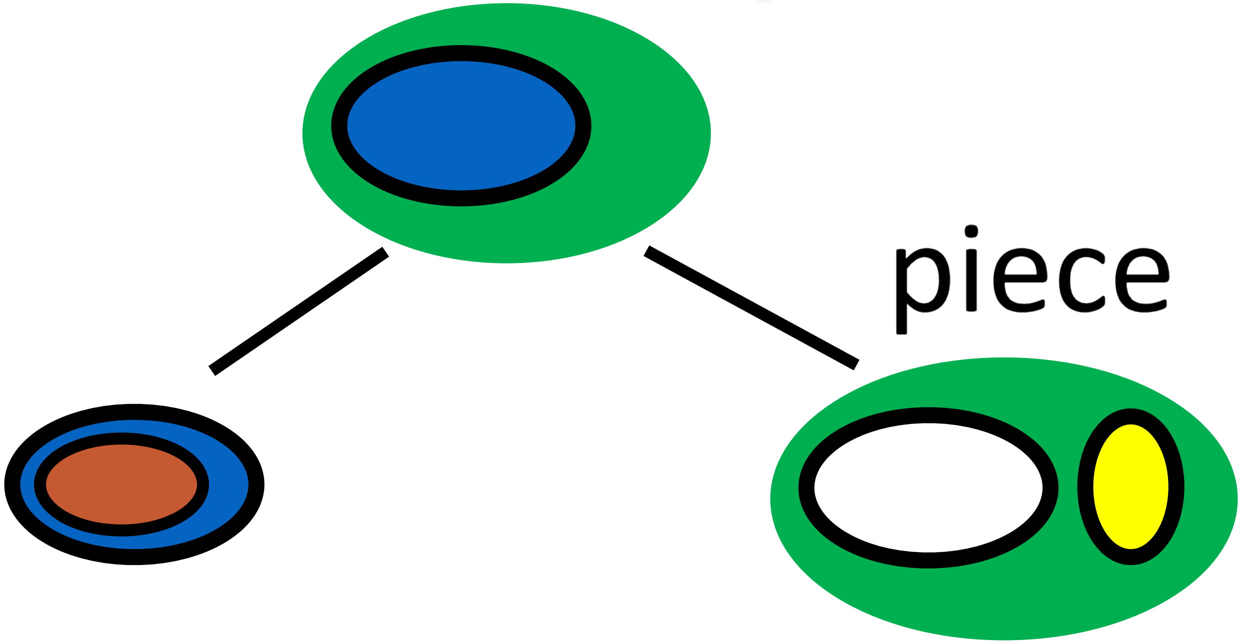

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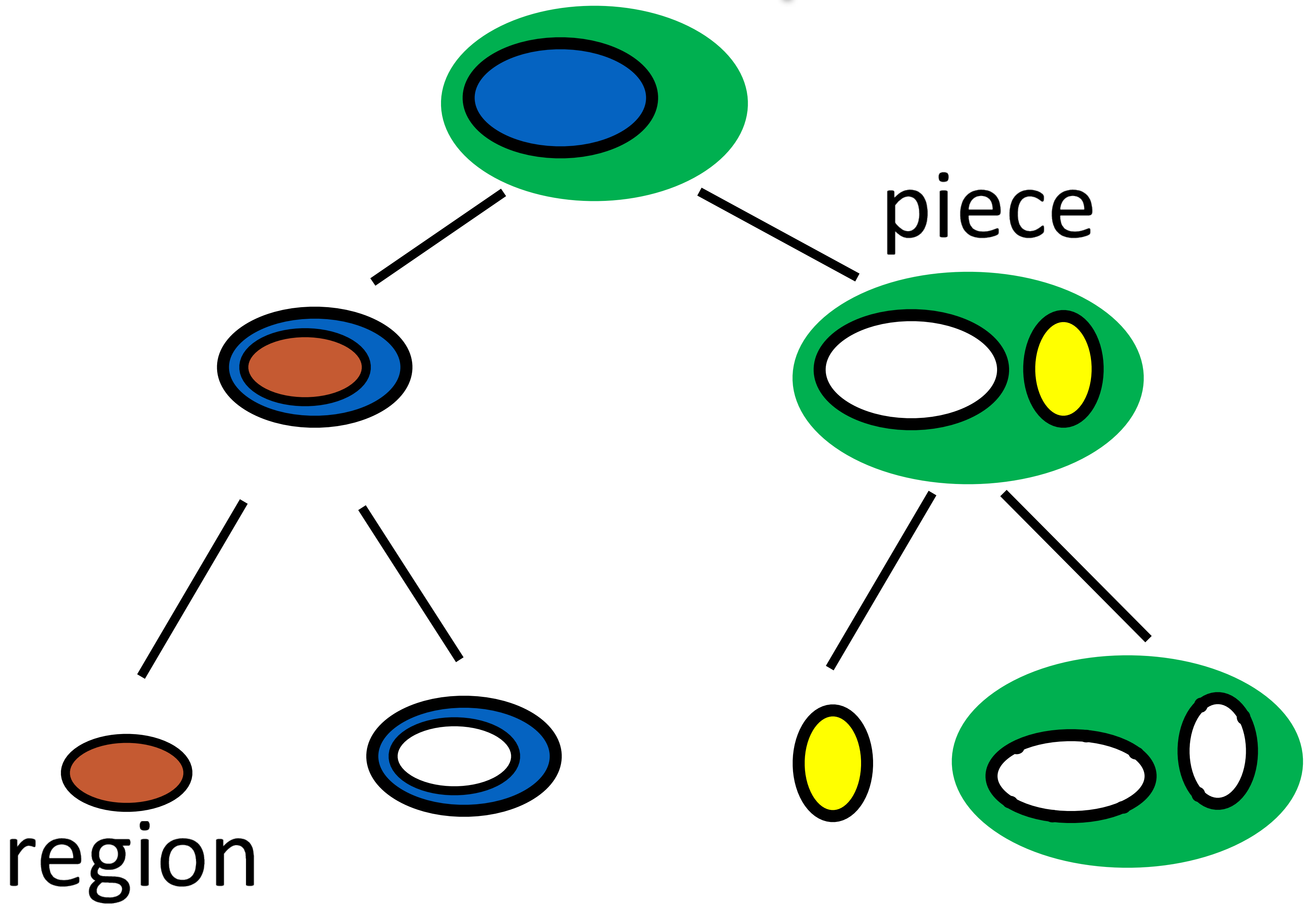
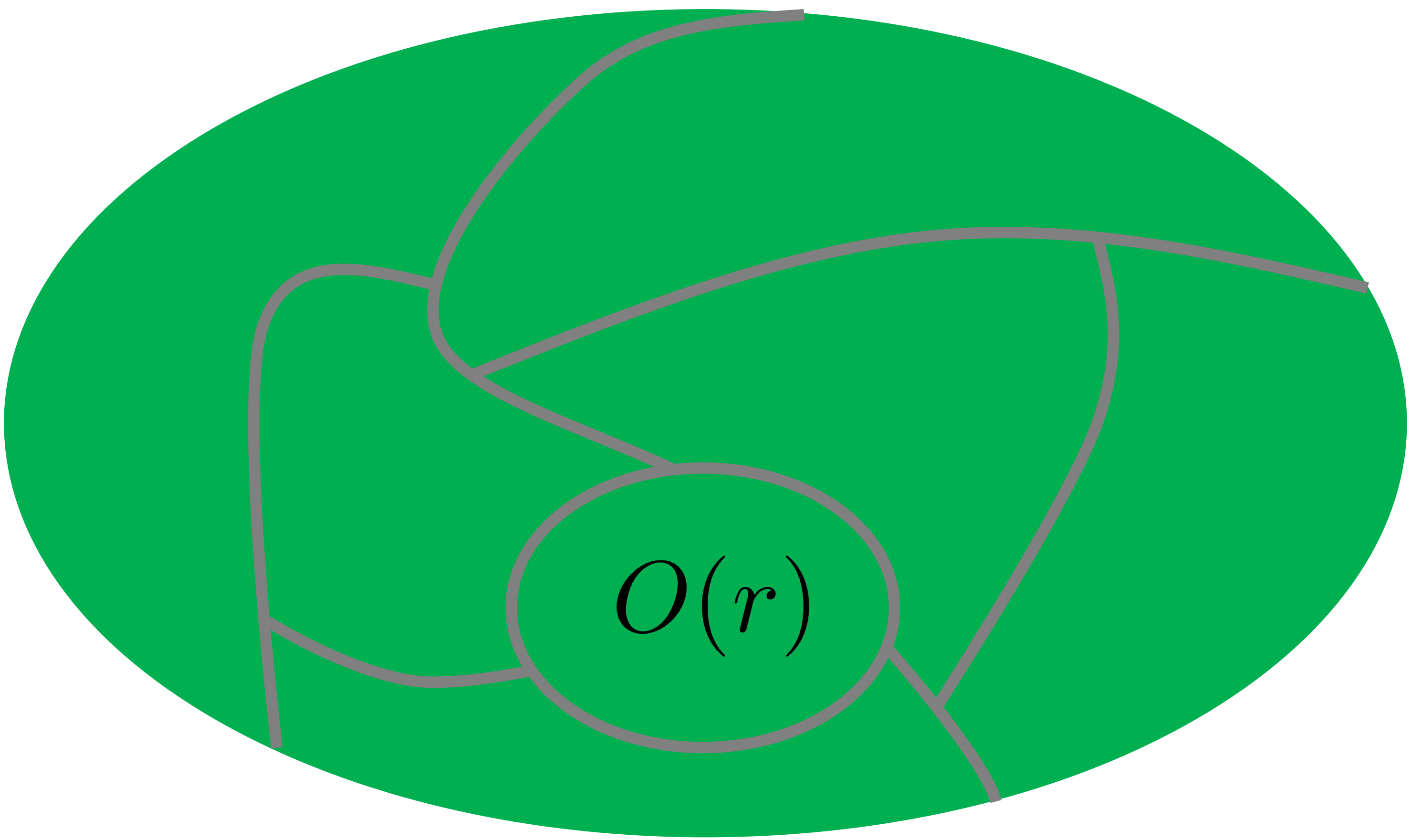


r-division

$O(n/r)$ regions, each with $O(r)$ vertices and $O(\sqrt{r})$ boundary vertices



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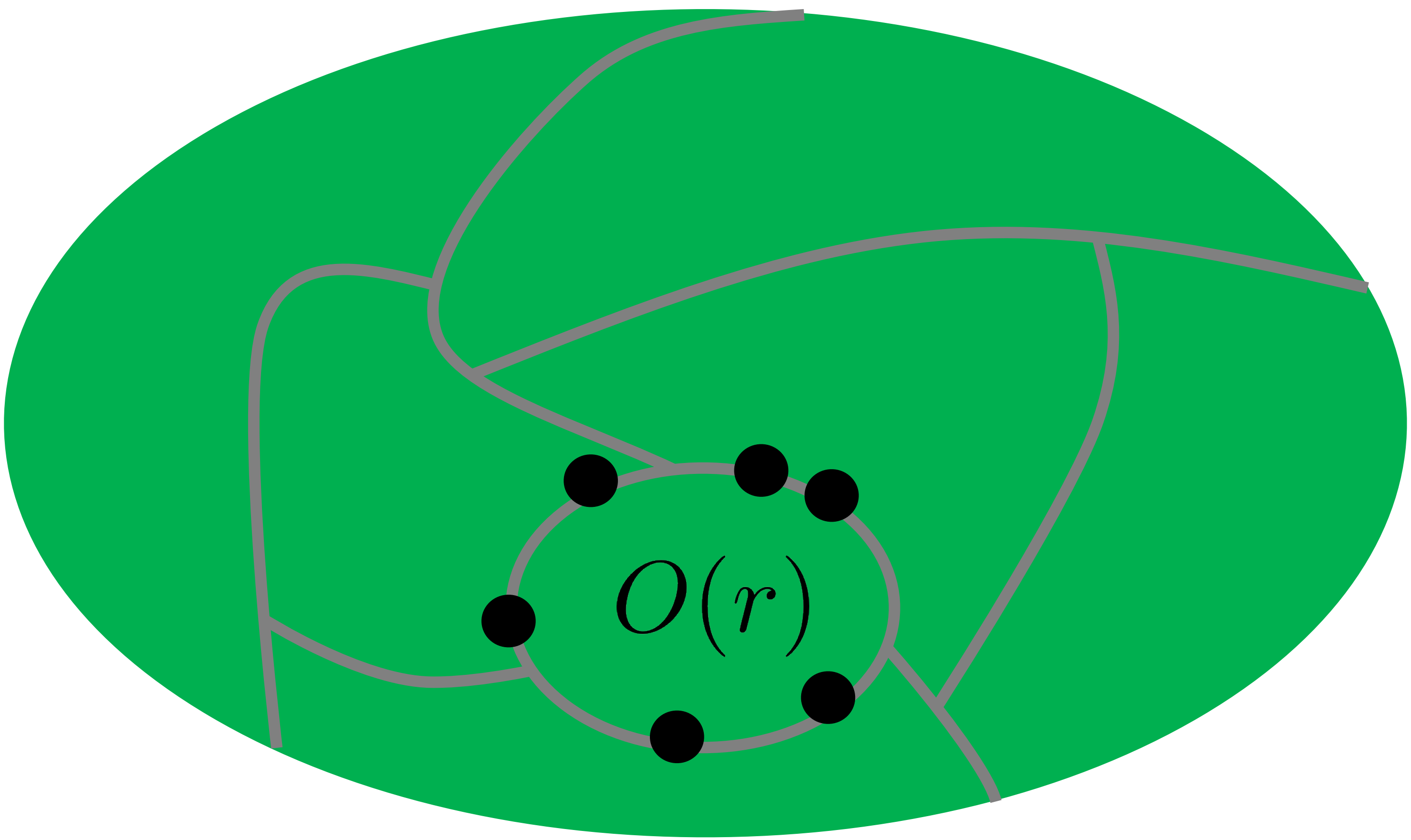

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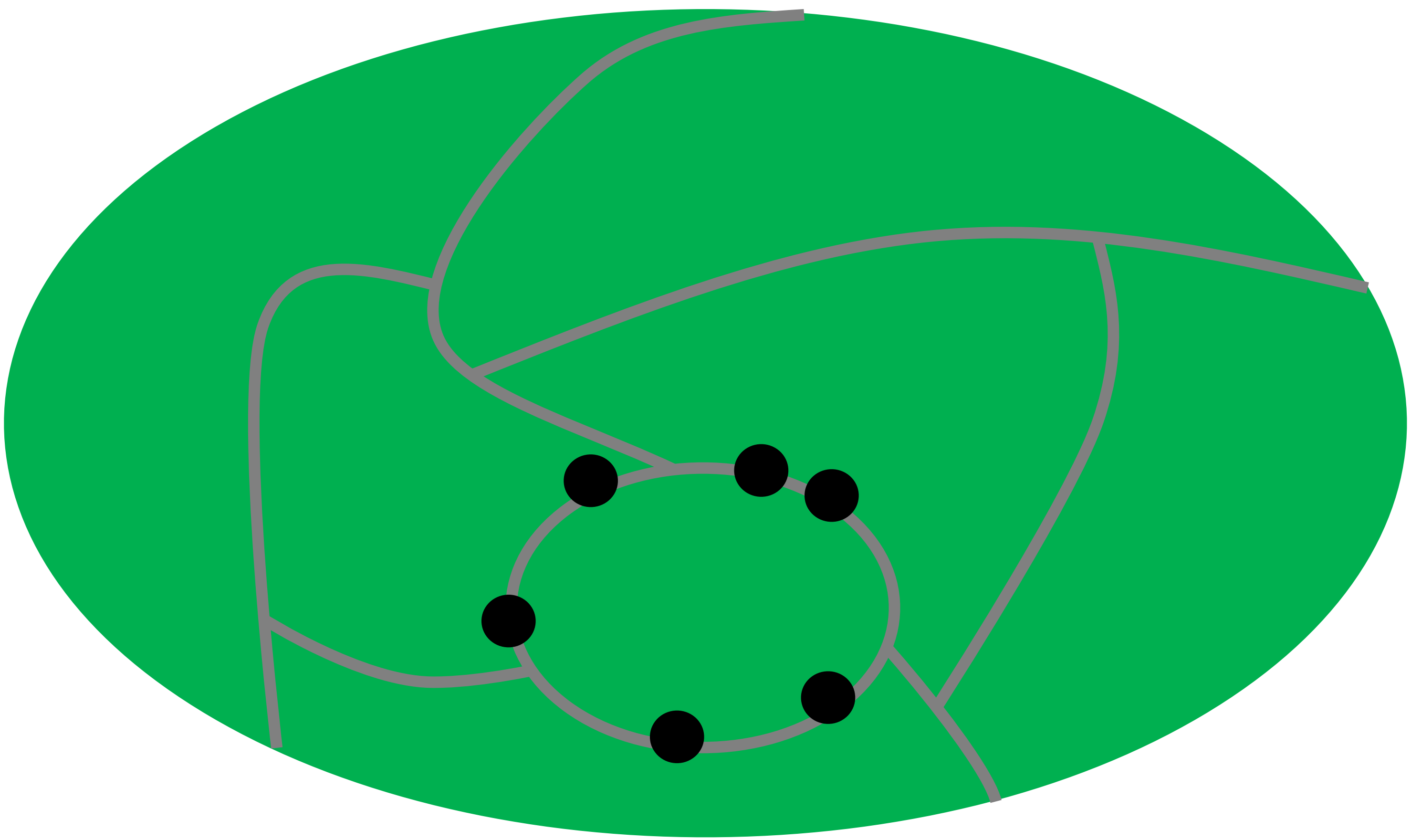

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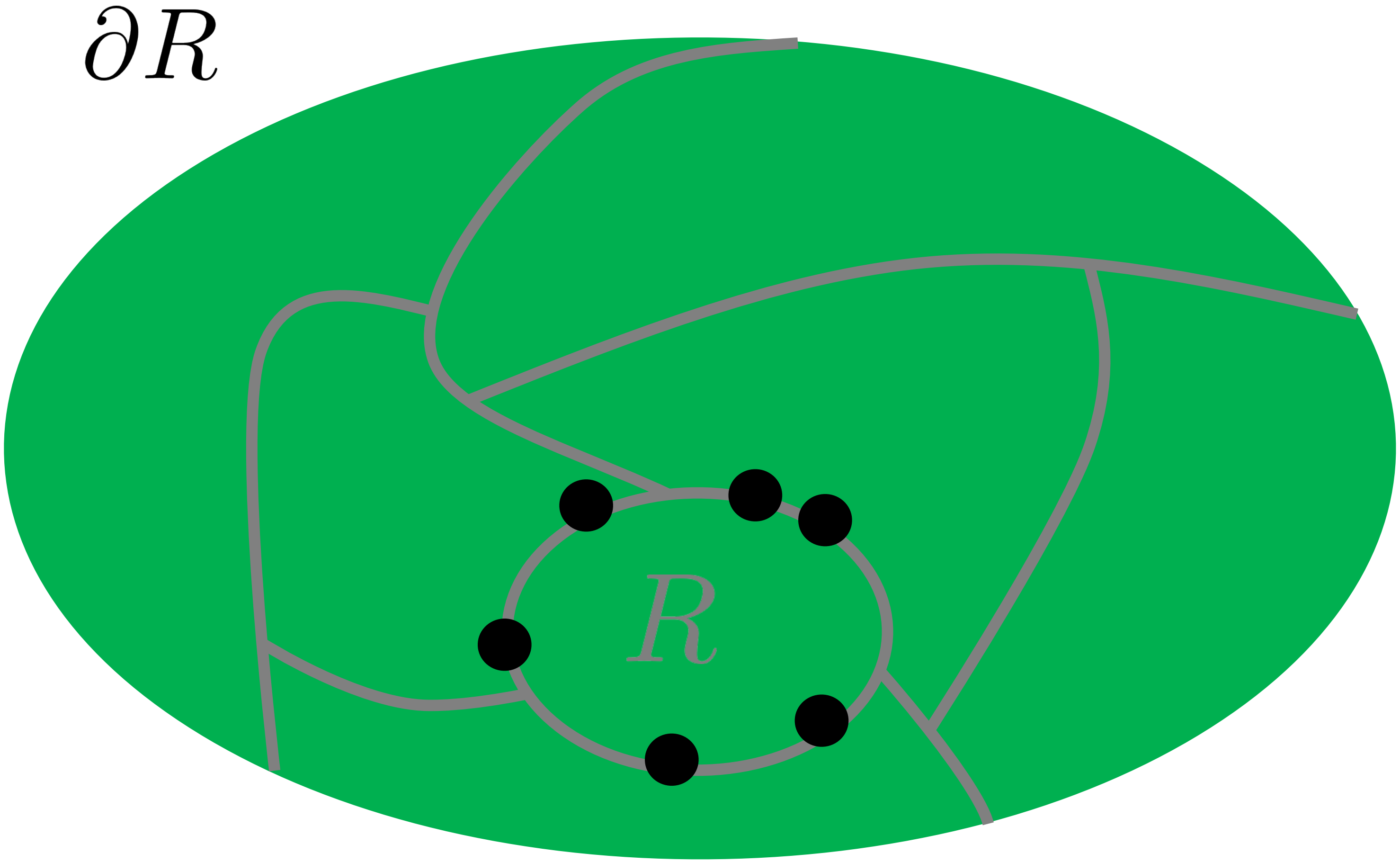

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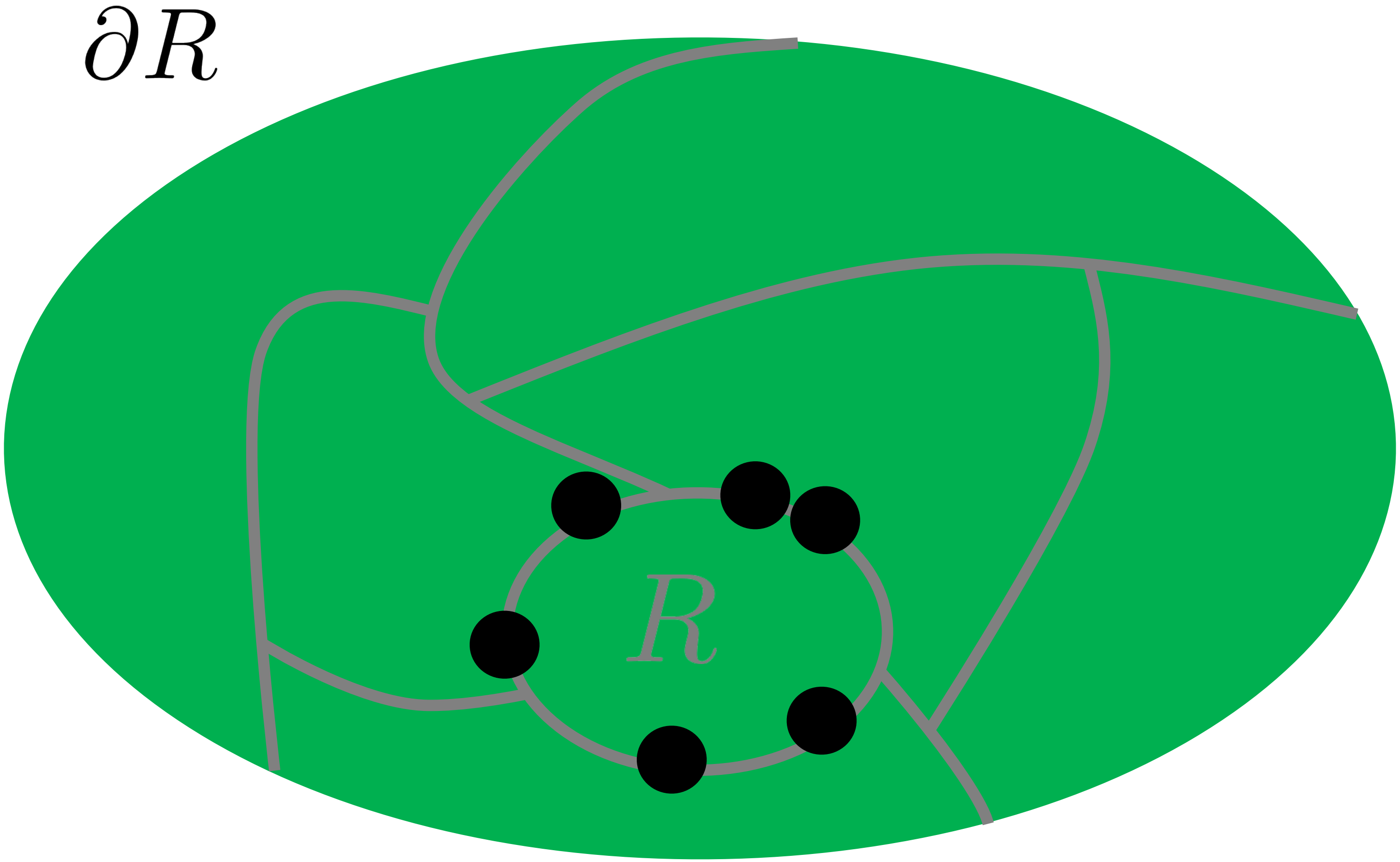
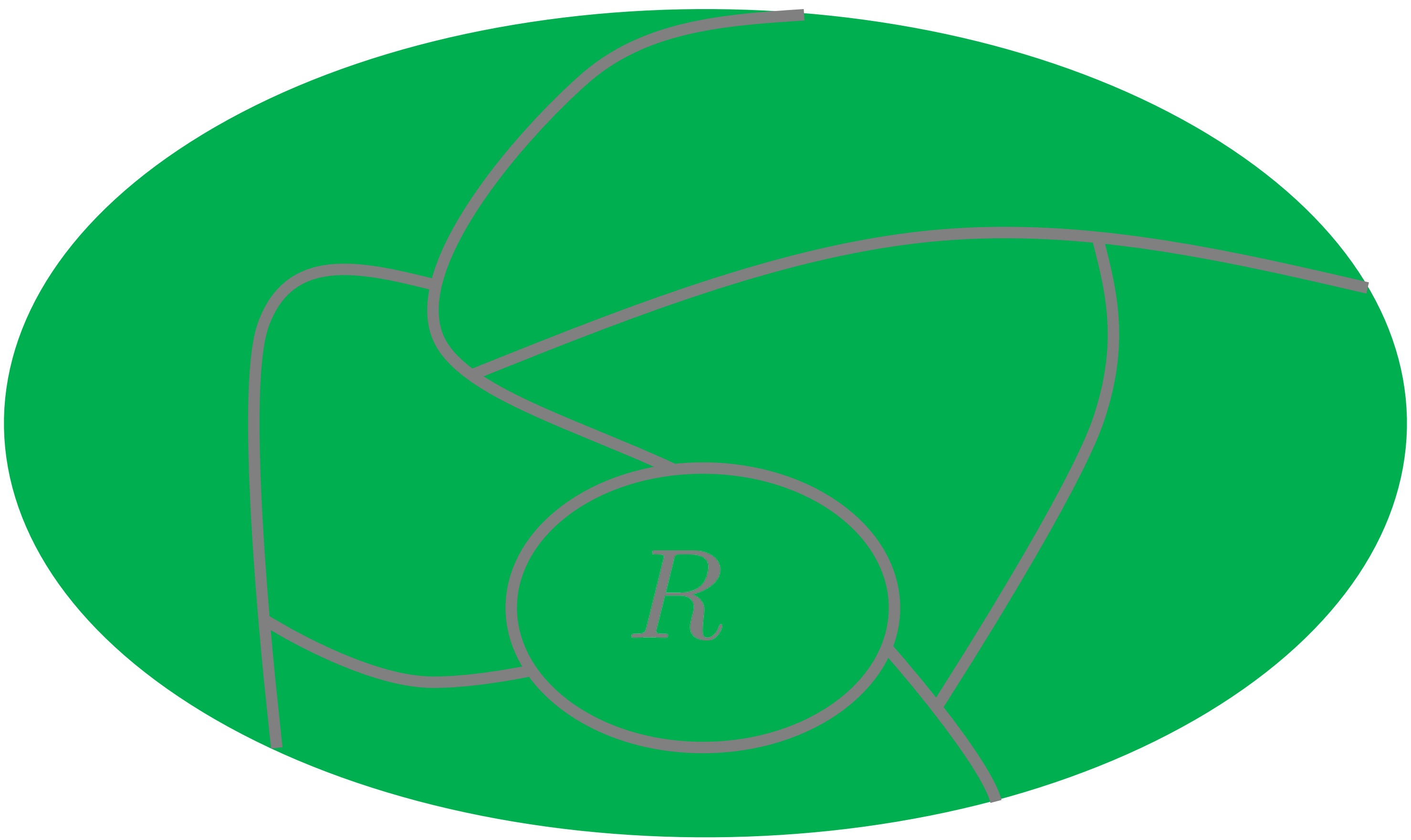


Fault tolerant distance labels



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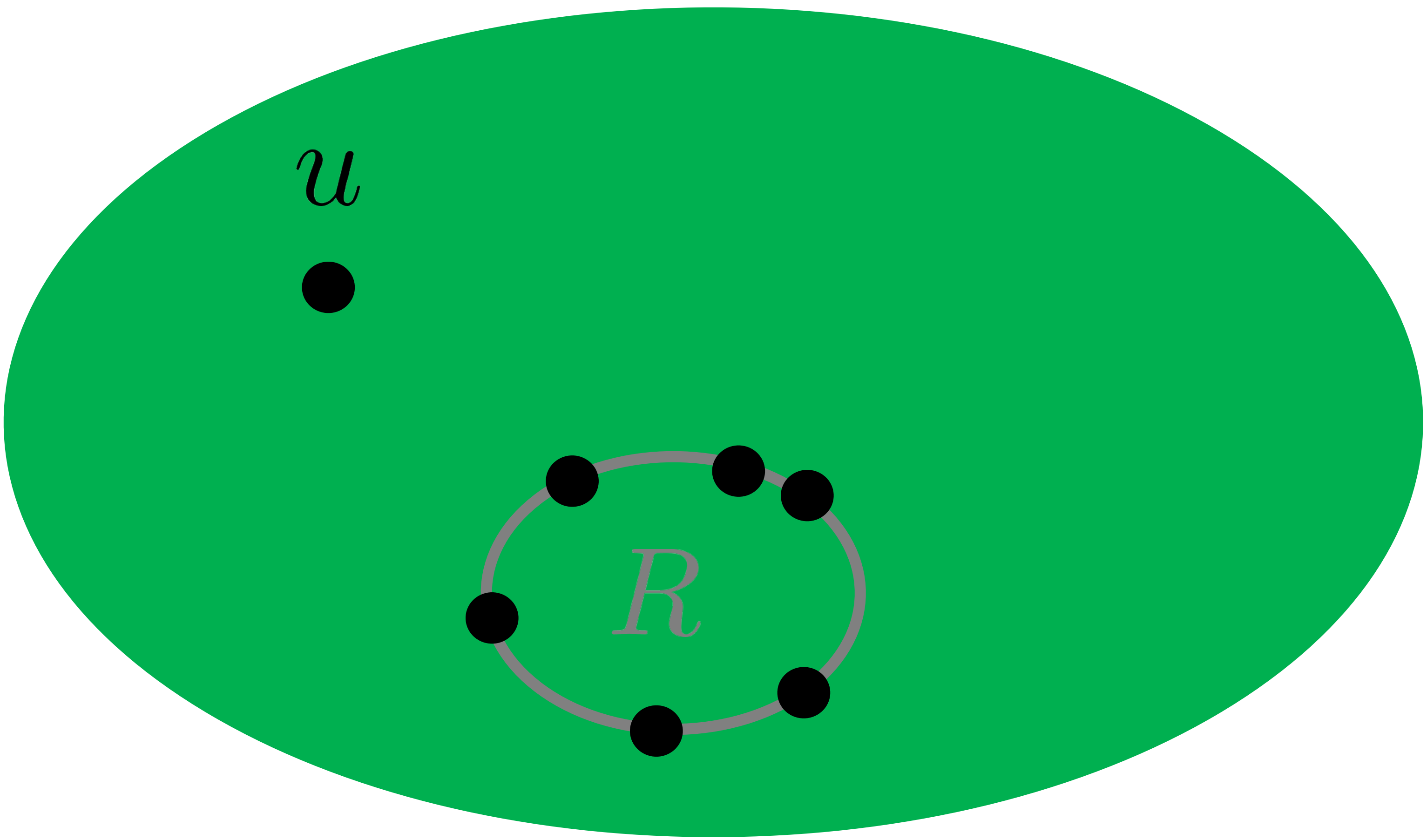
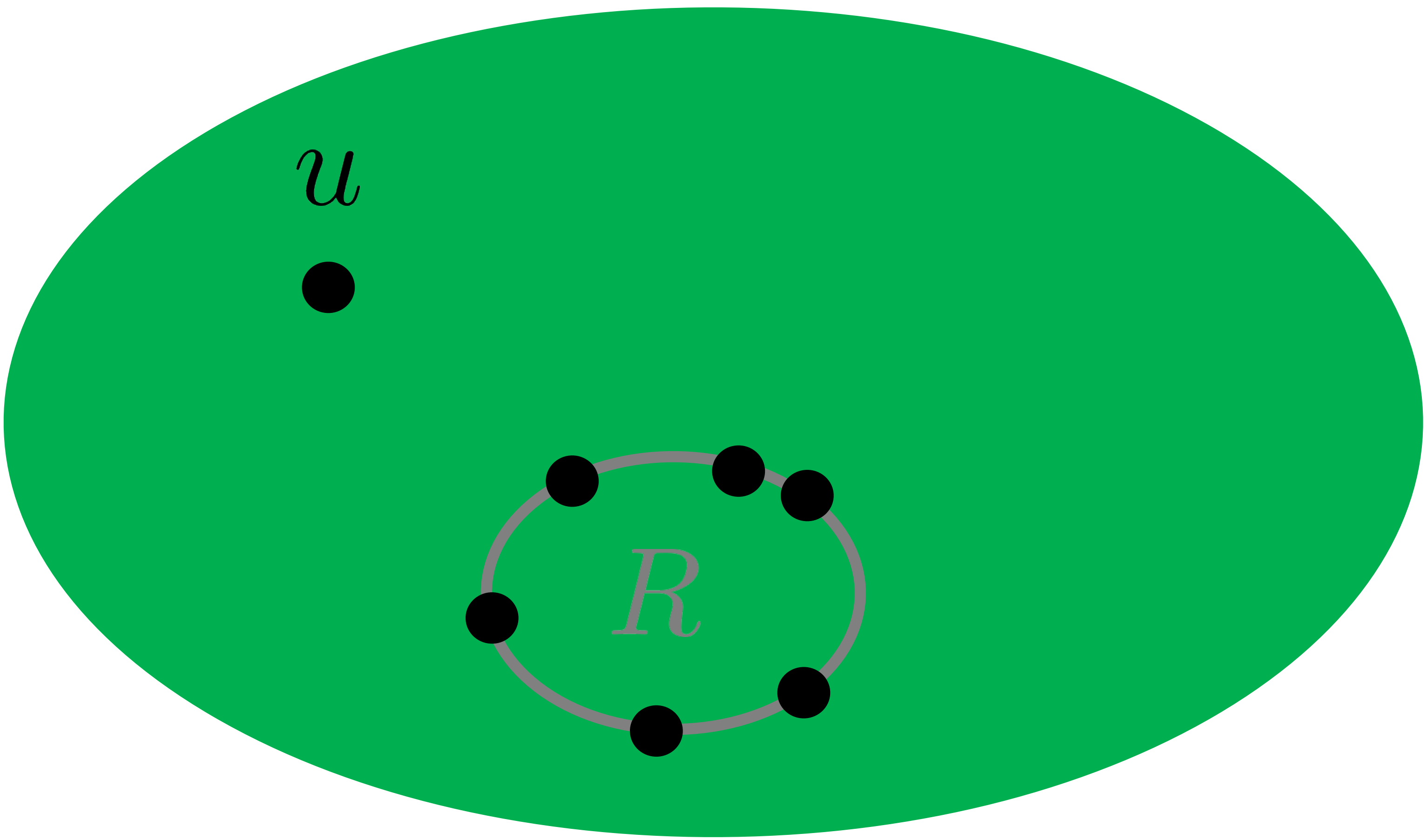


u's label:

u to ∂R internally disjoint from R for every region R



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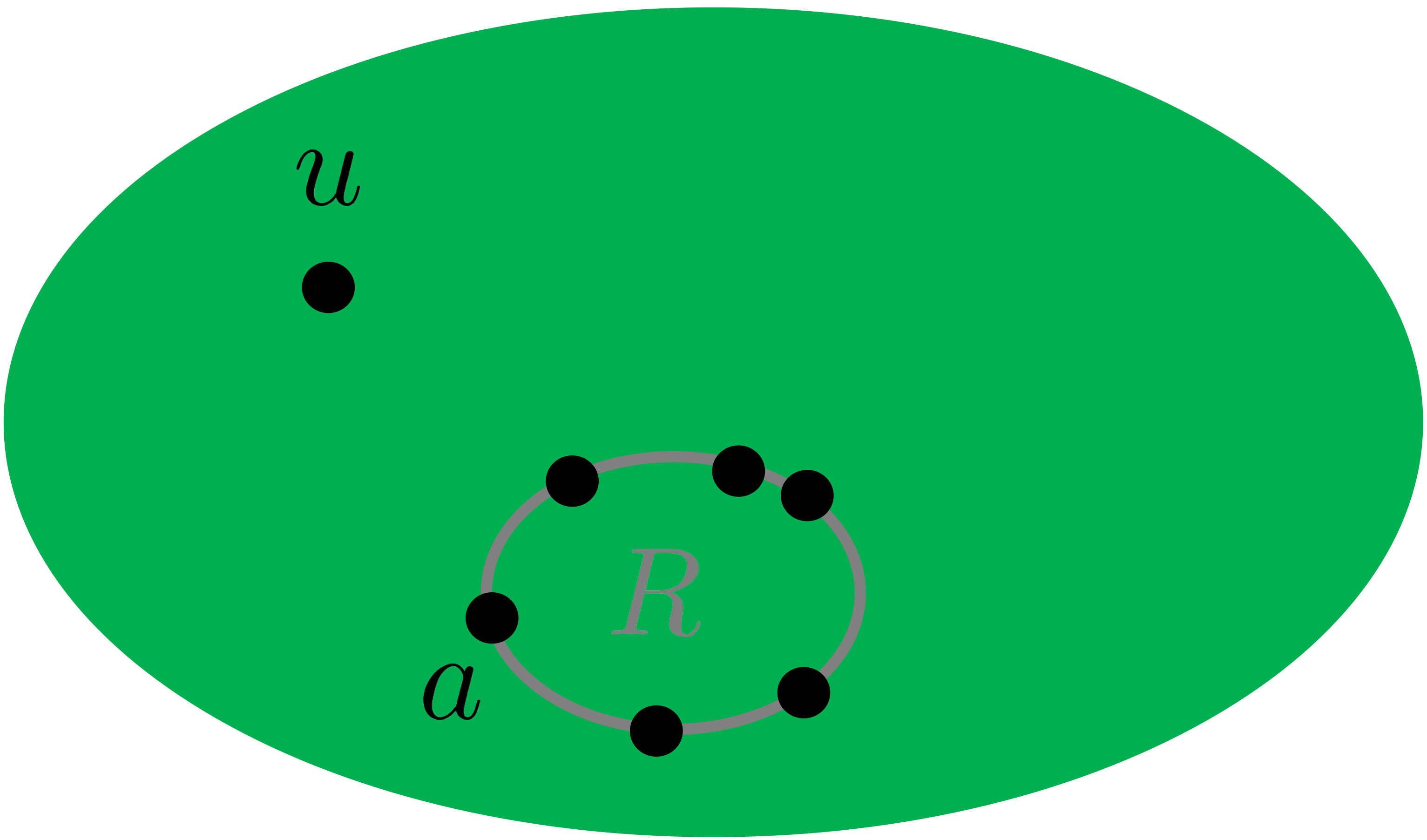

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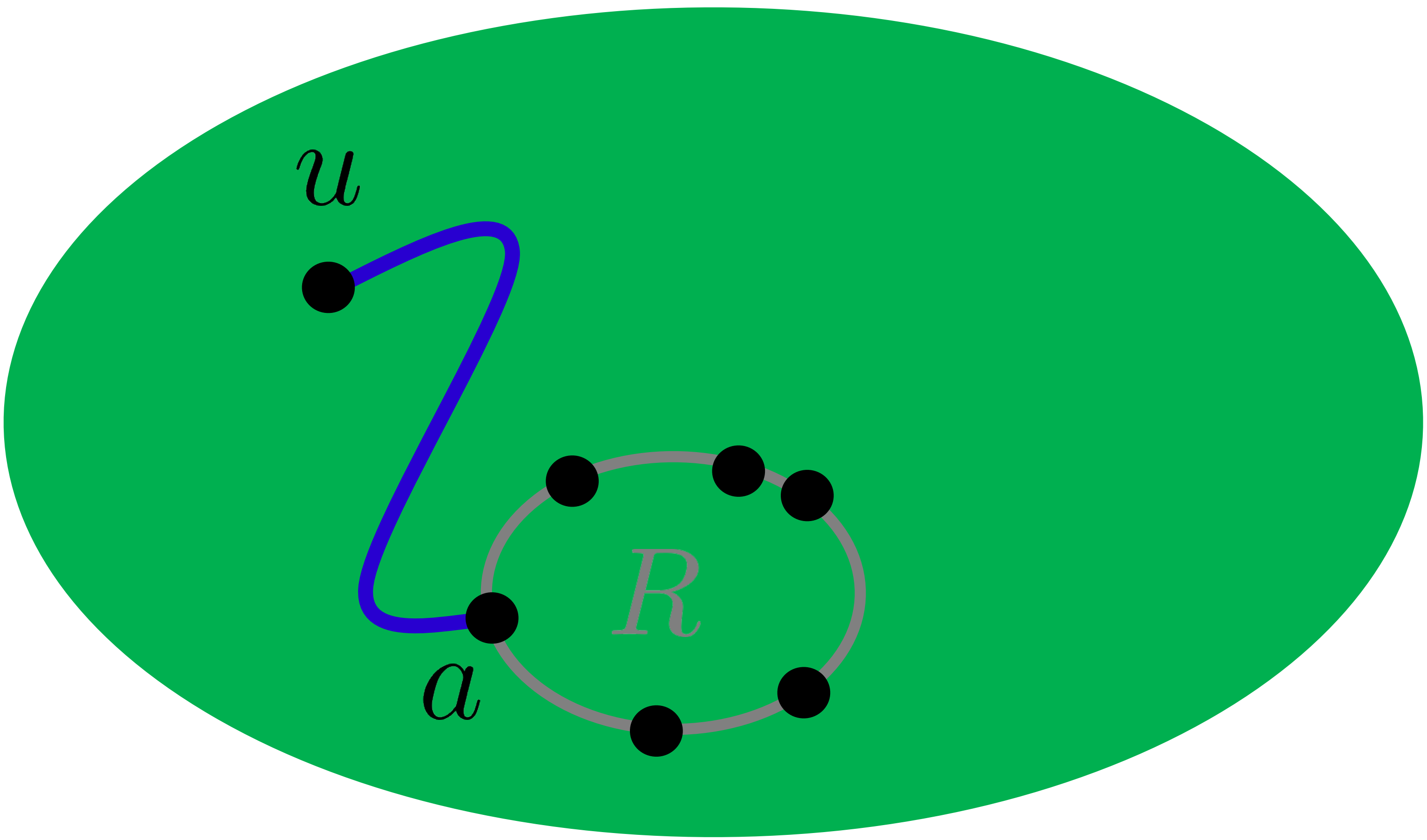

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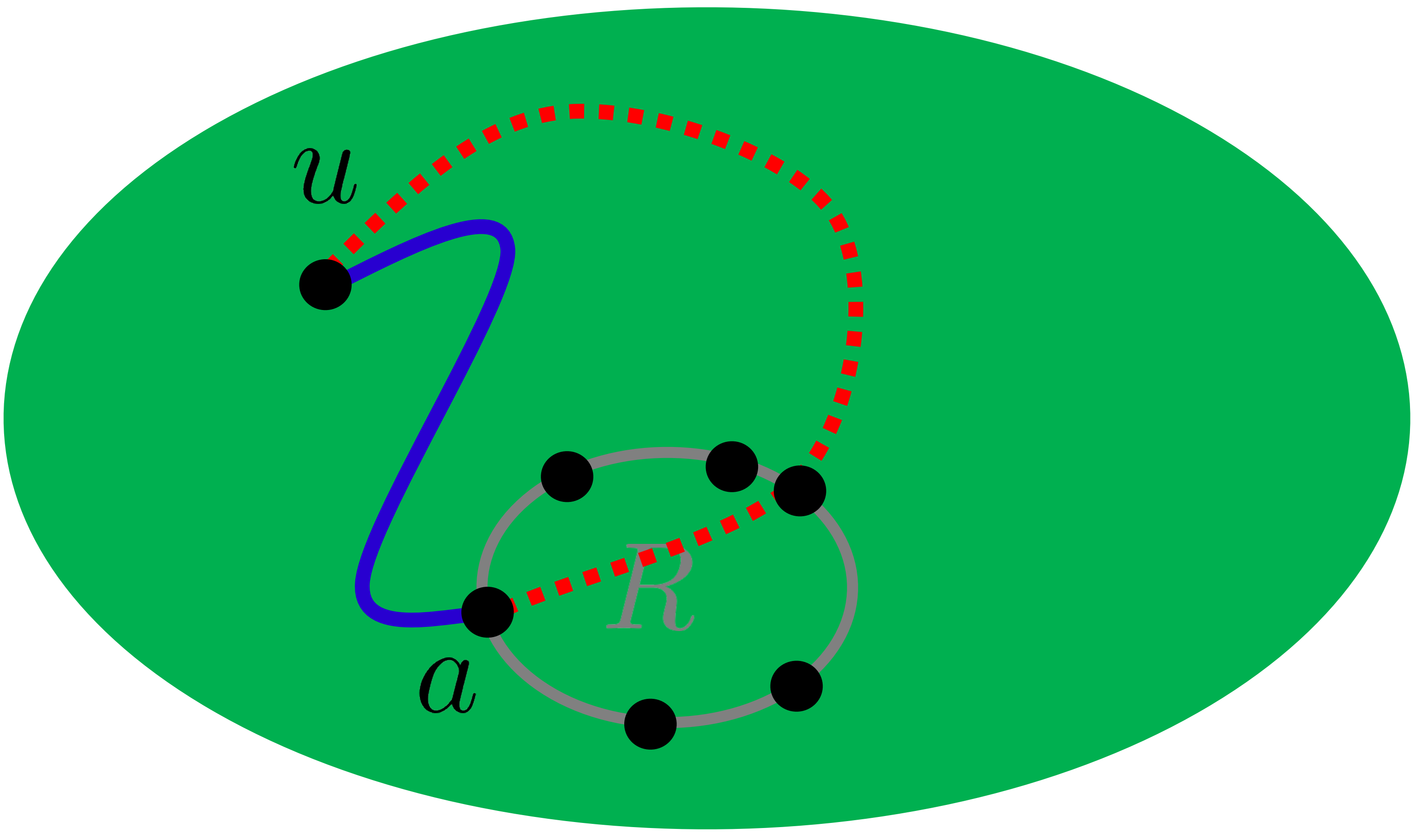

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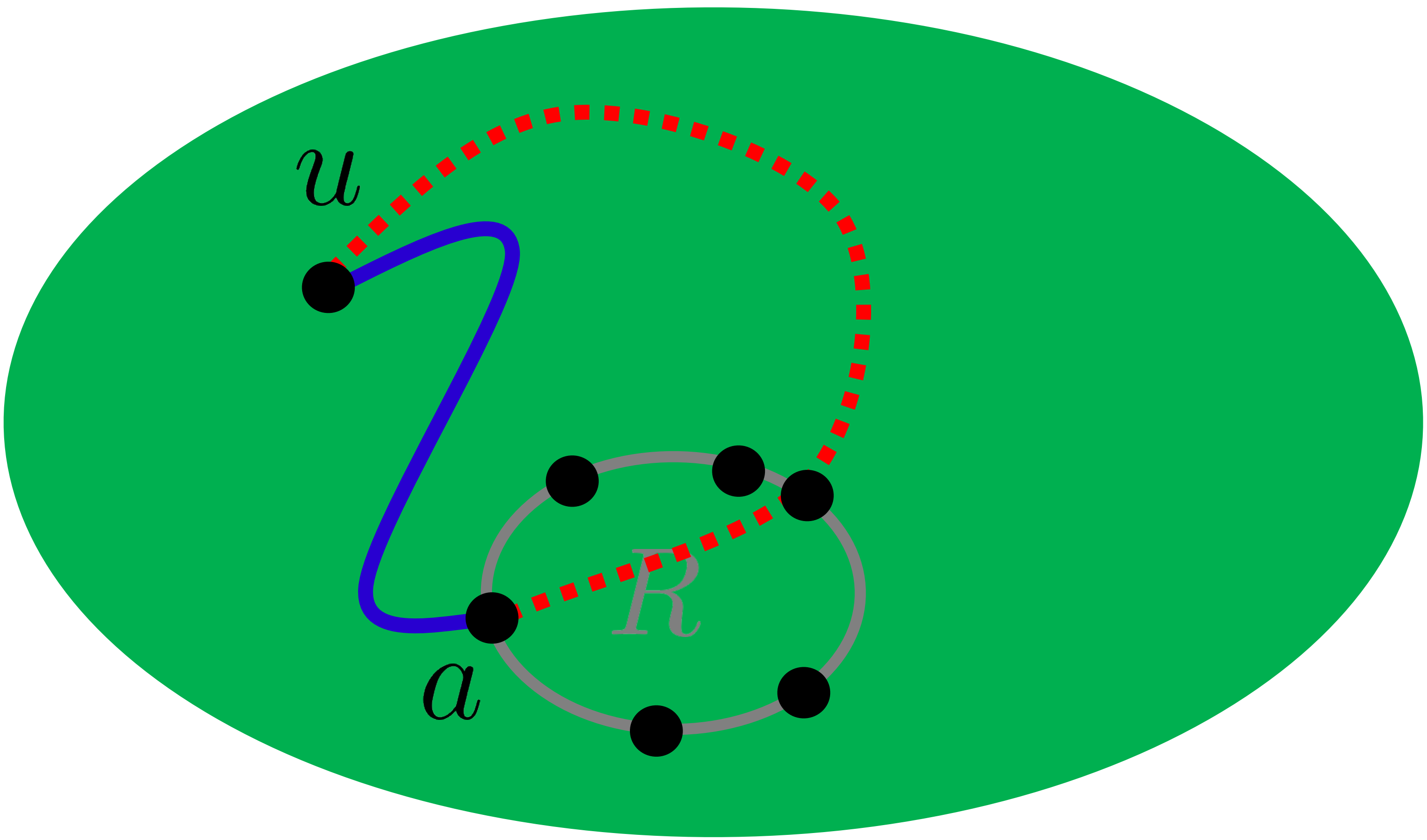

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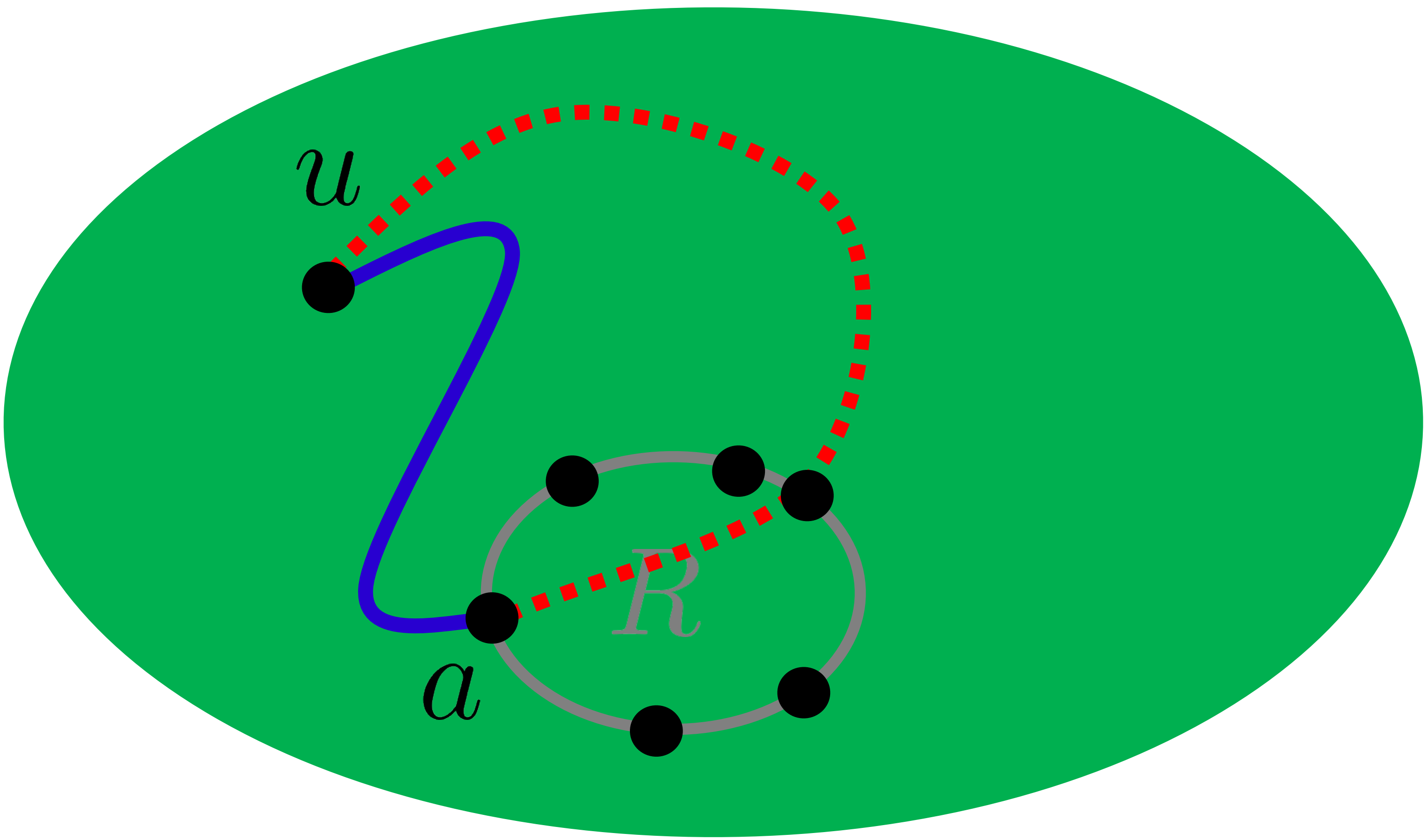
$\text{space} = \#regions \cdot |\partial R|$



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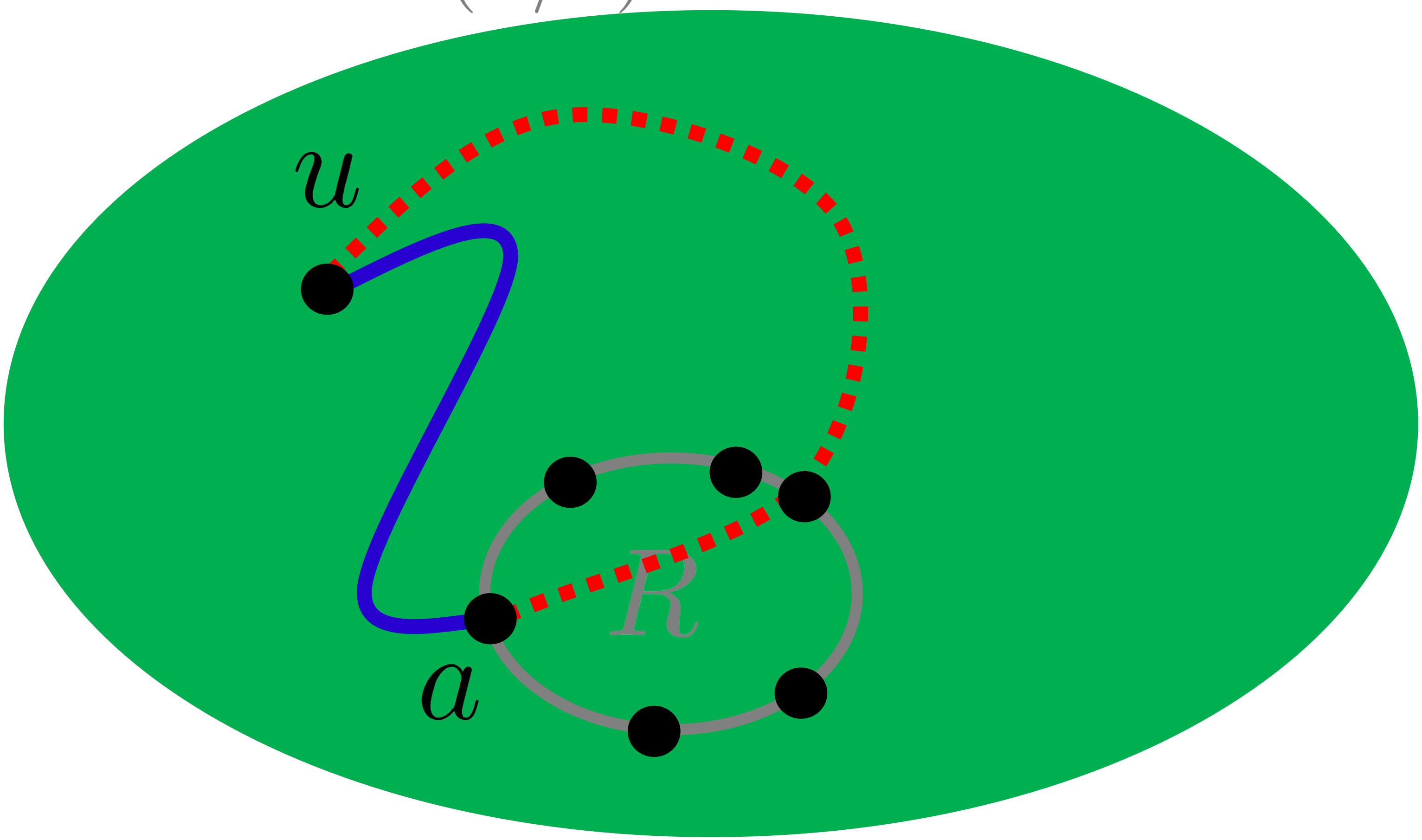
$O(n/r)$



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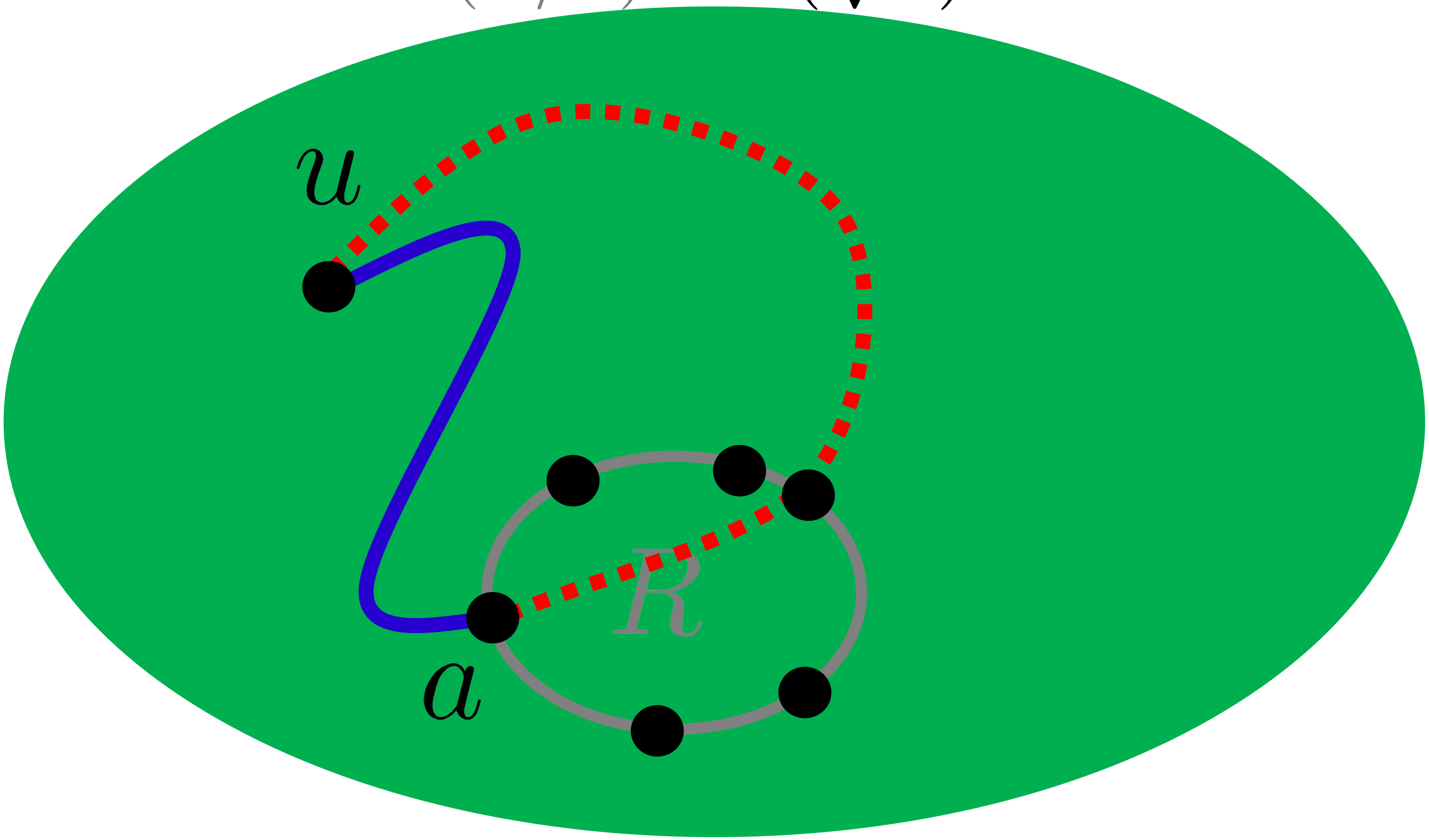
$$O(n/r) \cdot O(\sqrt{r})$$



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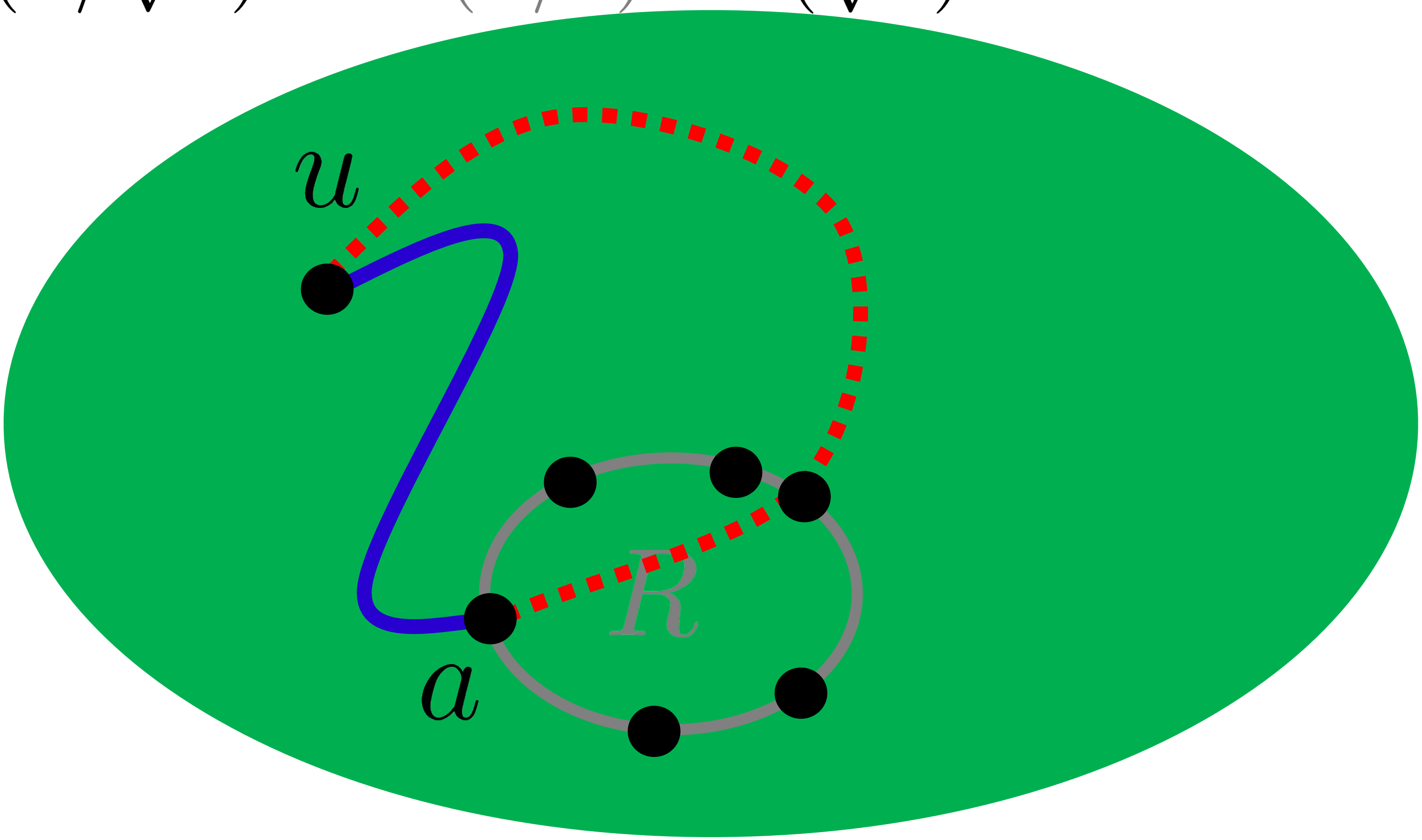
$$\tilde{O}(n/\sqrt{r}) = O(n/r) \cdot O(\sqrt{r})$$



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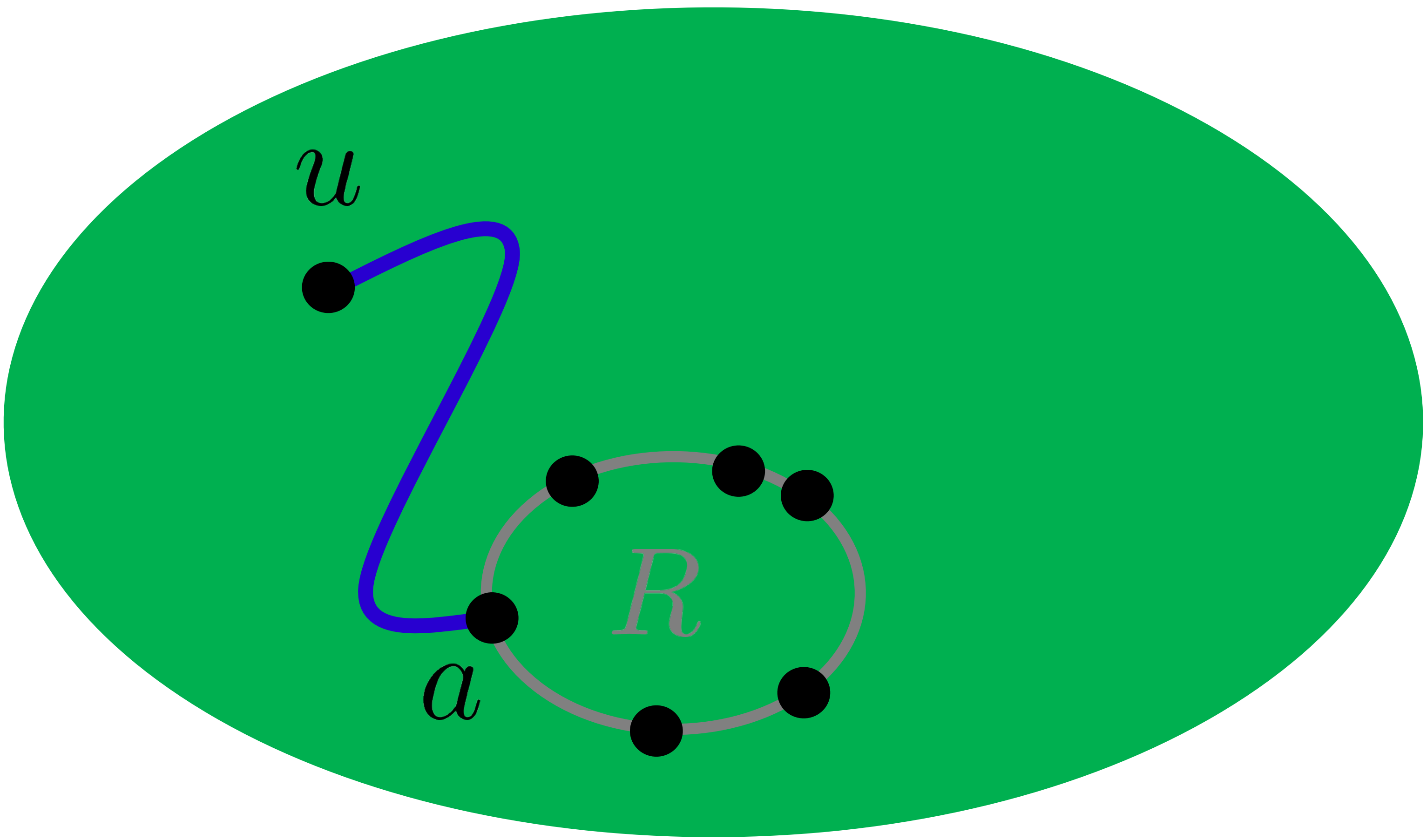

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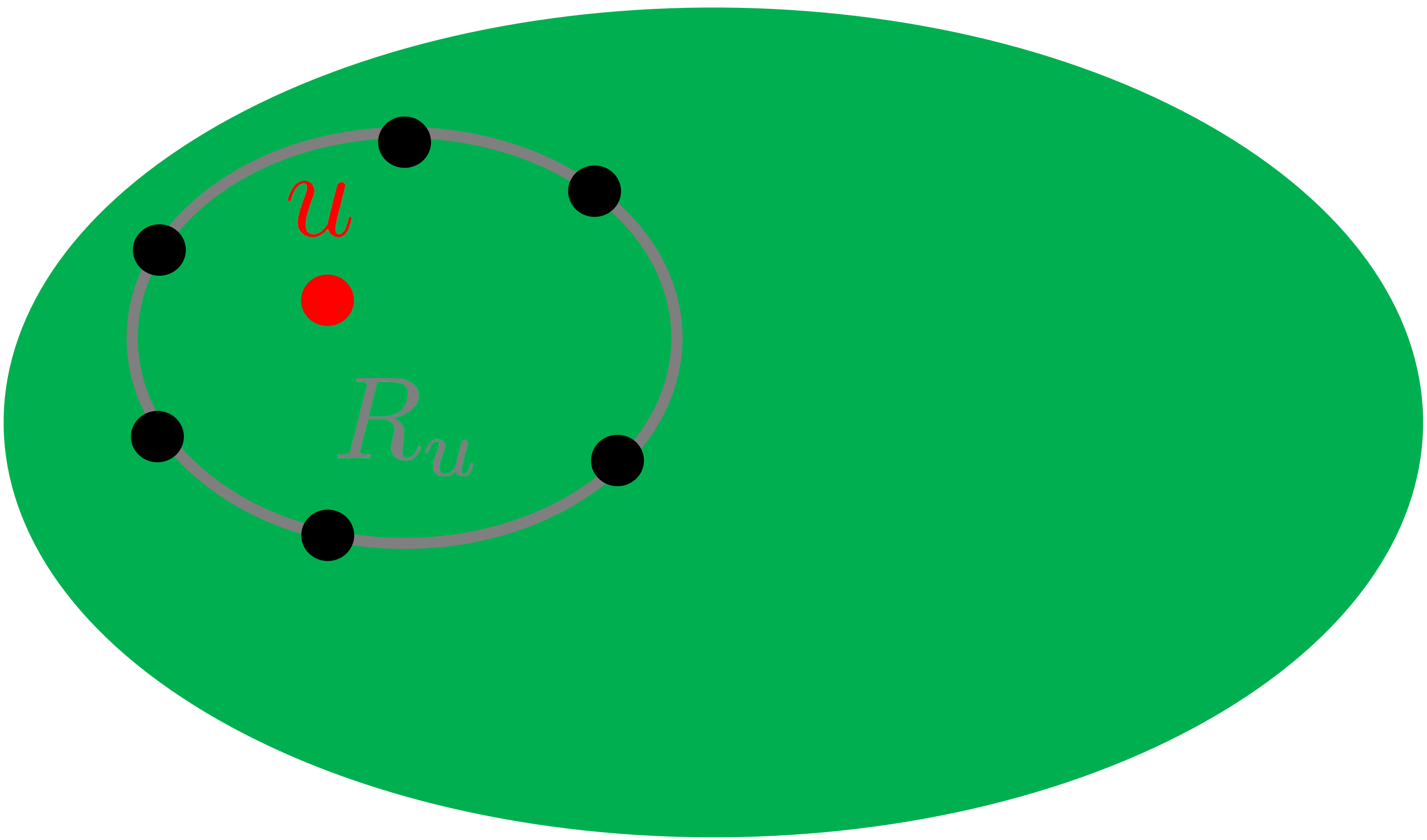
∂R_u to ∂R_u in $G \backslash \{u\}$



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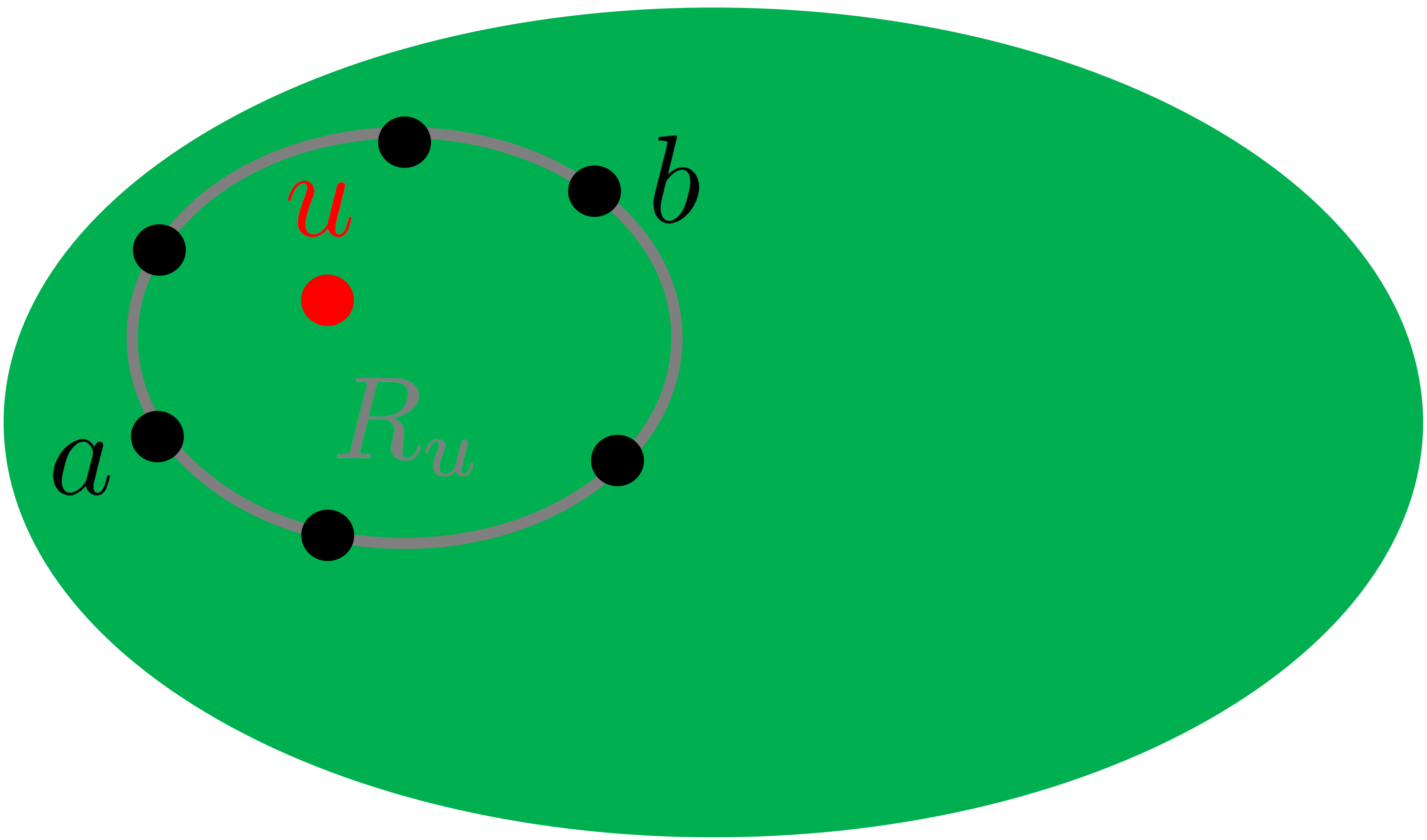

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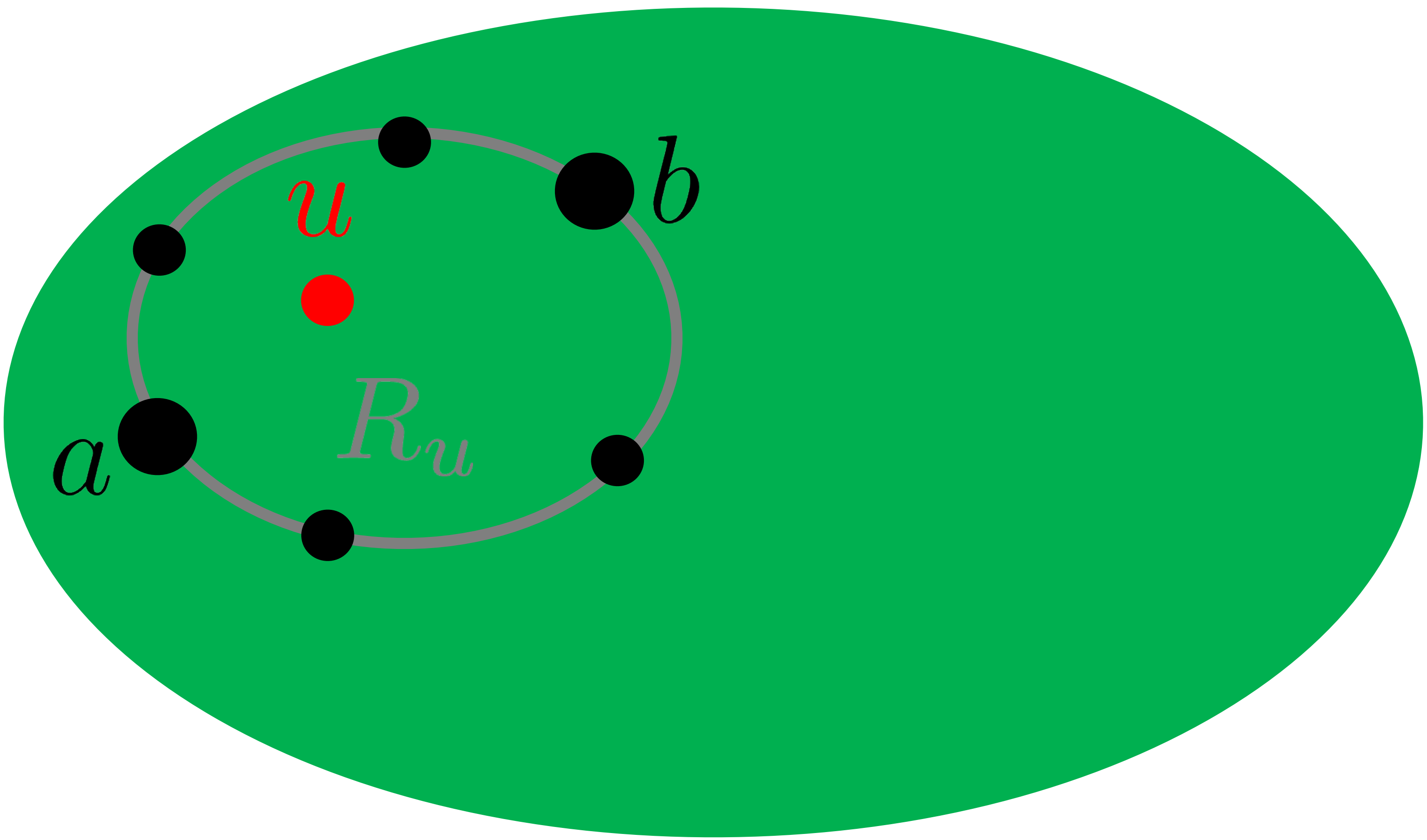

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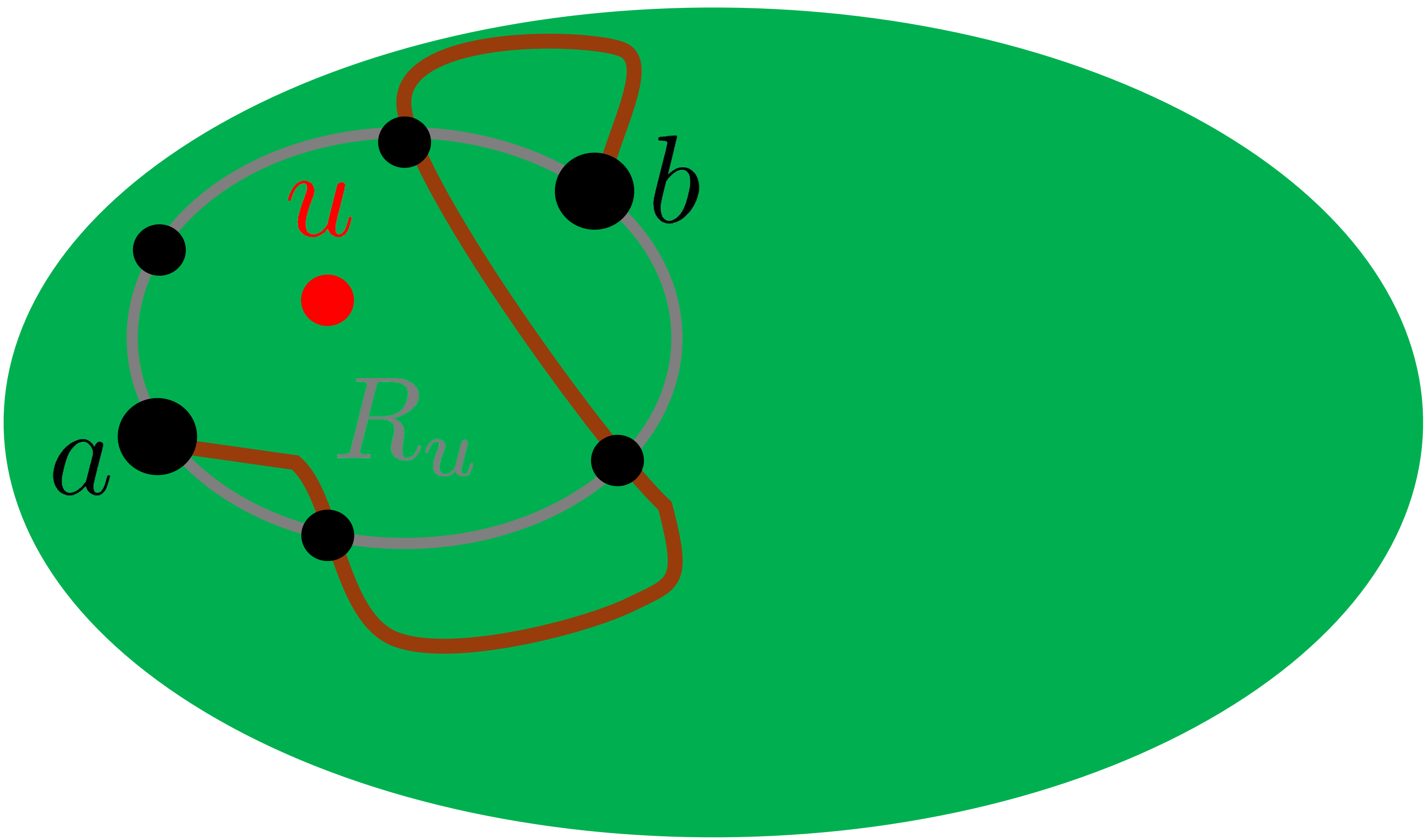

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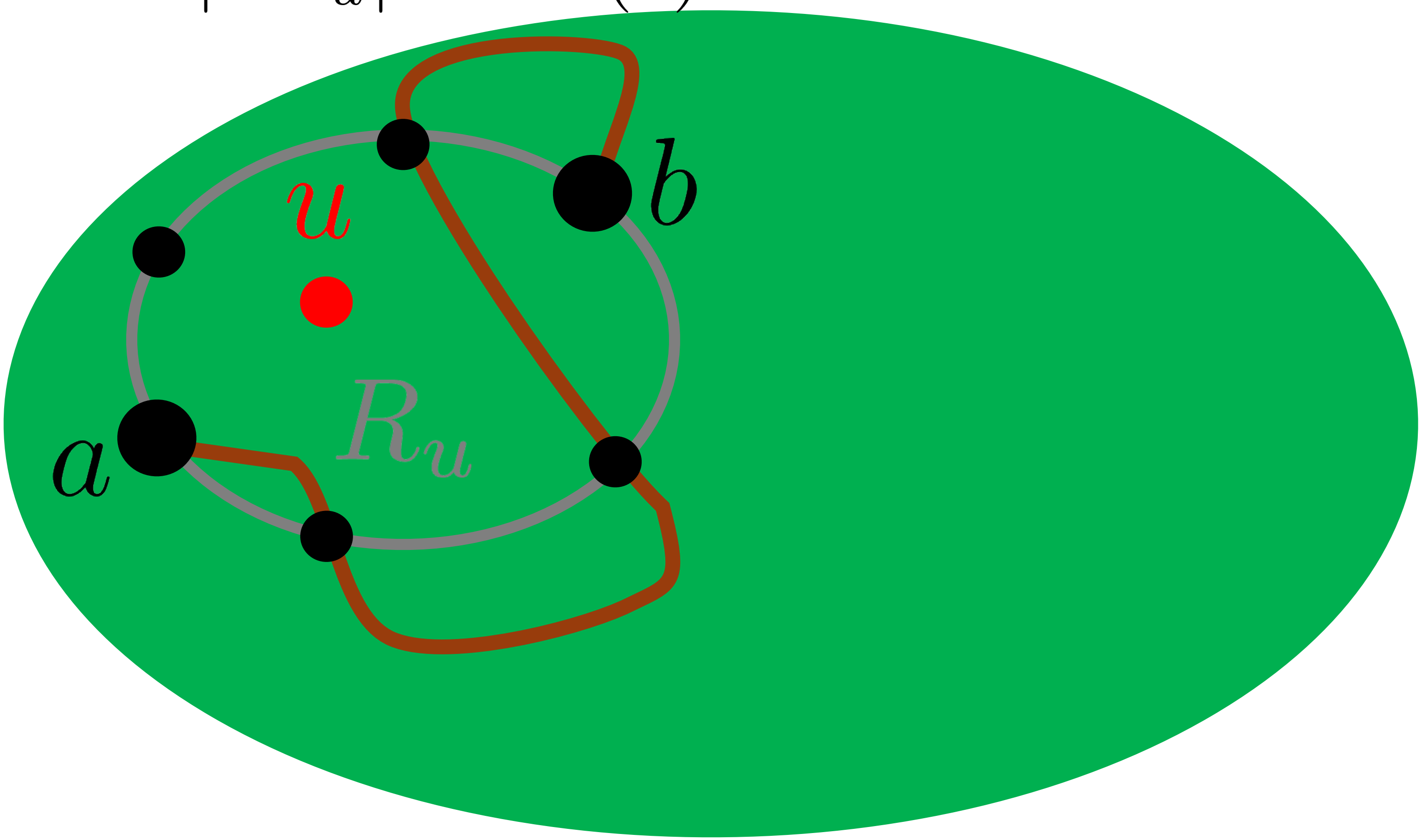
$\text{space} = |\partial R_u|^2 = \tilde{O}(r)$



Case 1: Shortest path touches ∂R_f

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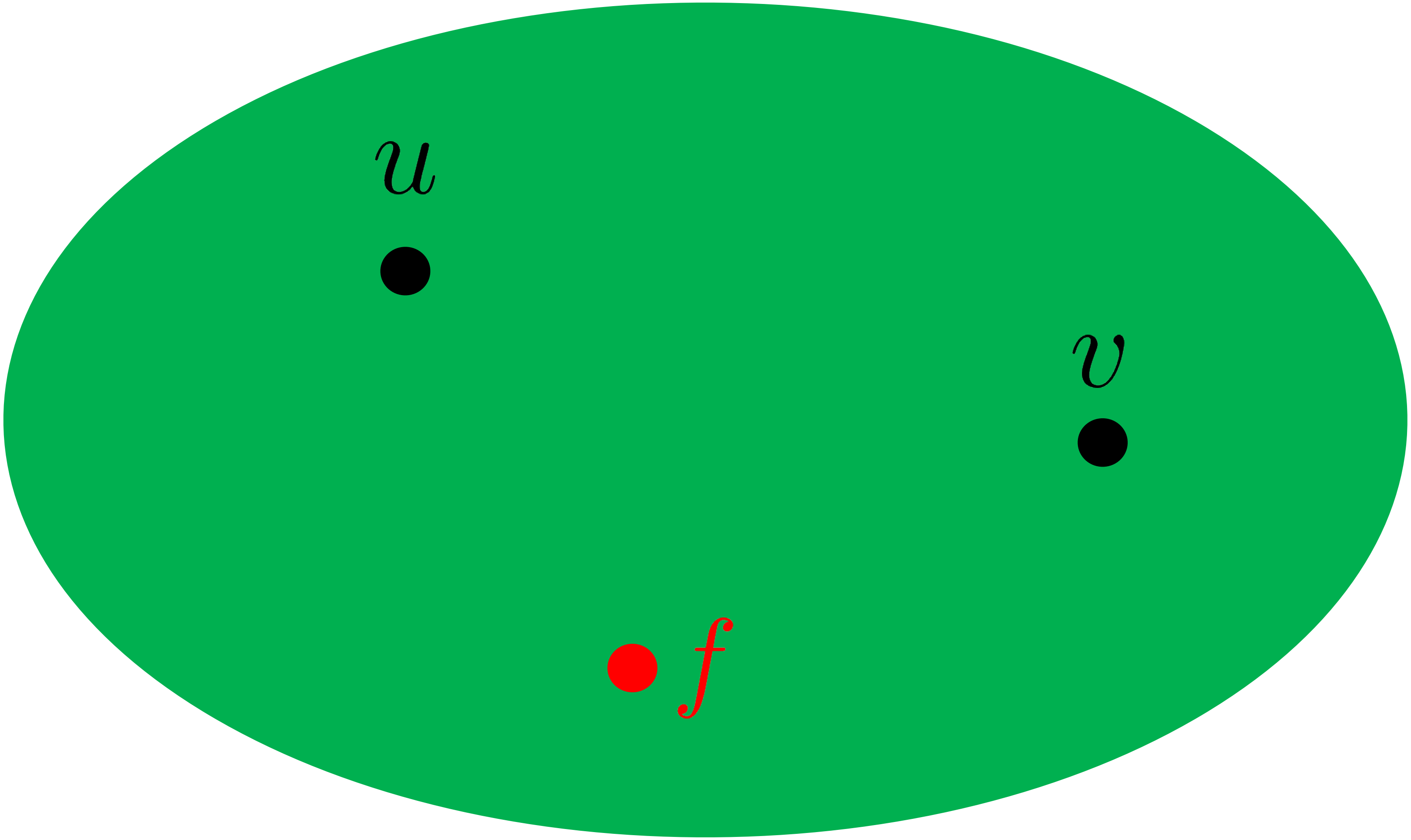

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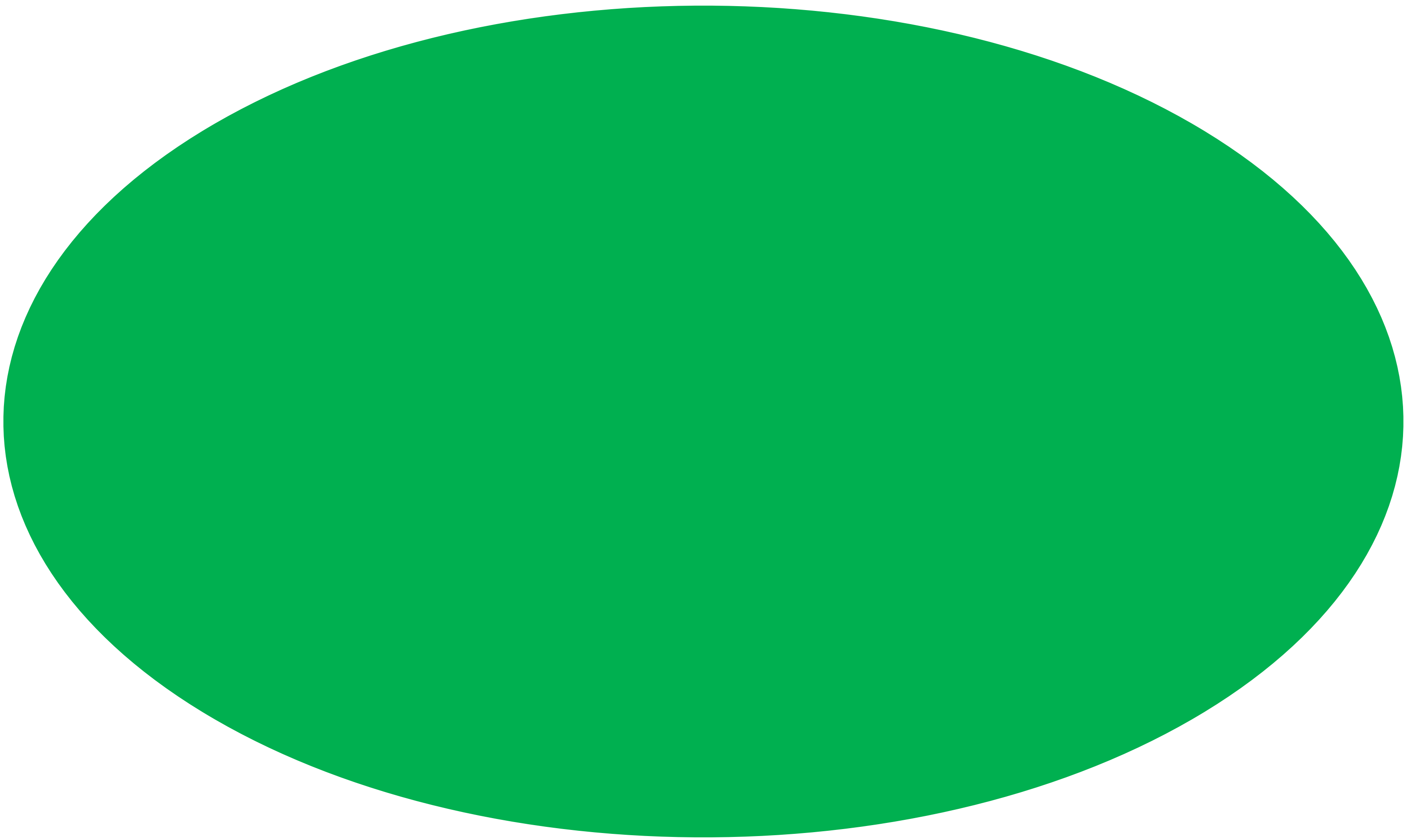

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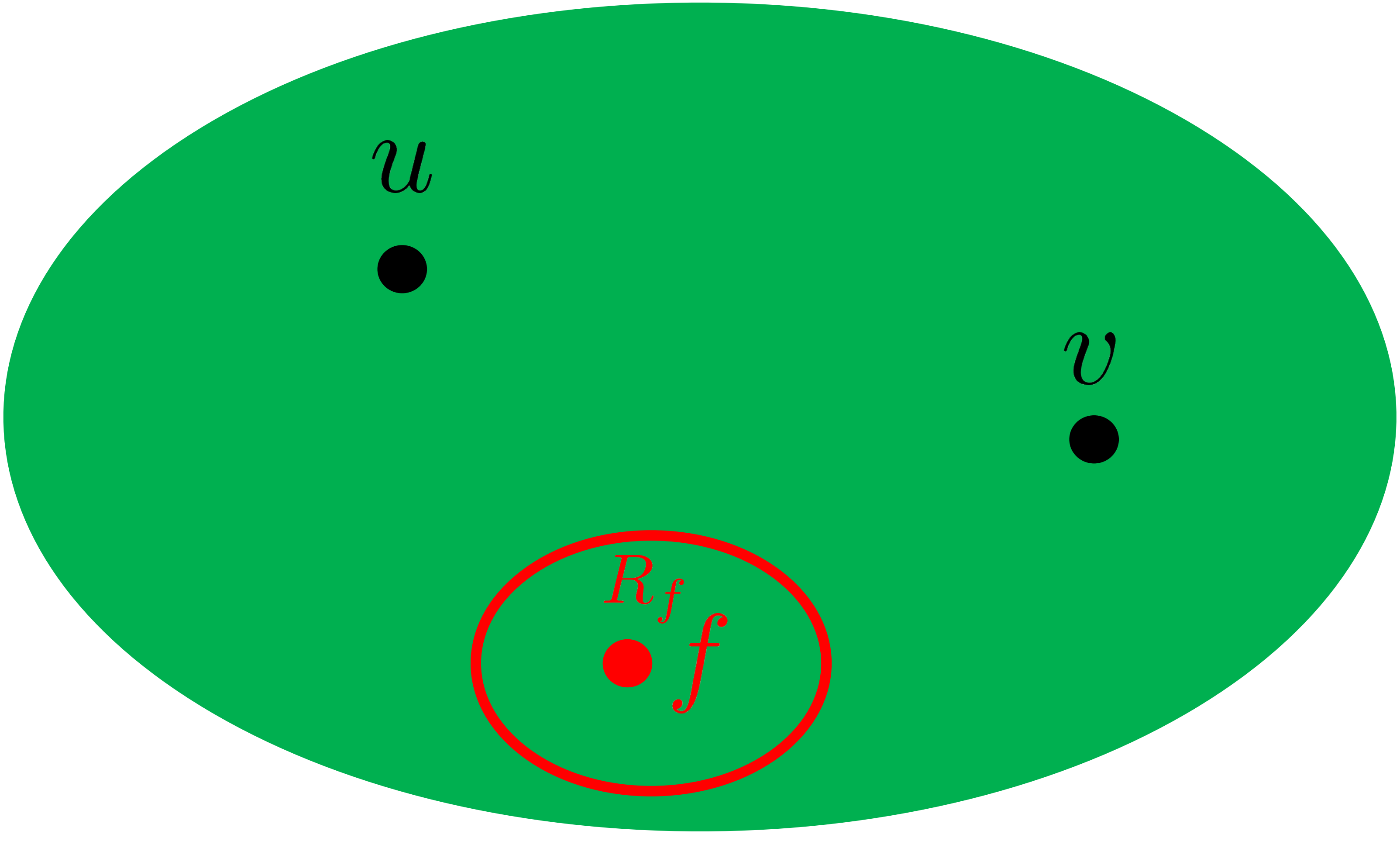

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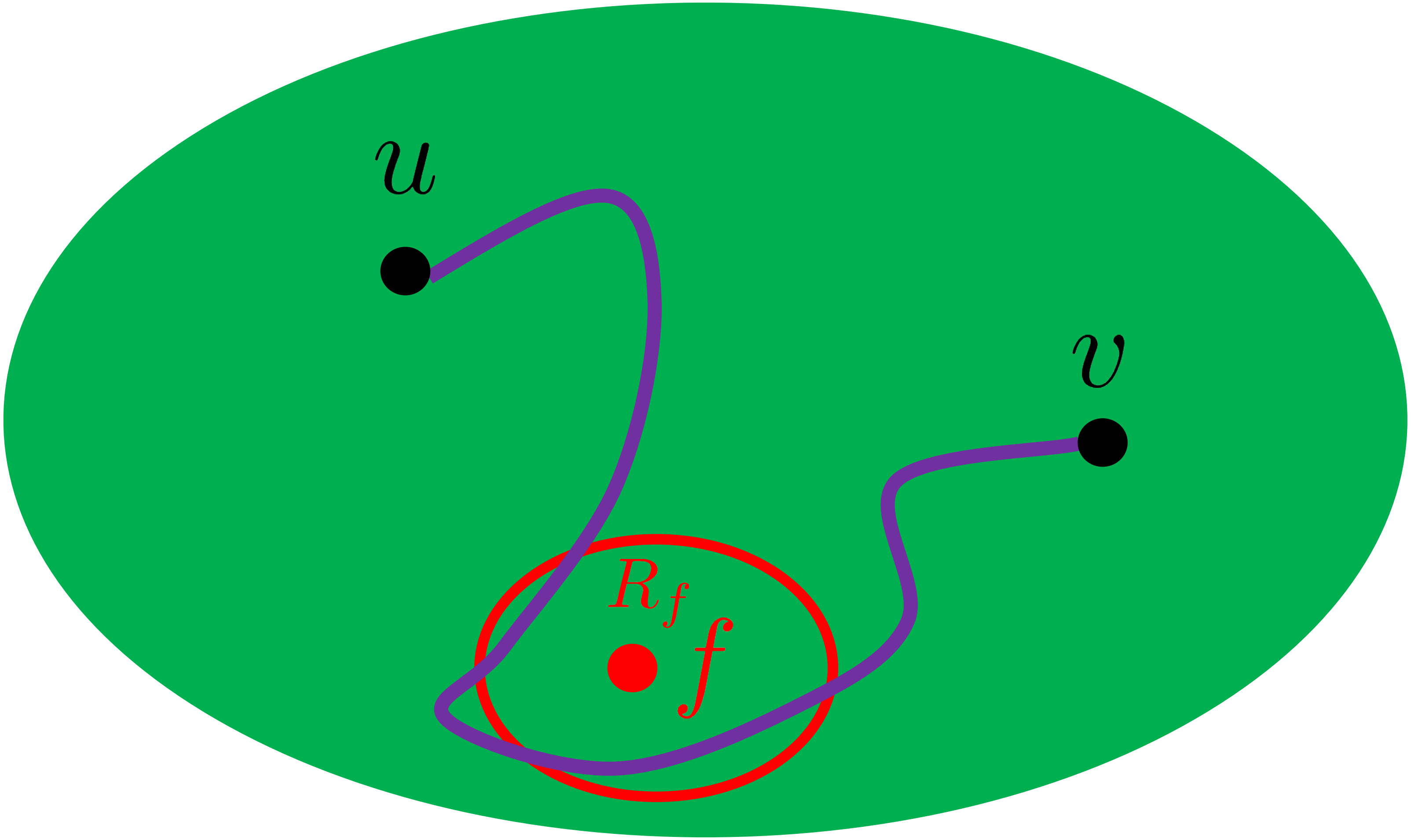

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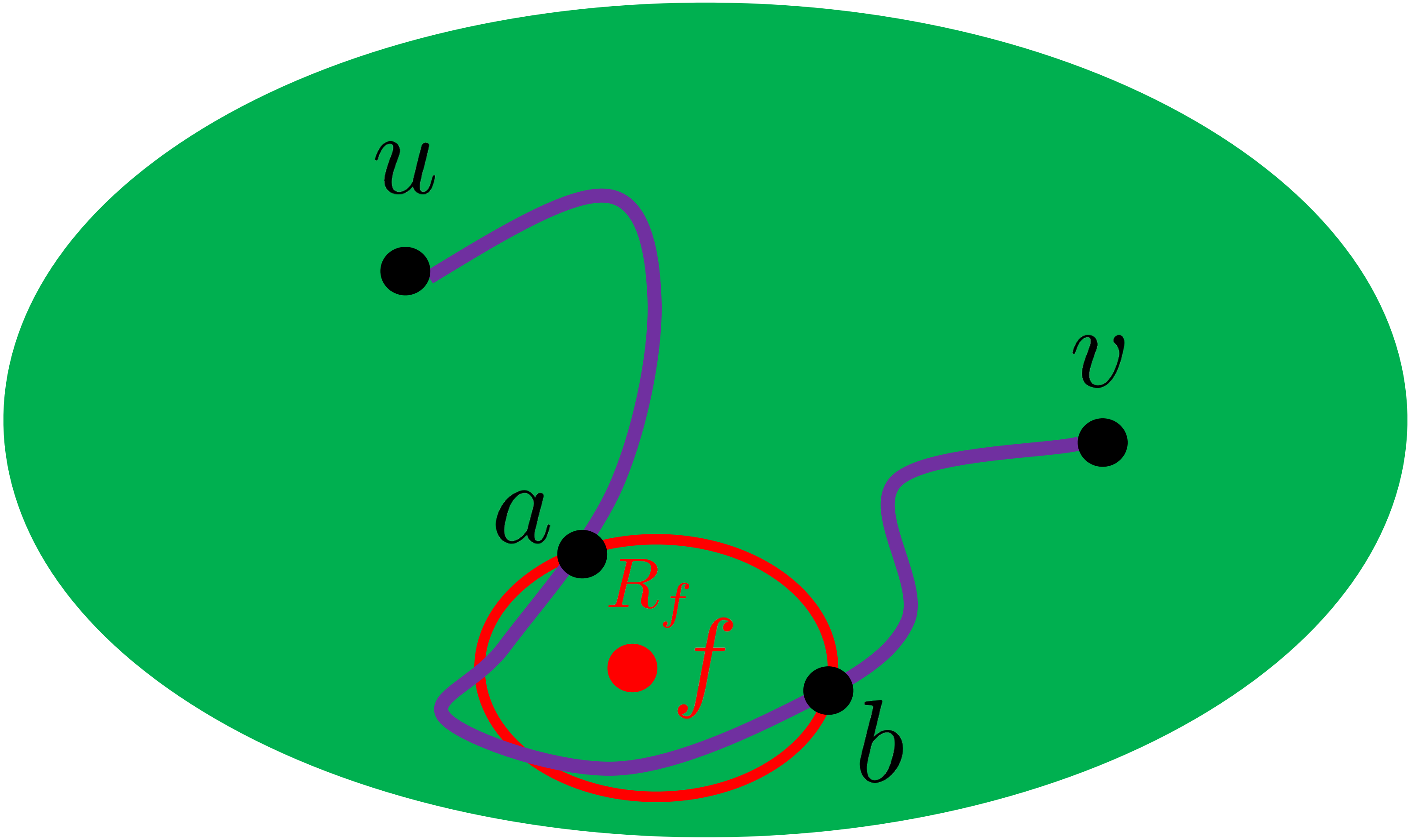

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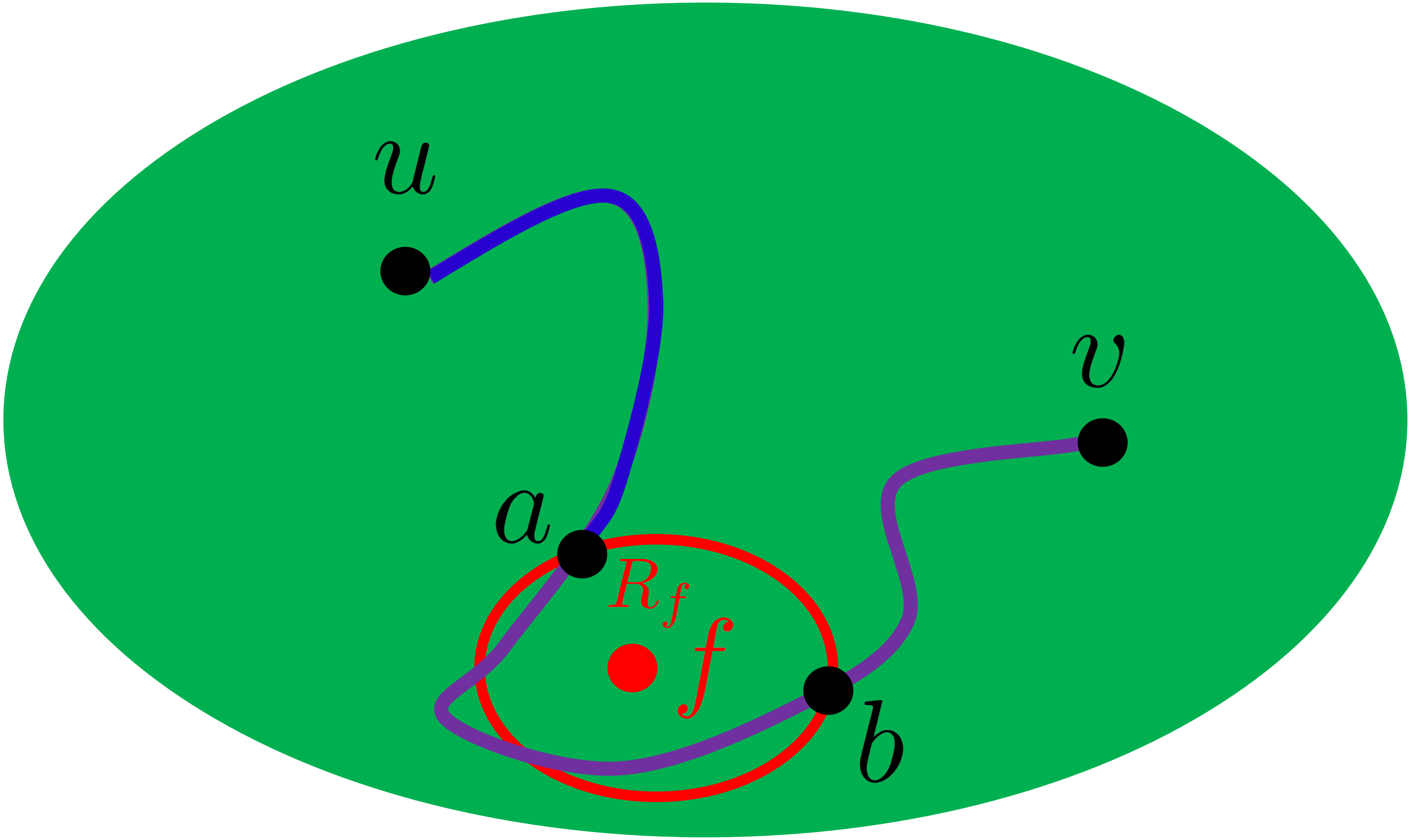

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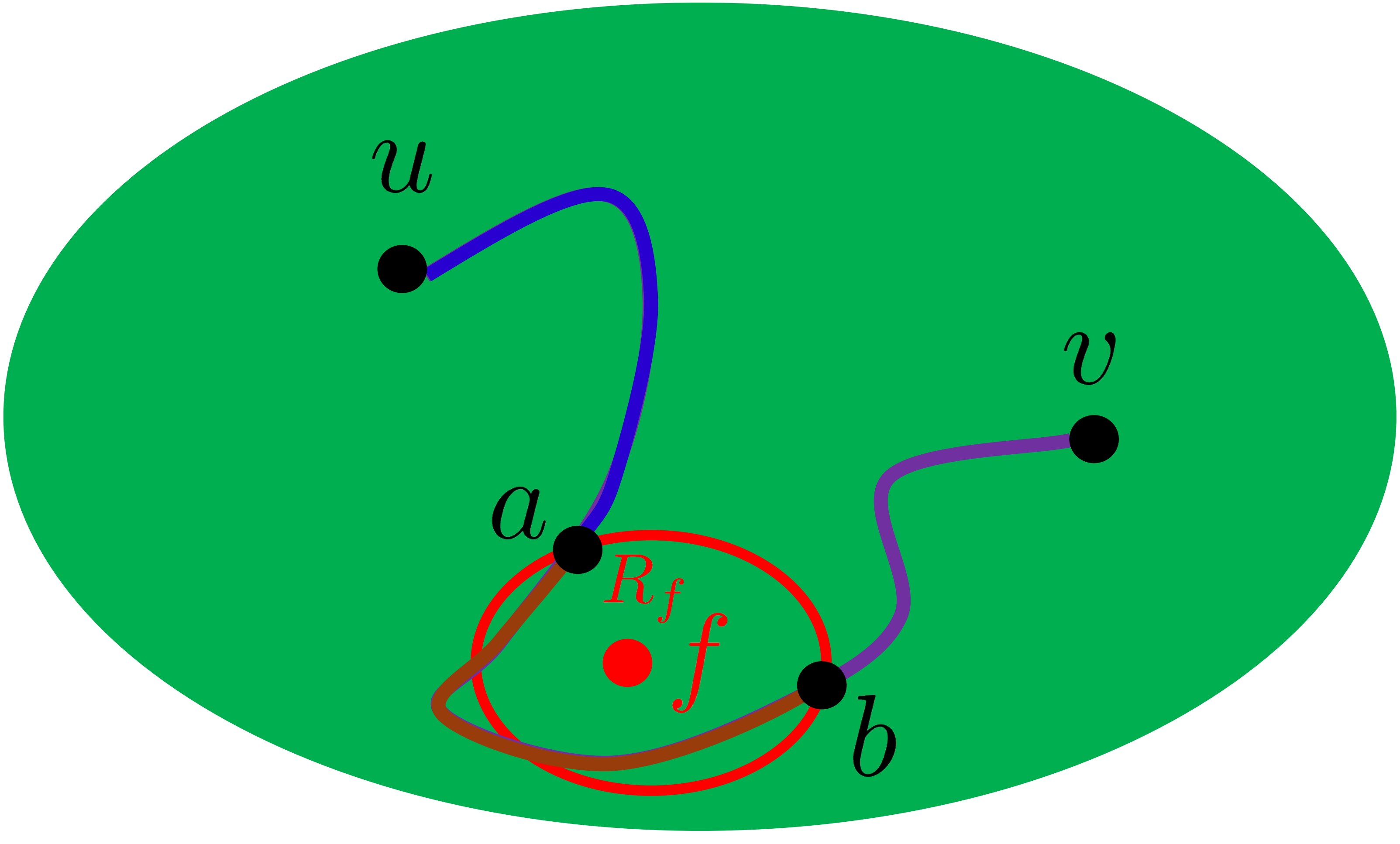

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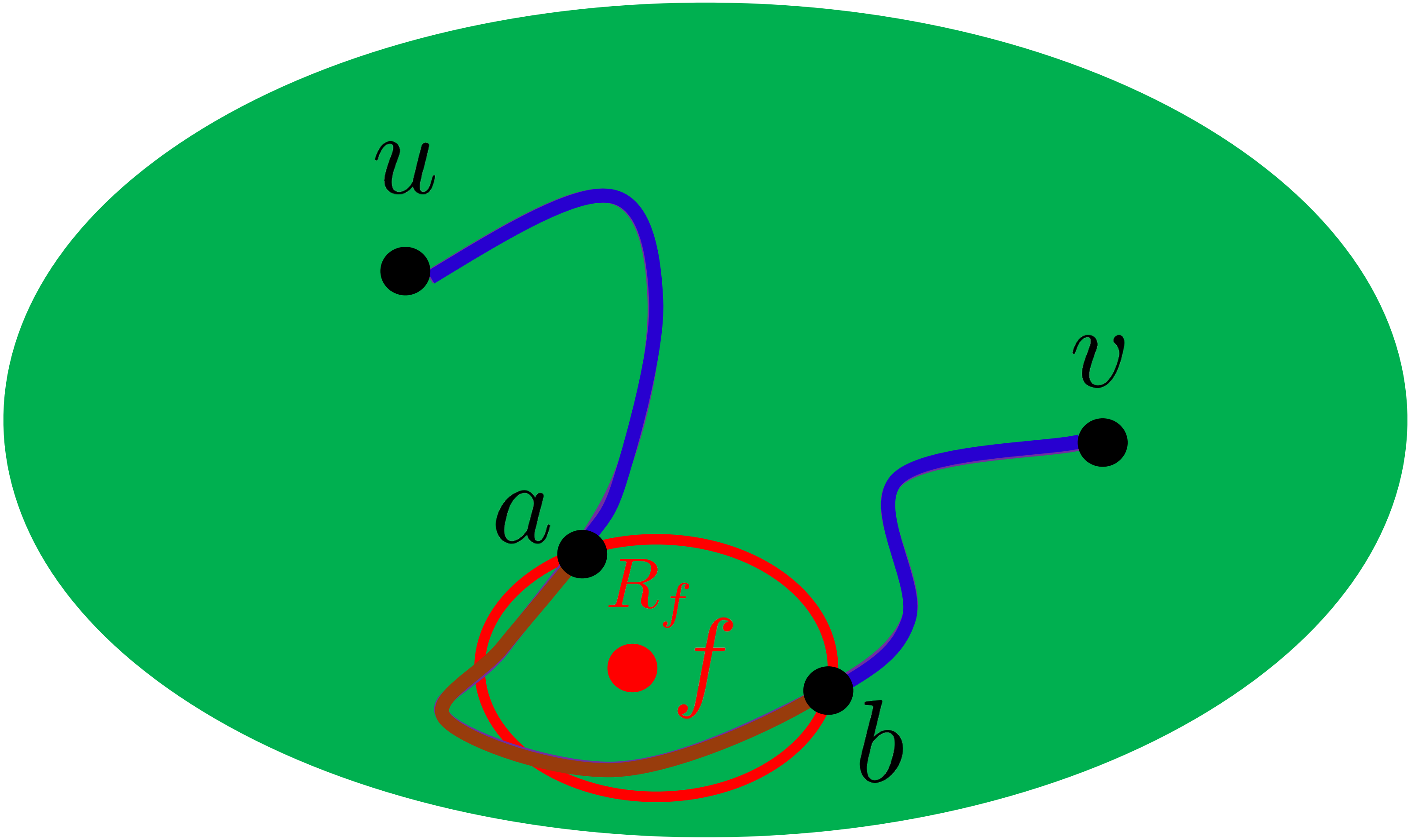

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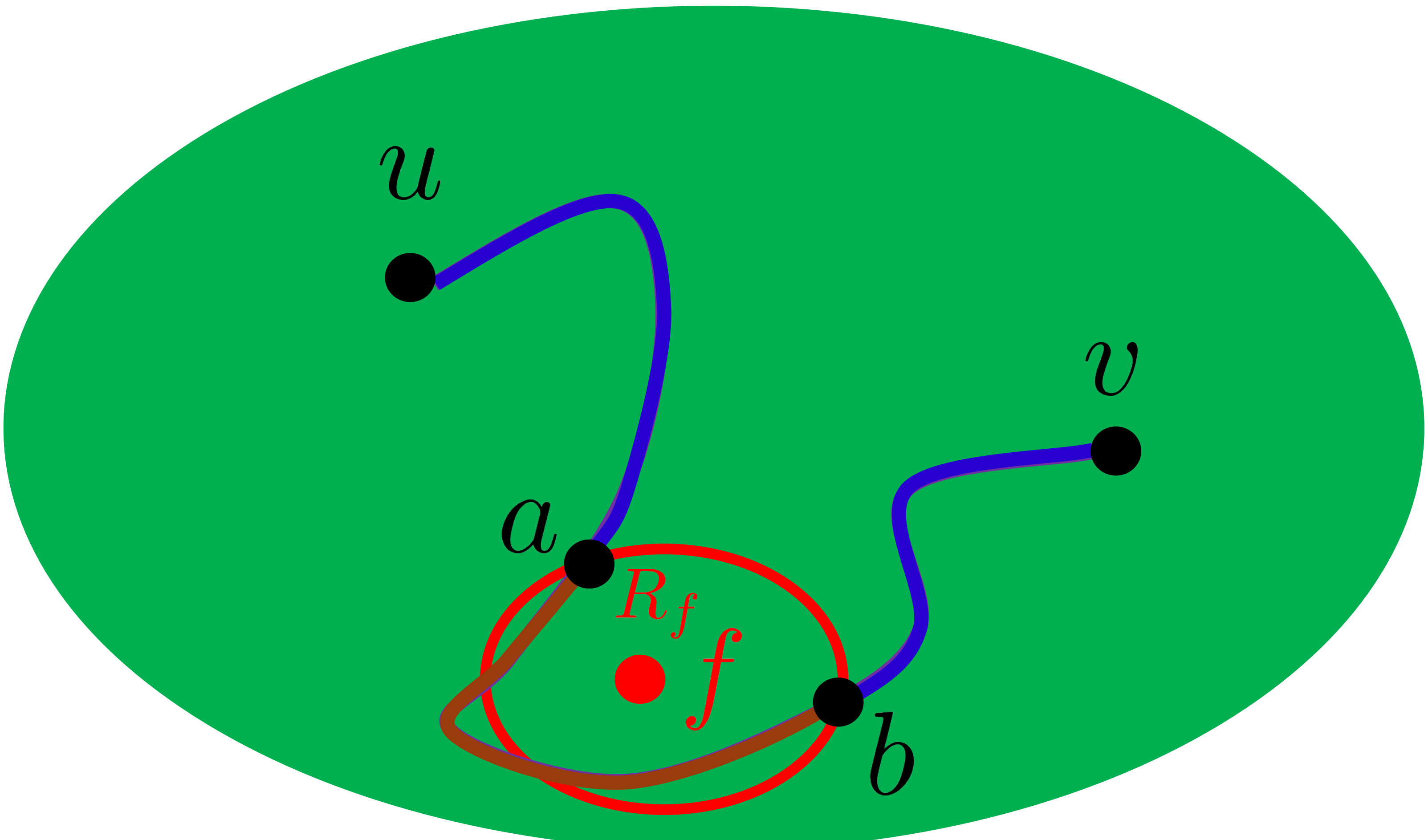

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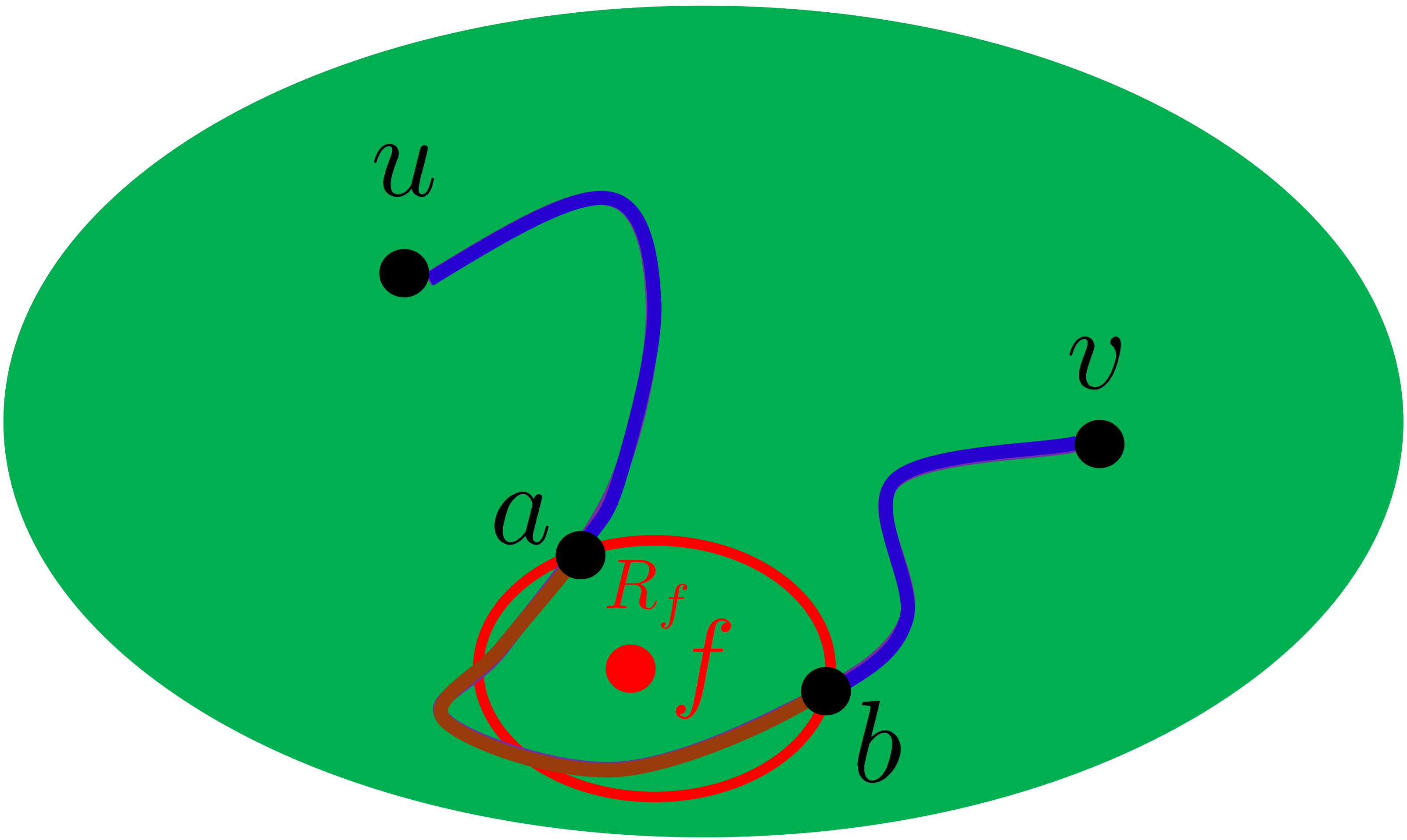

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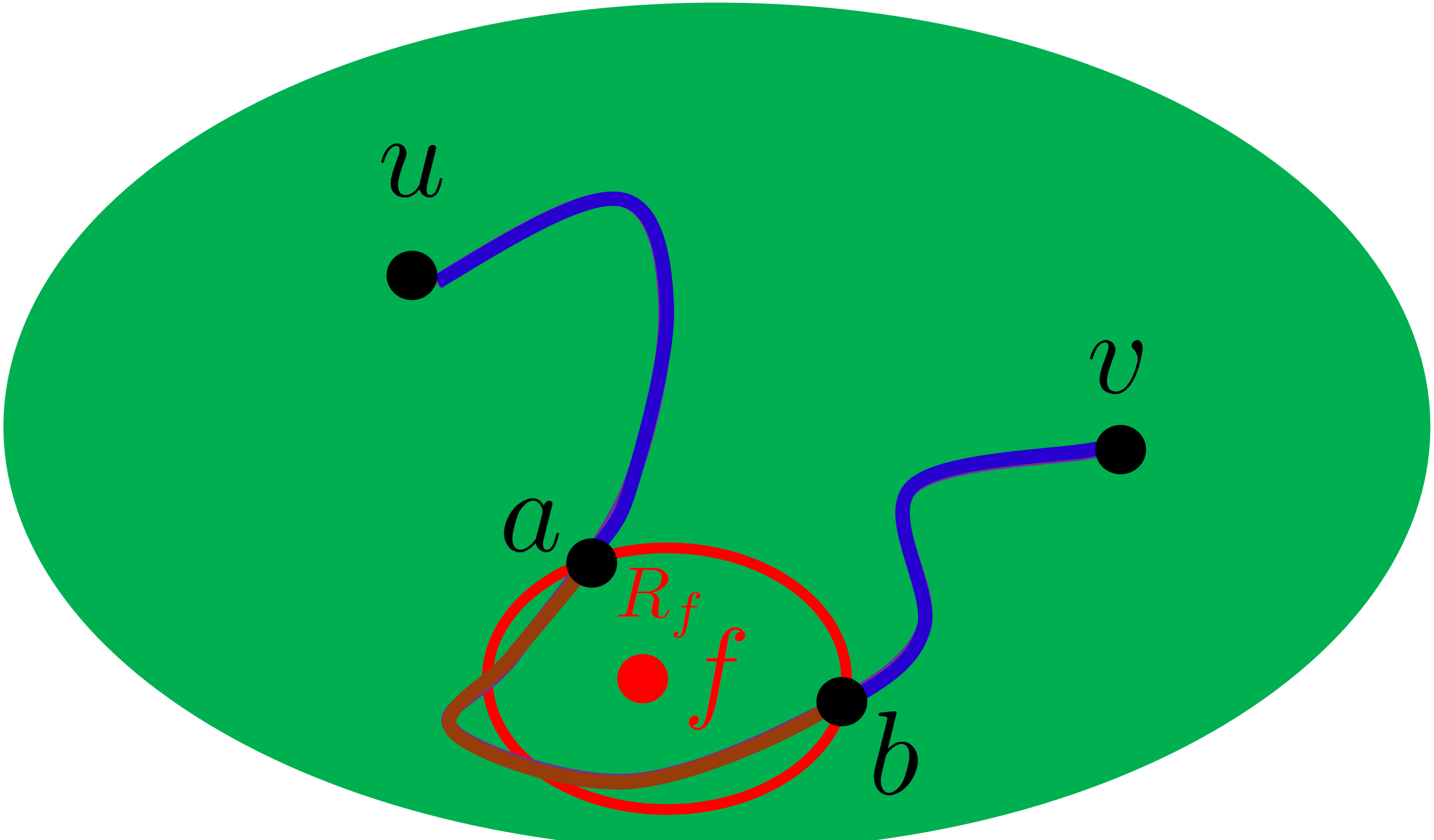

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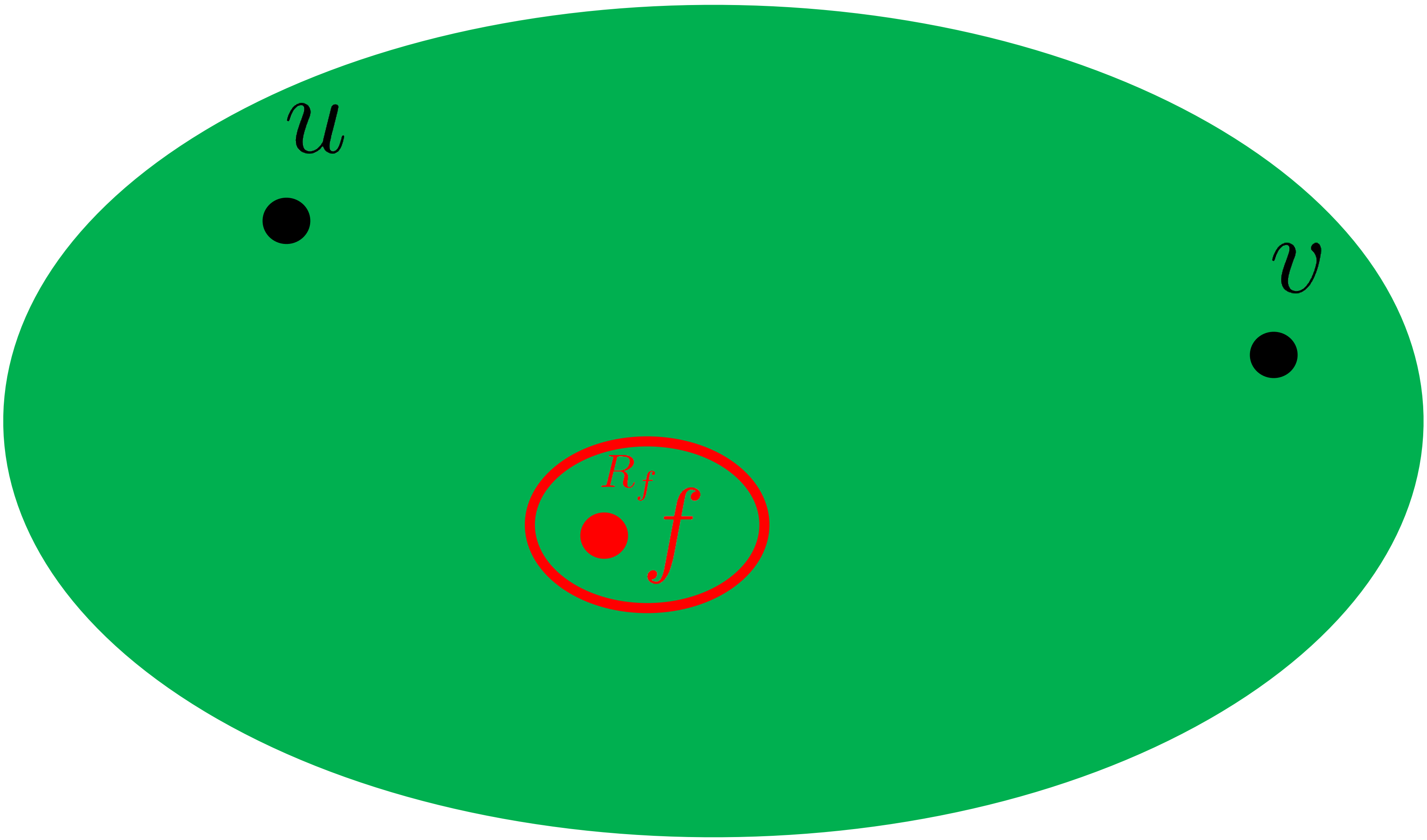

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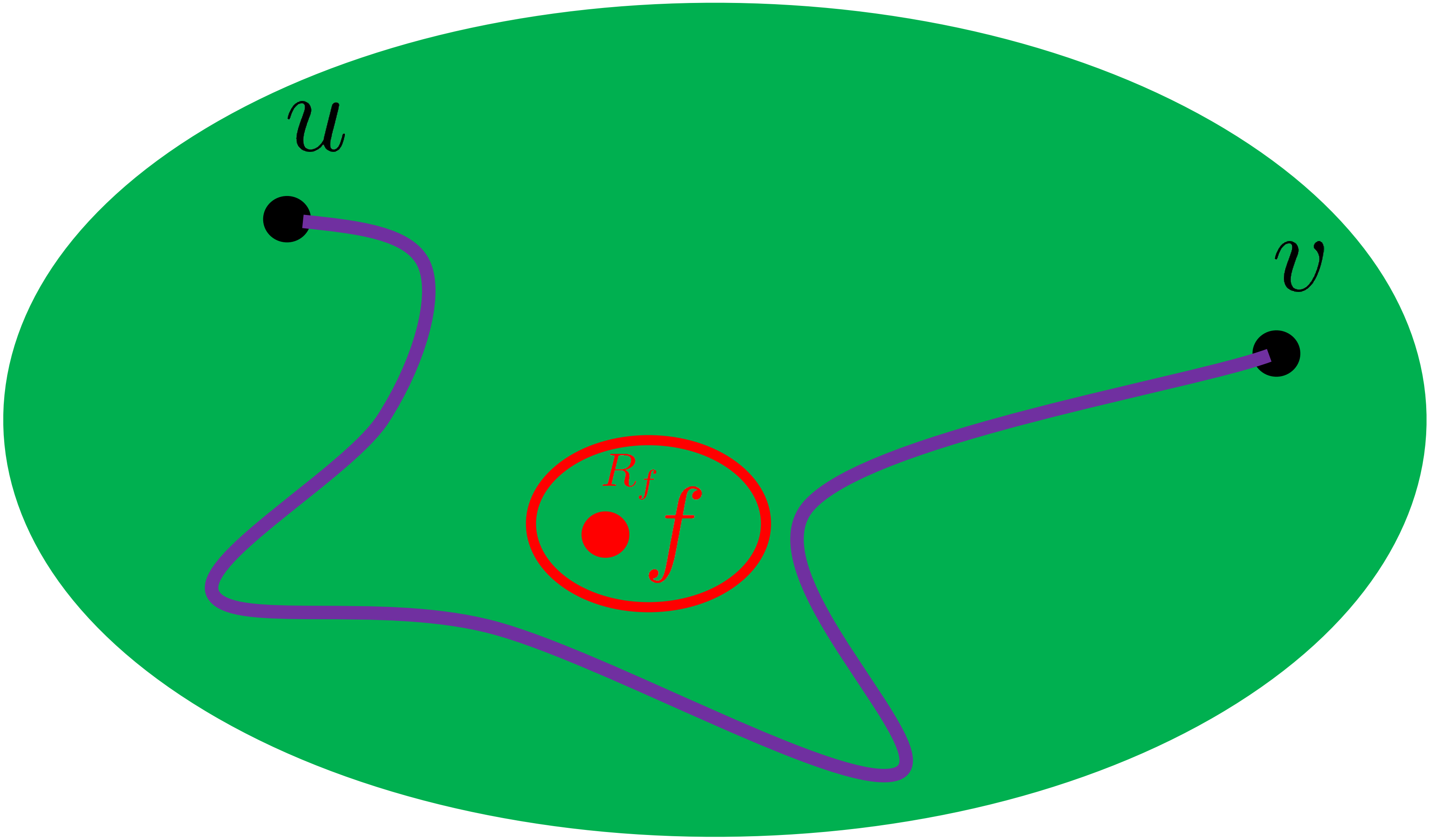

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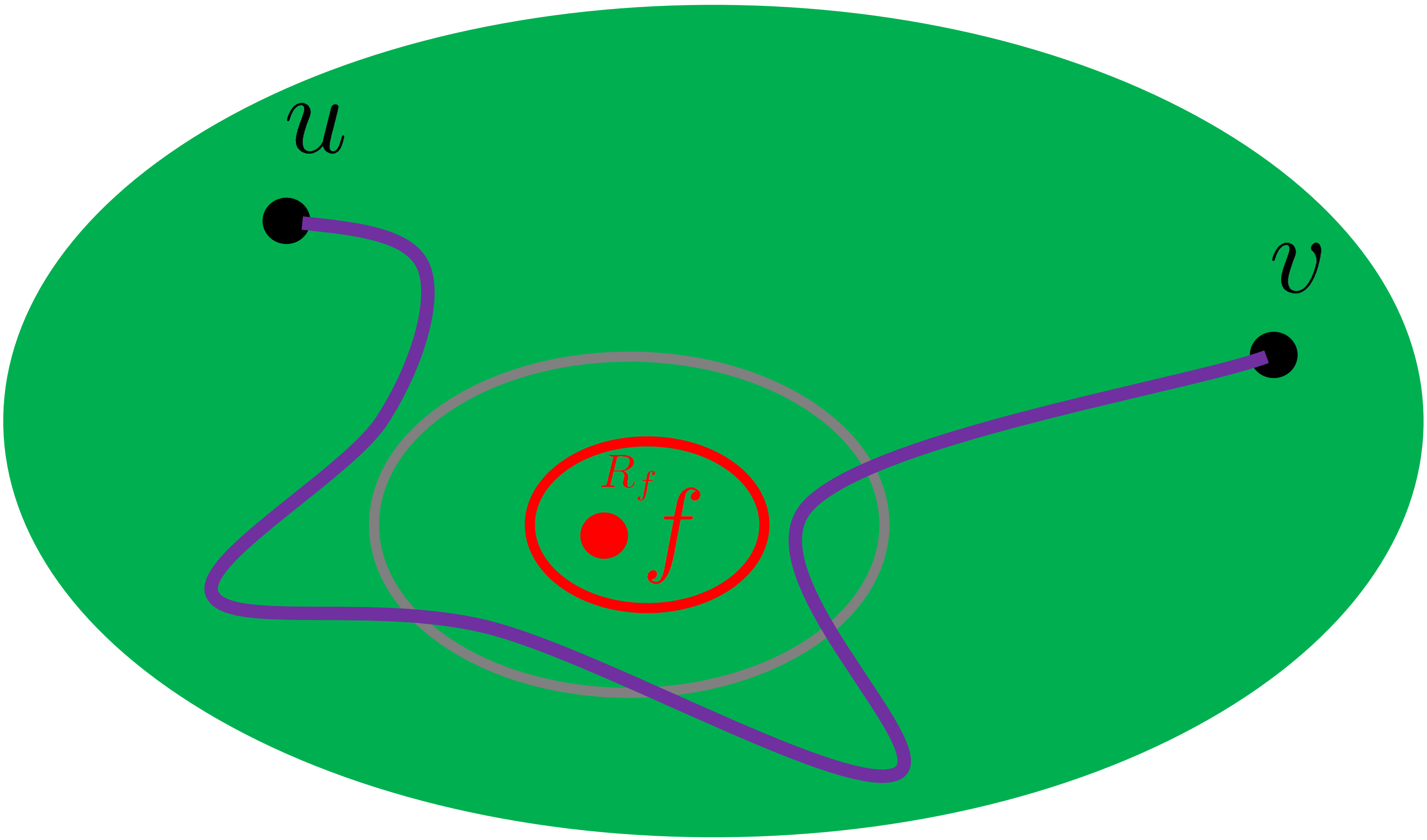

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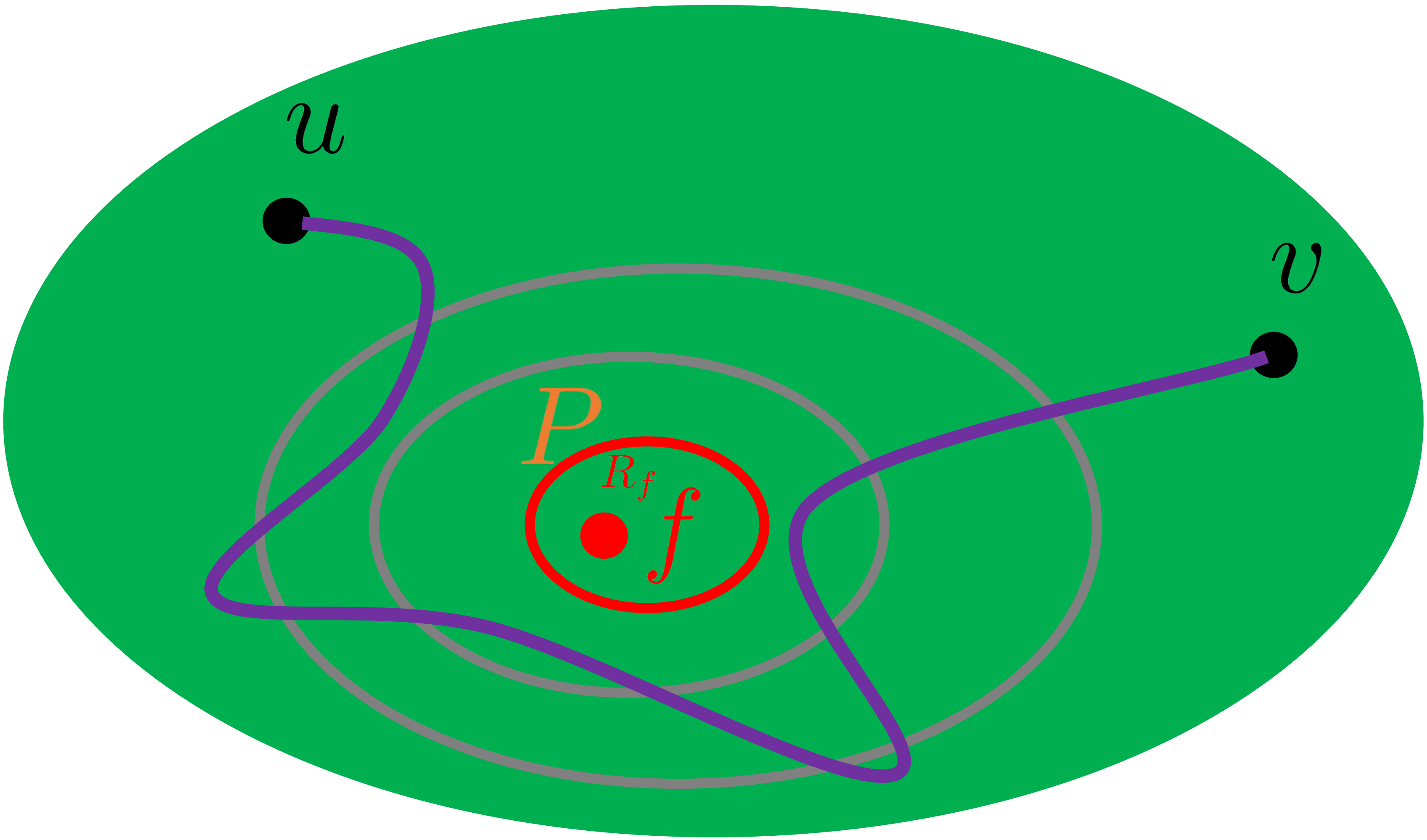

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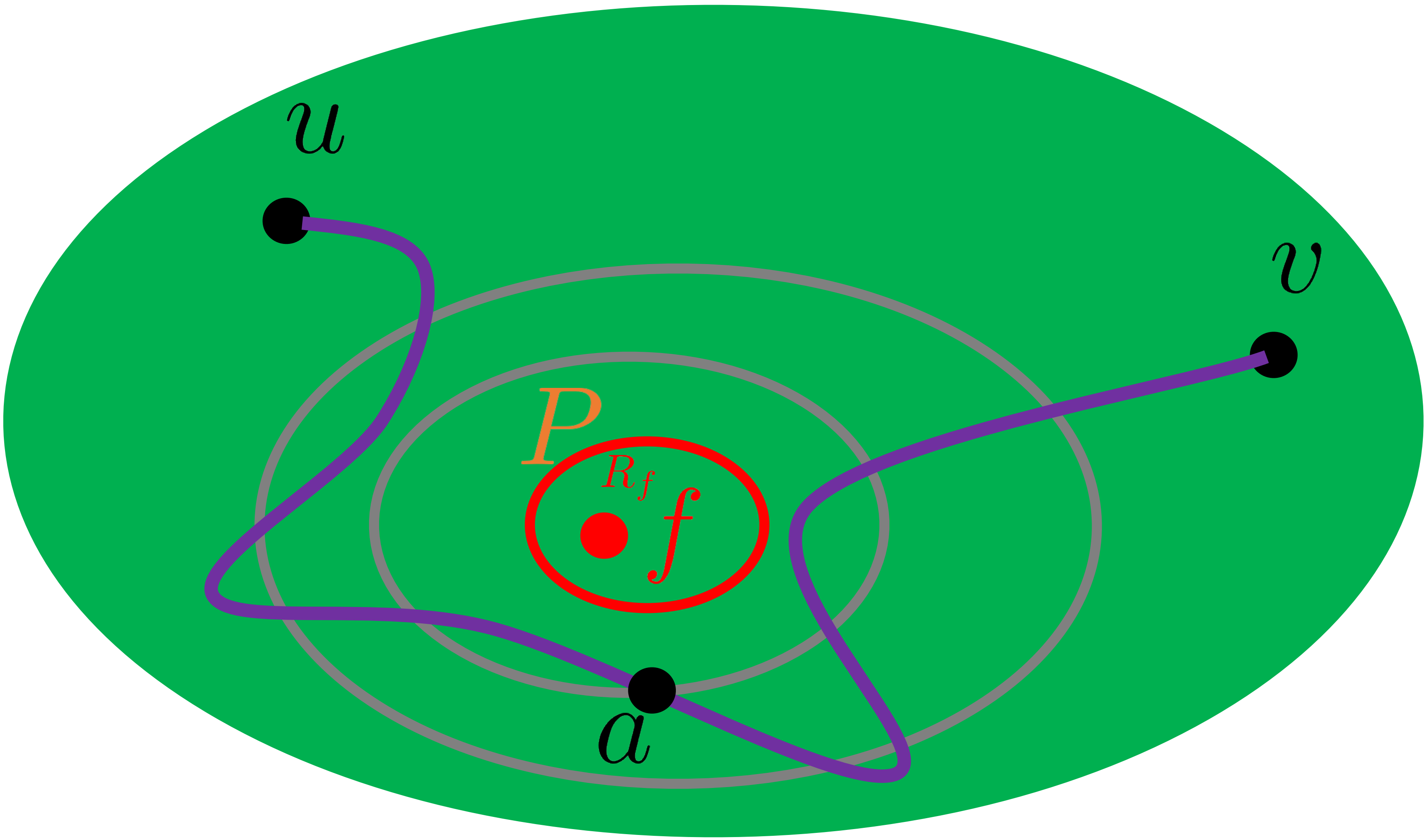

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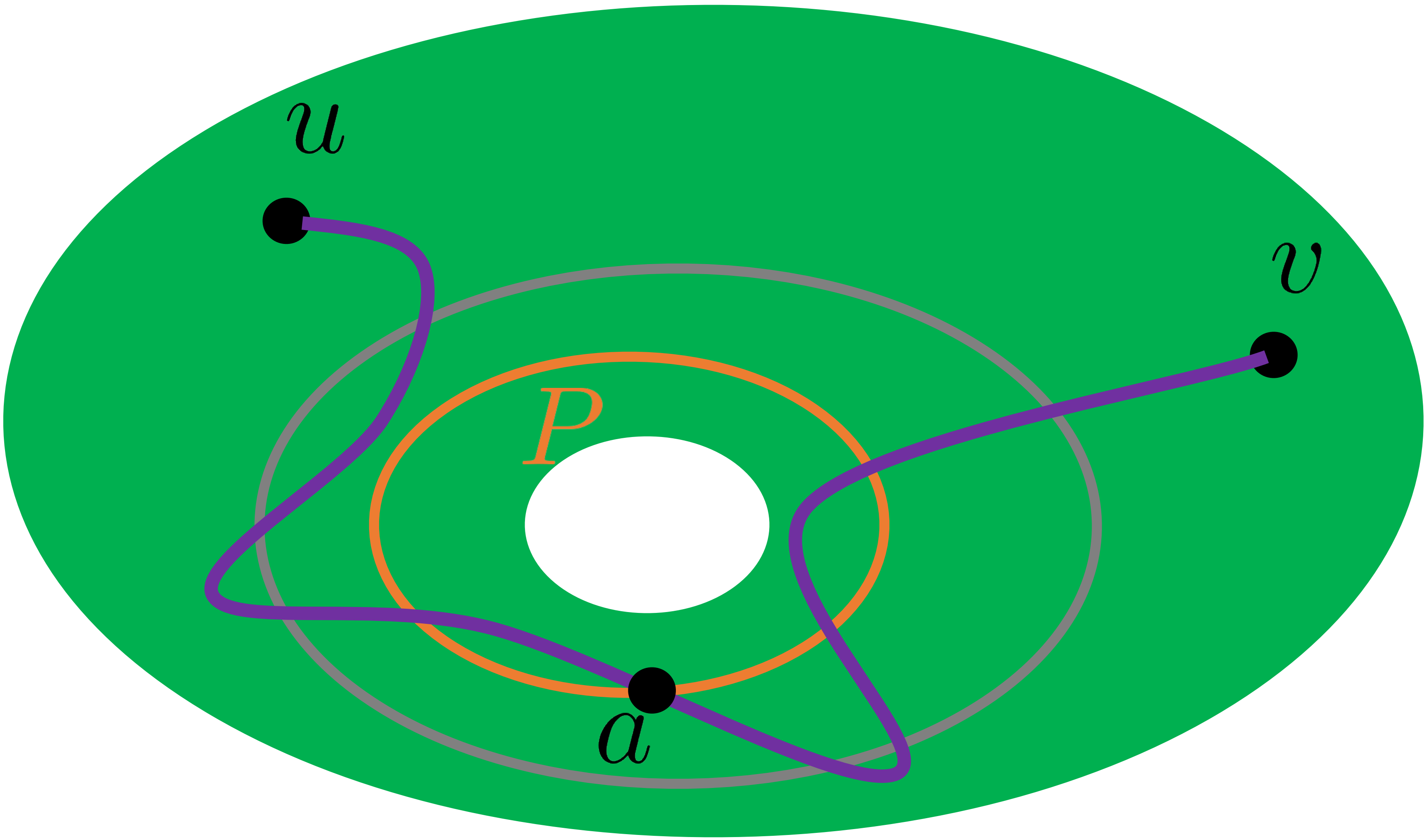

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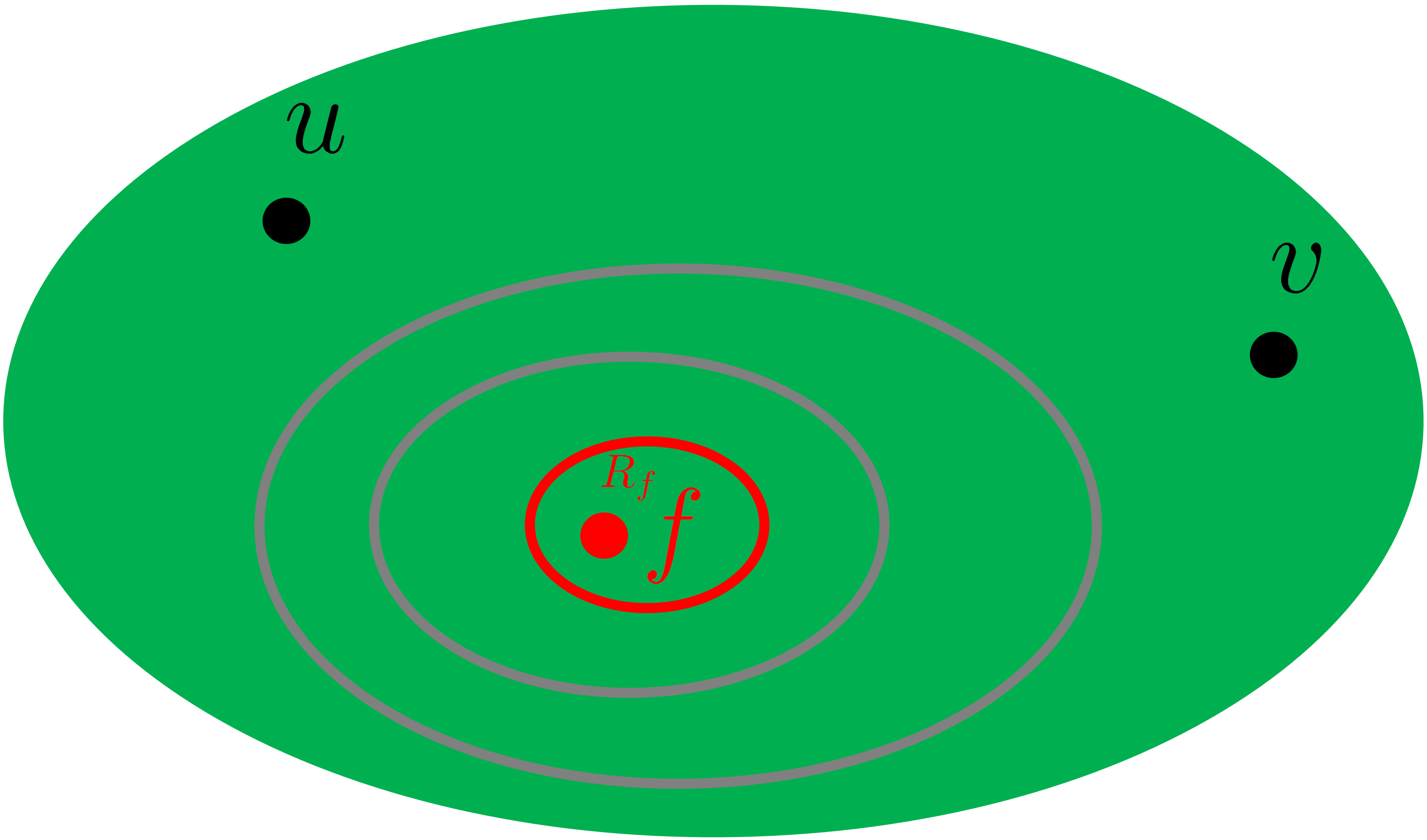

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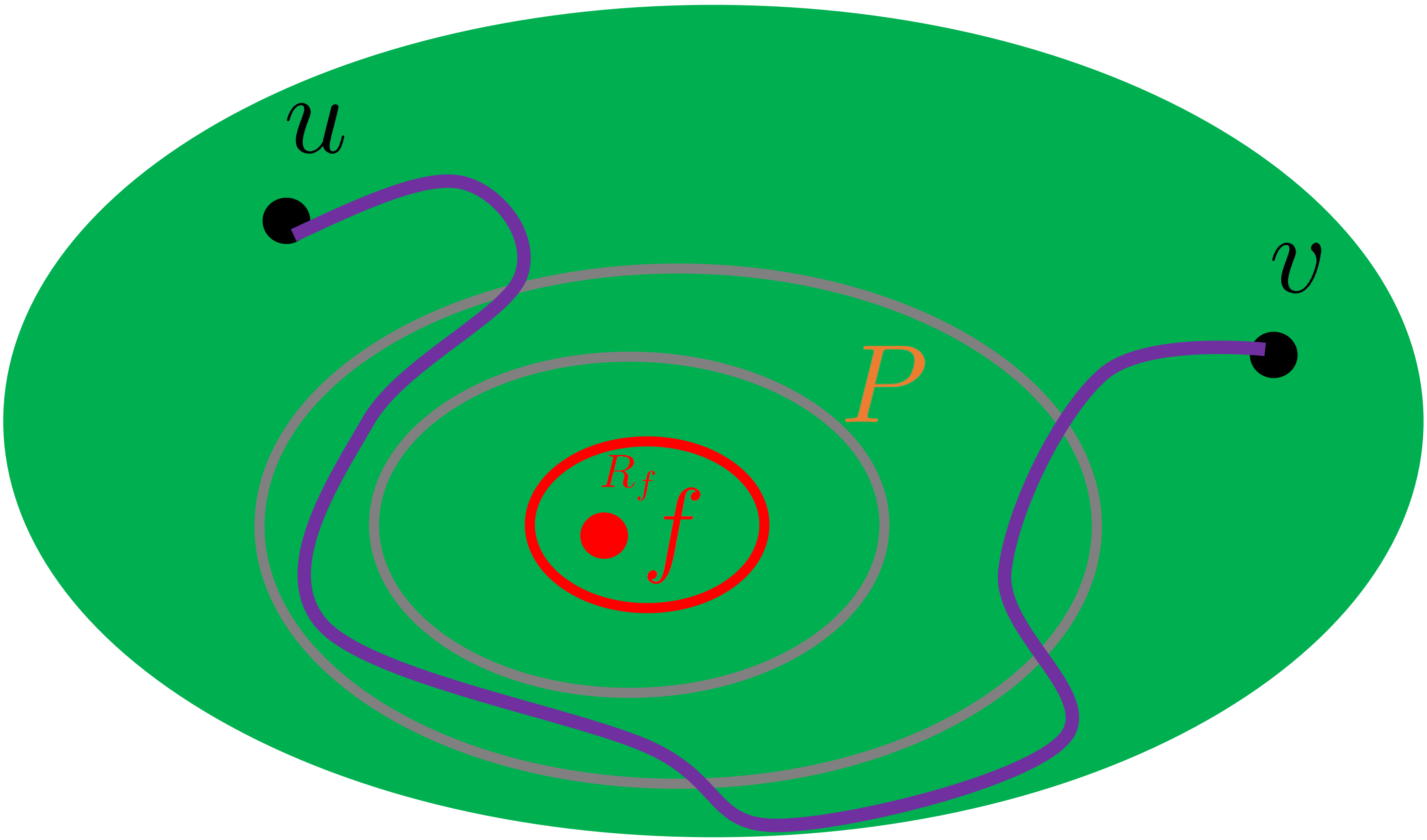

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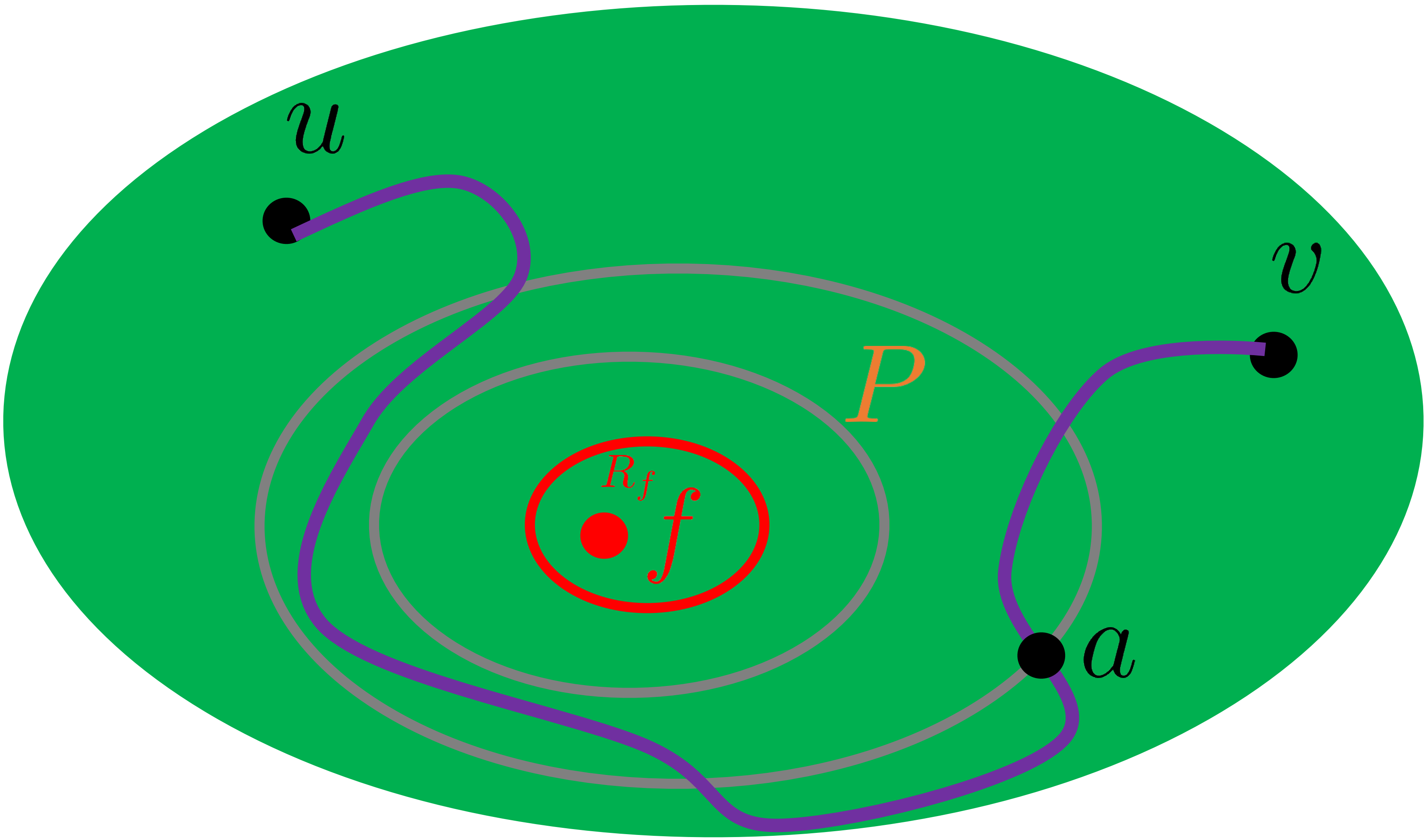

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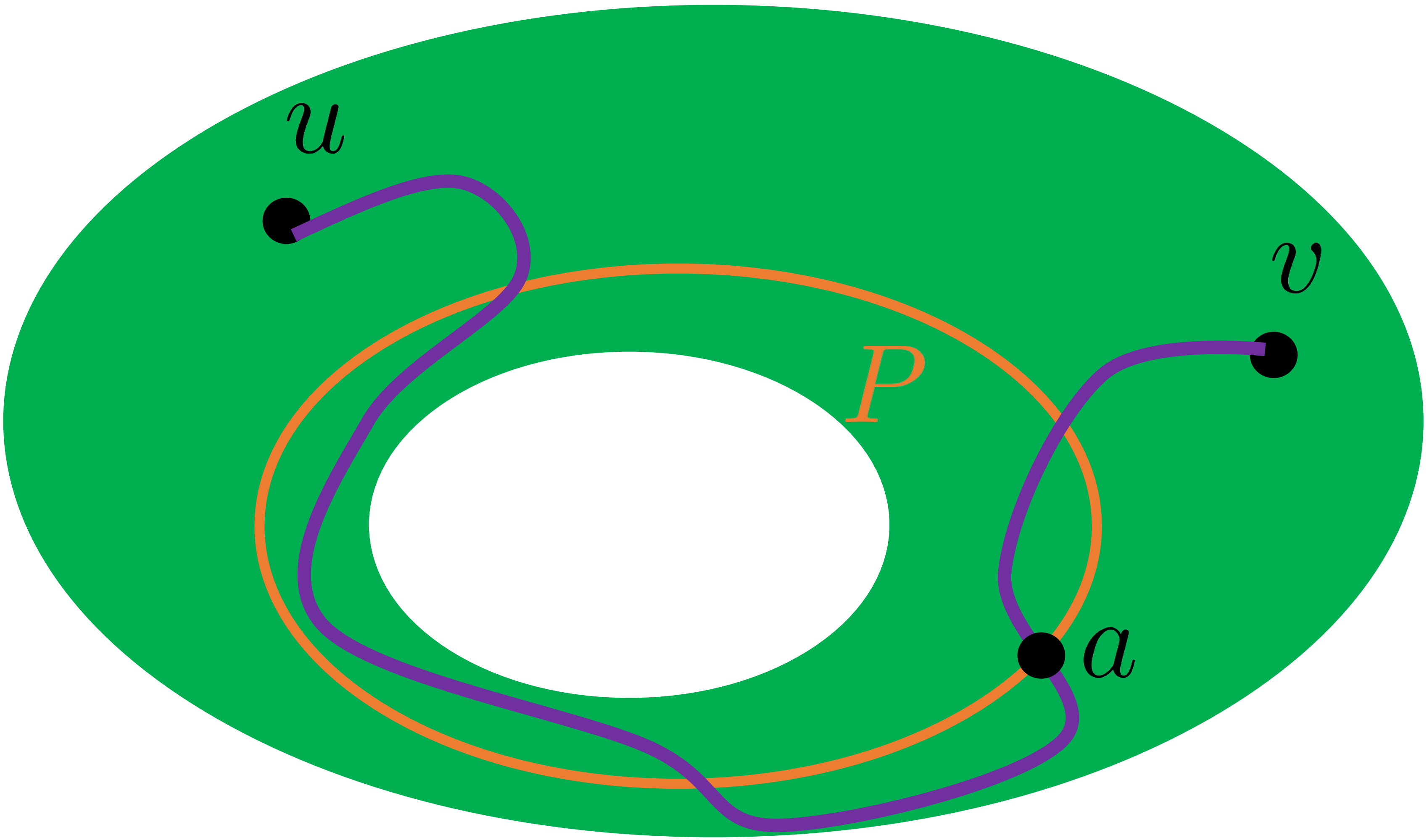

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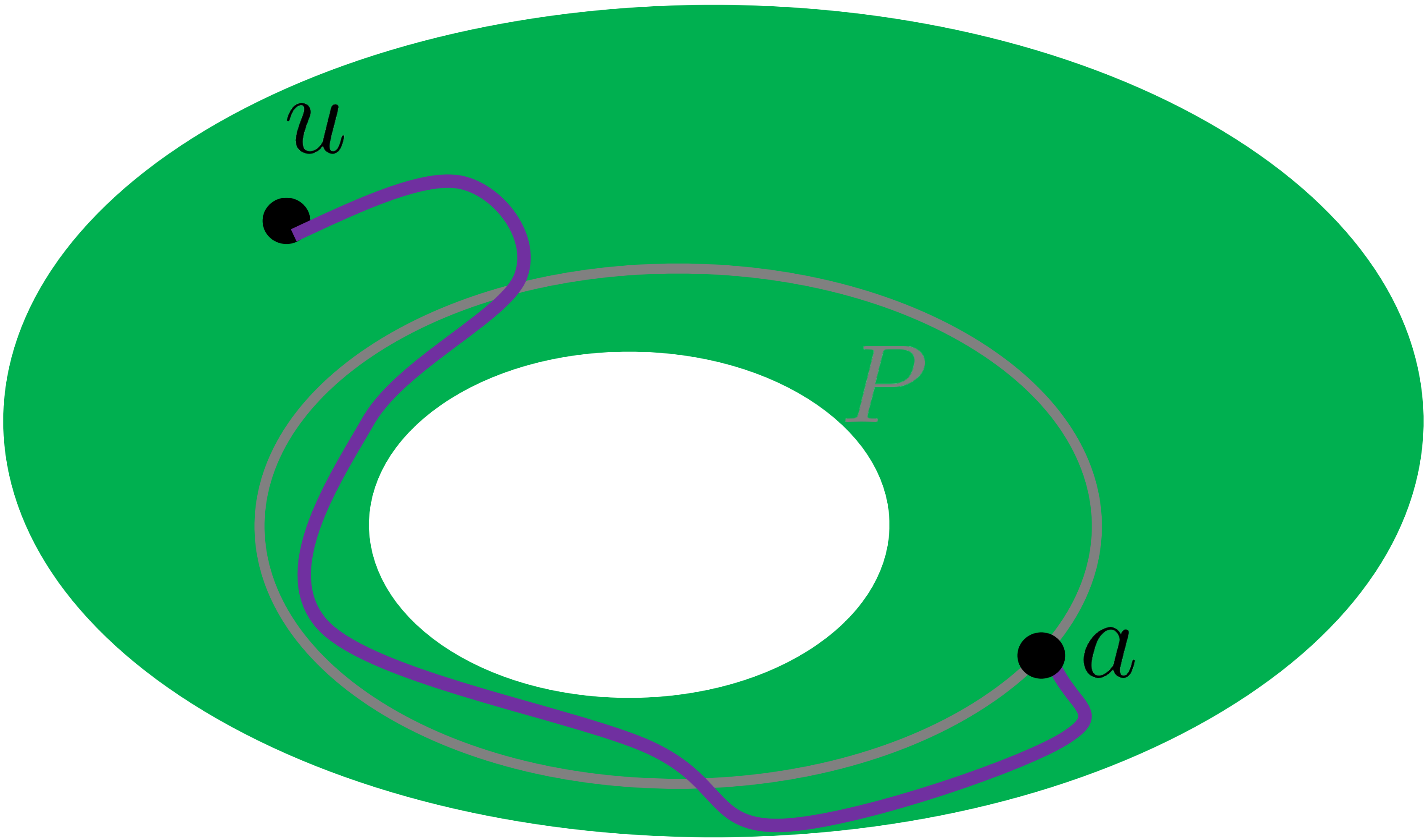

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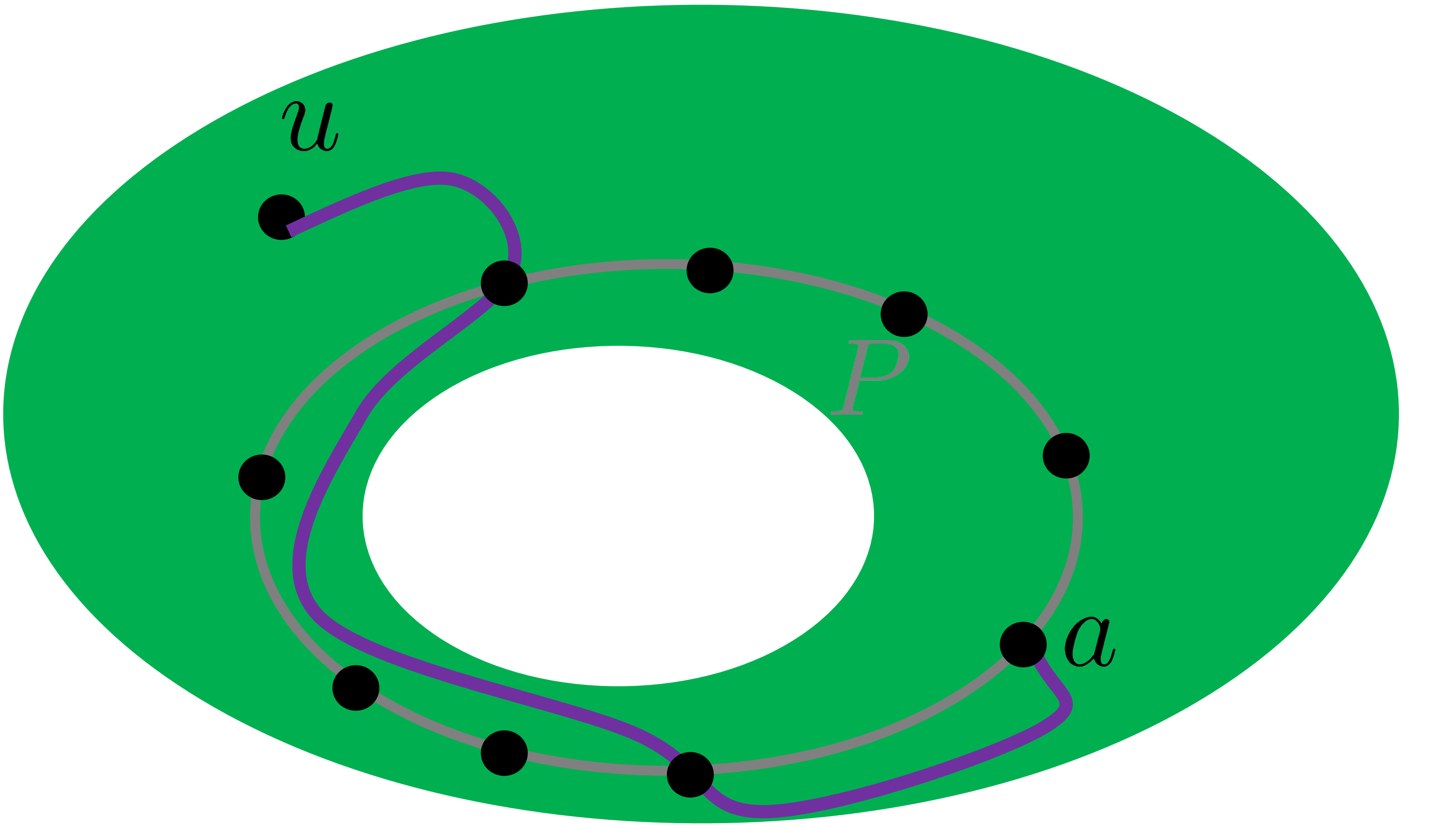
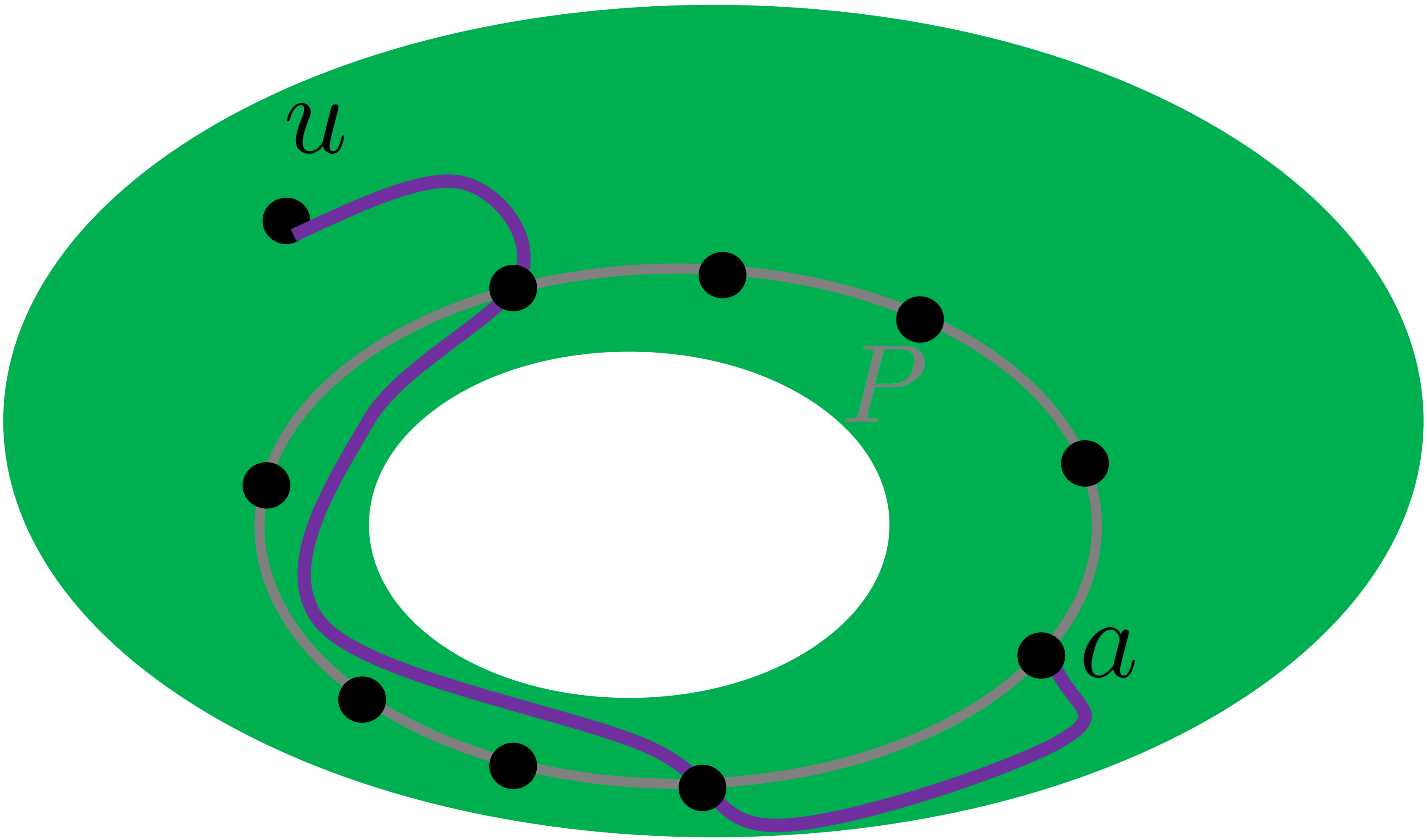
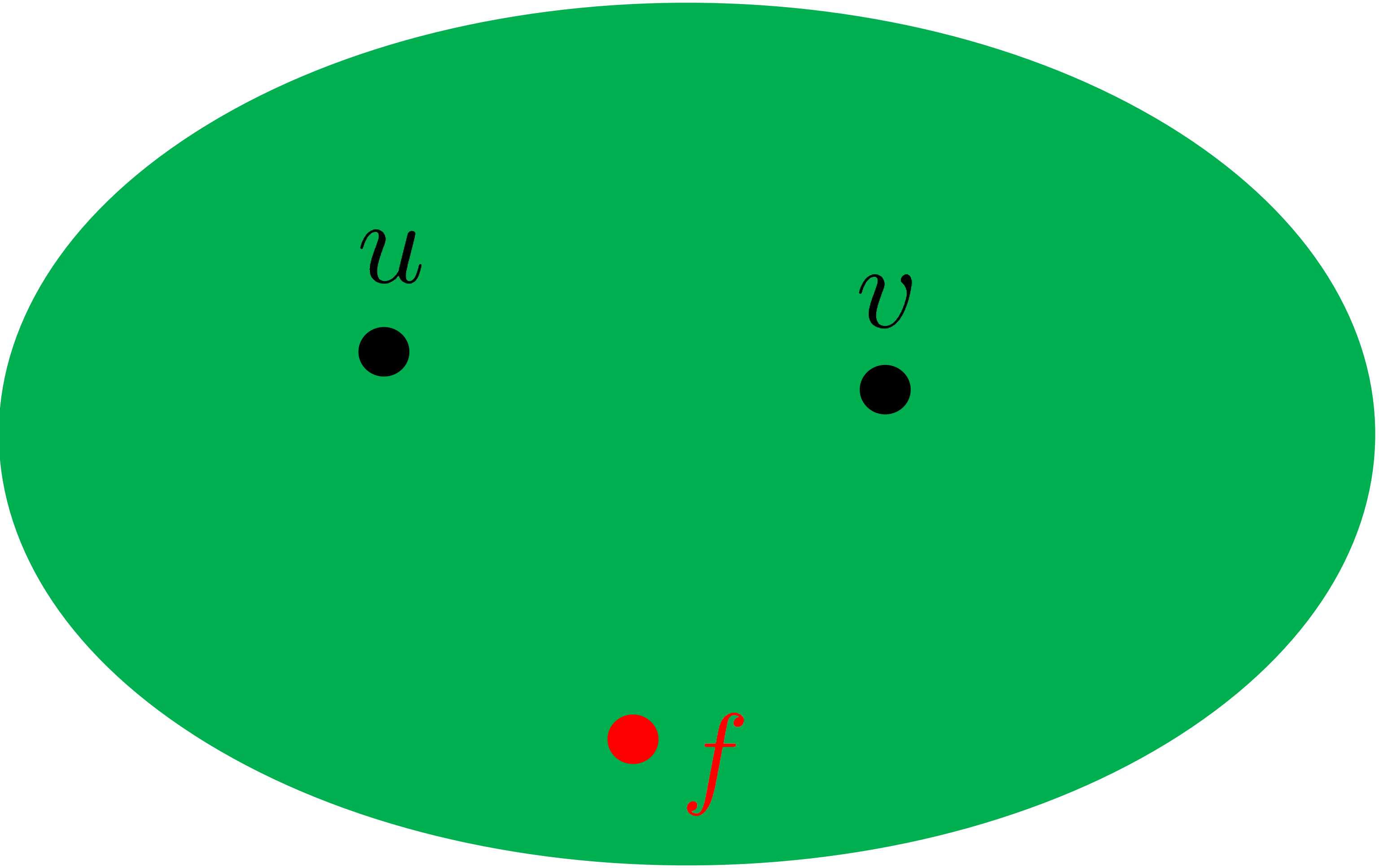


u's label in case 2:

$$\text{space} = \tilde{O}(\sum_{P \in Pieces} |\partial P|) = \tilde{O}(n/\sqrt{r})$$



Case 3: Shortest path doesn't touch ∂P for any ancestor piece P of R_f



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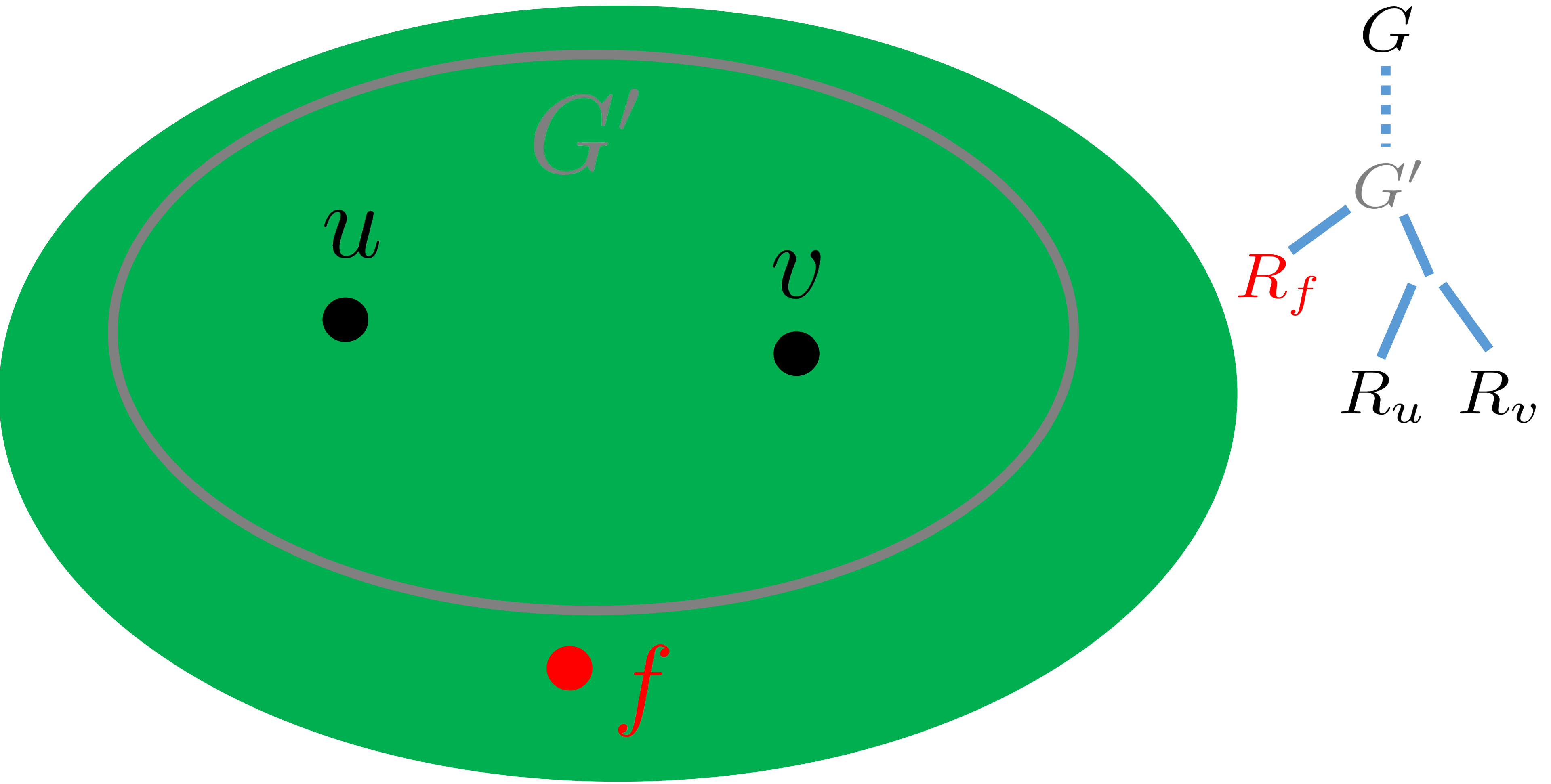

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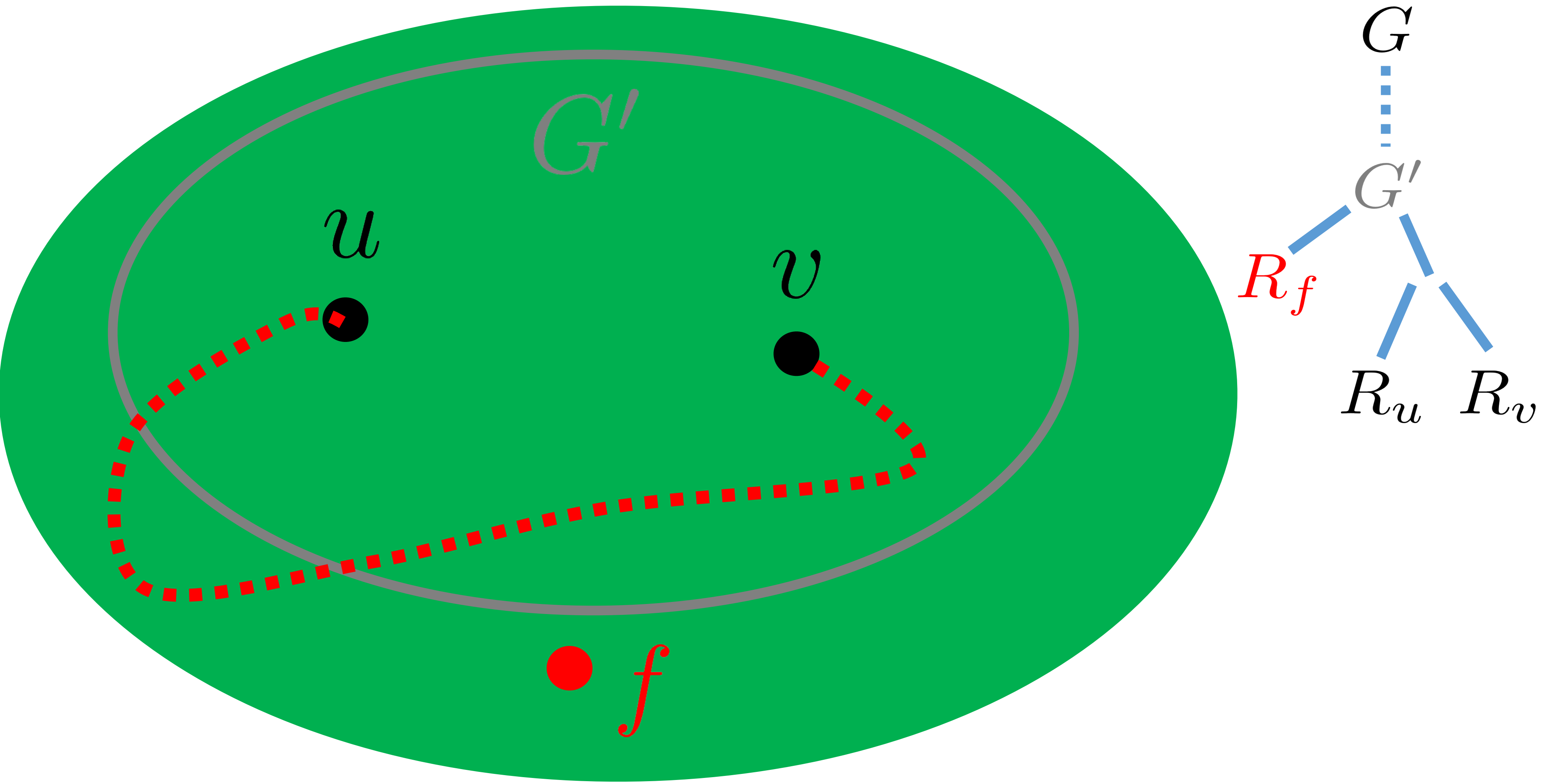

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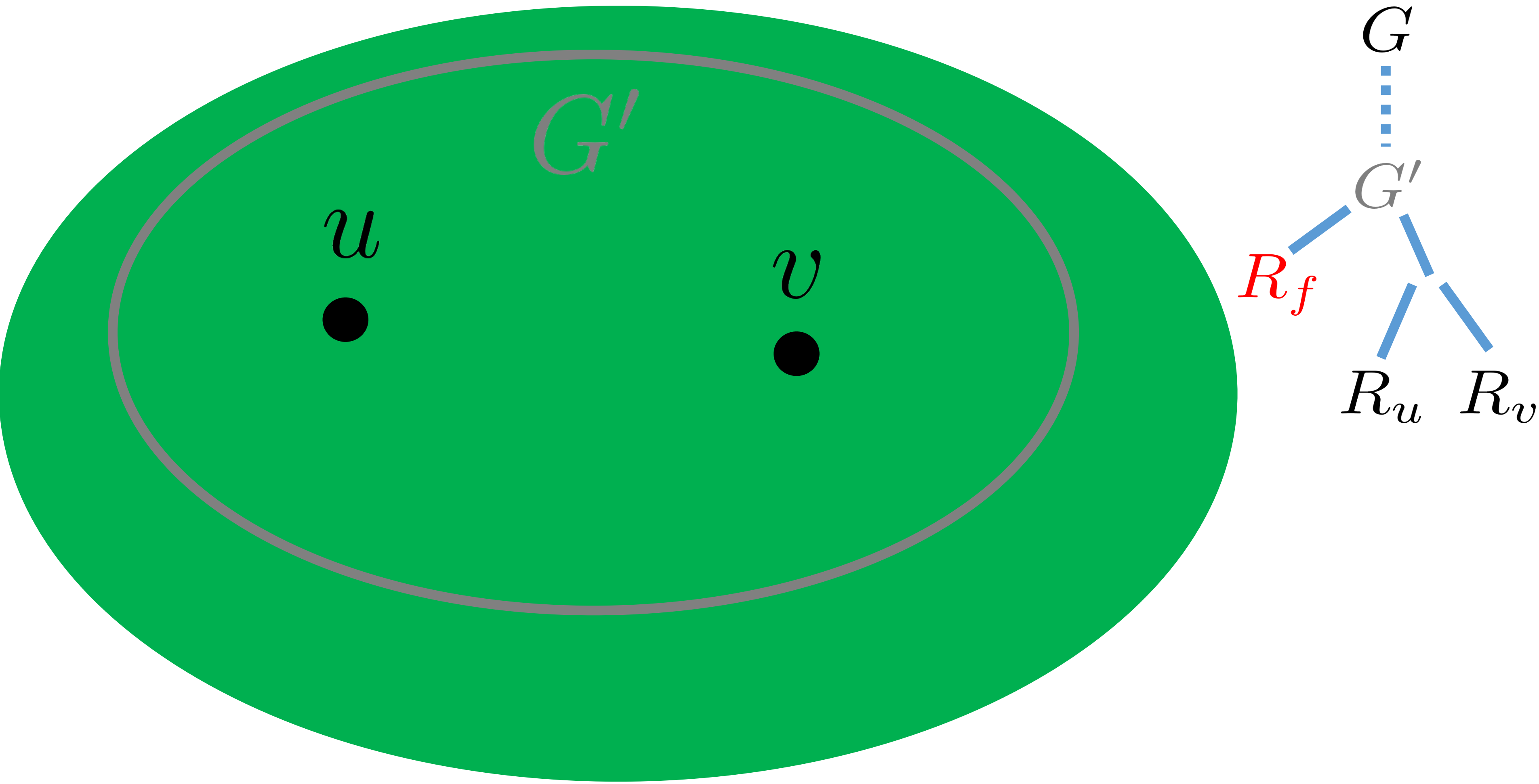

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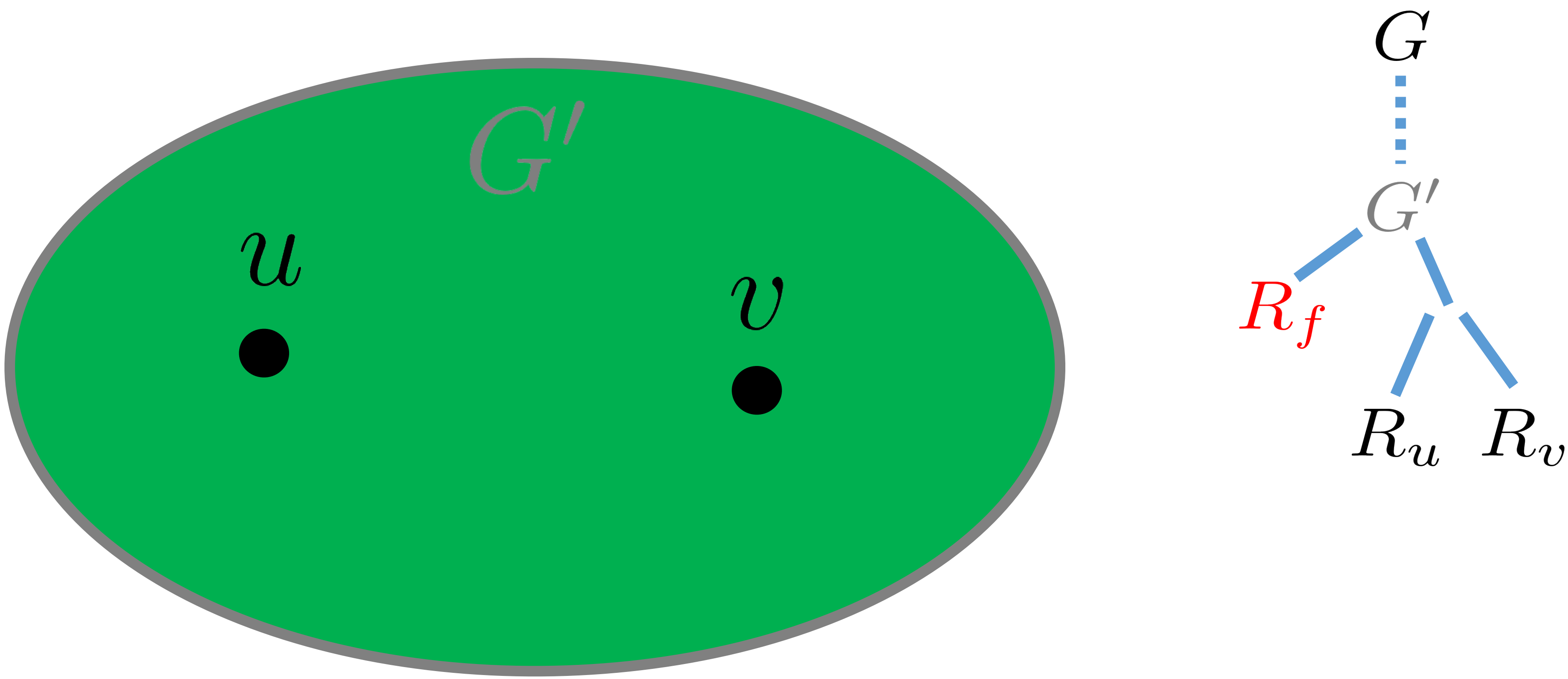

use labeling without faults in G'

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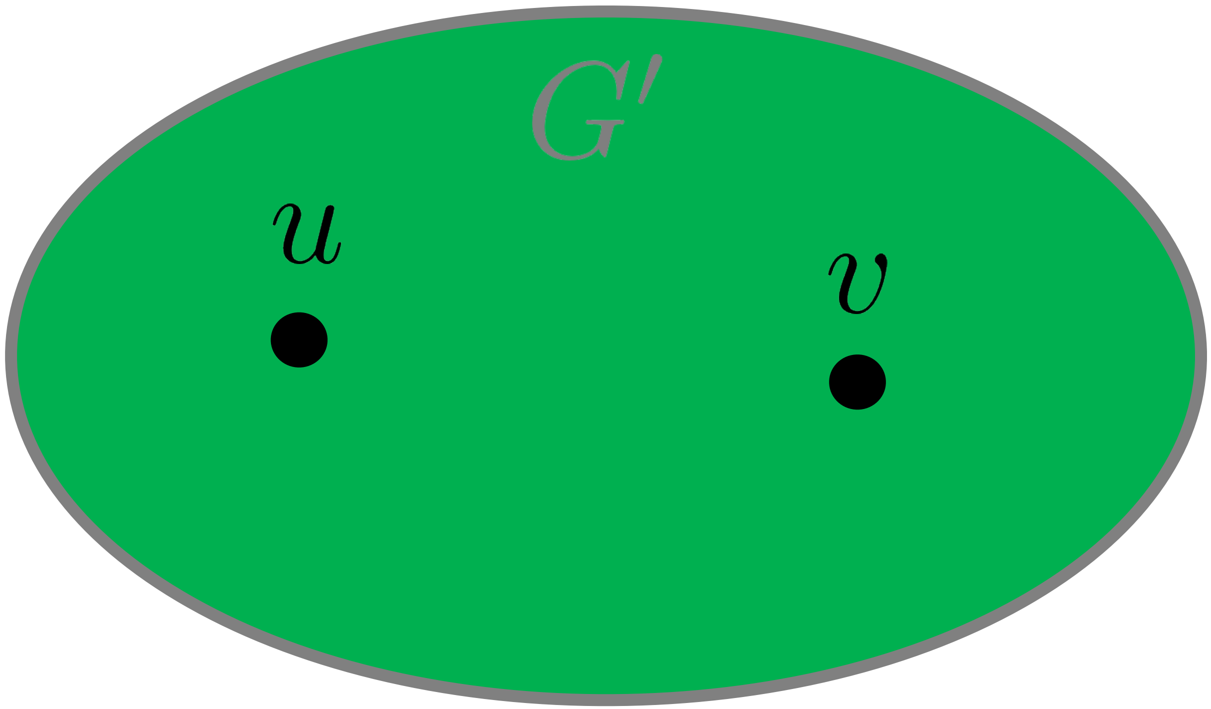
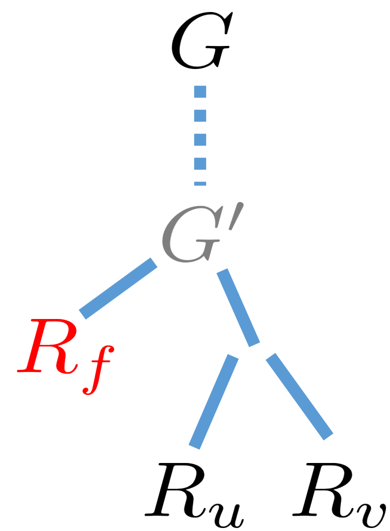
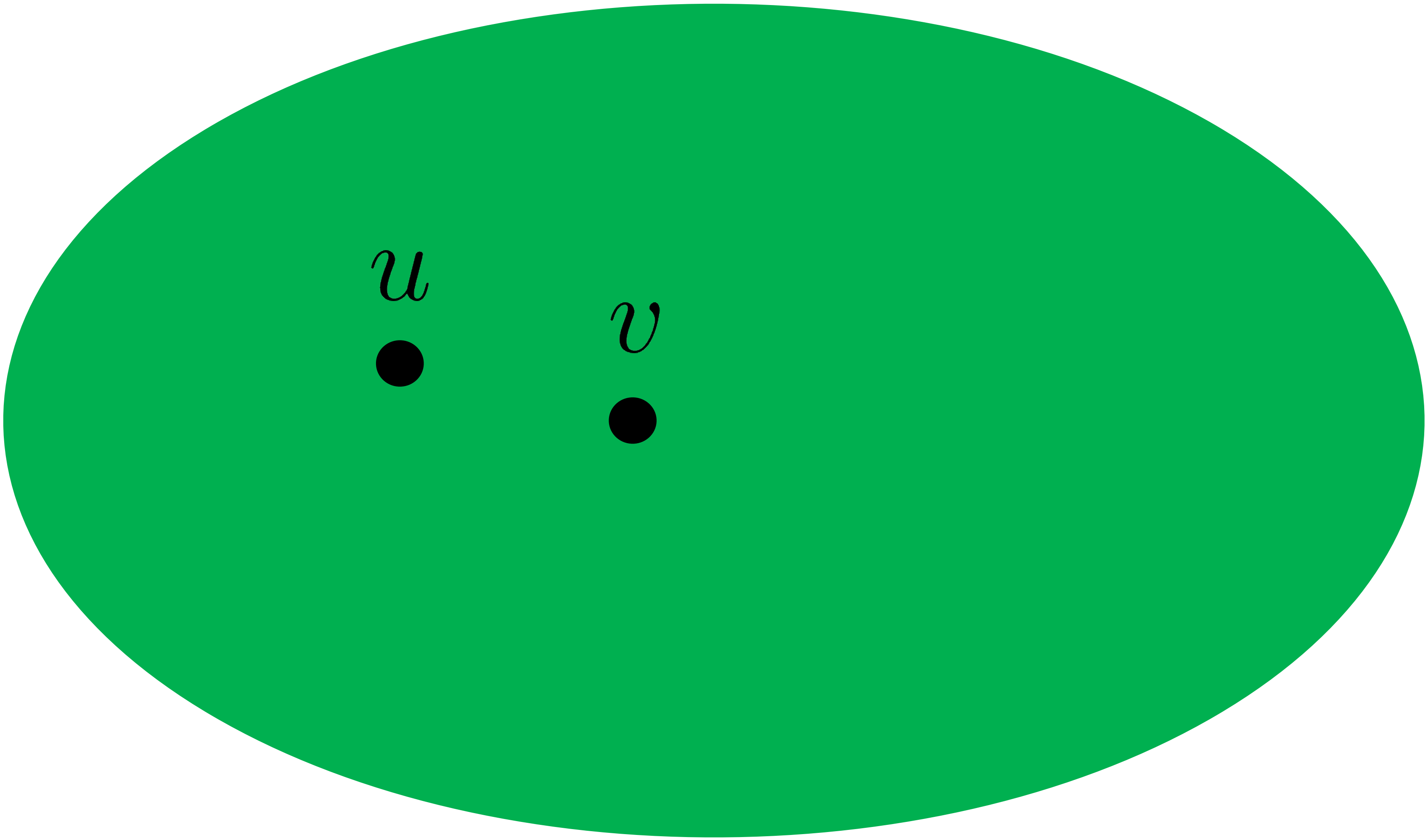




use labeling without faults in G'

$\tilde{O}(\sqrt{n})$ space

Last case: $R_u = R_v$ and the shortest path doesn't touch ∂R_u



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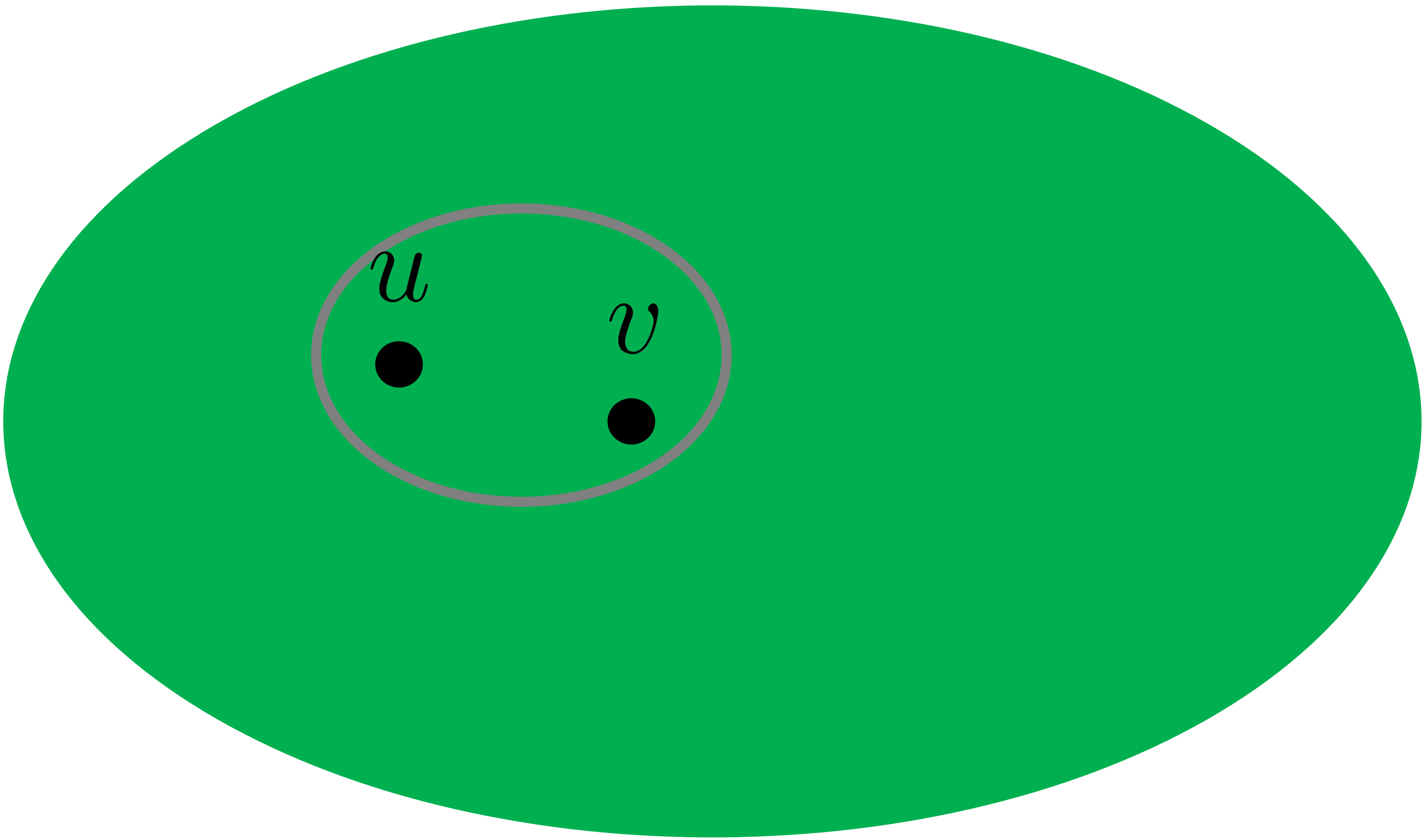

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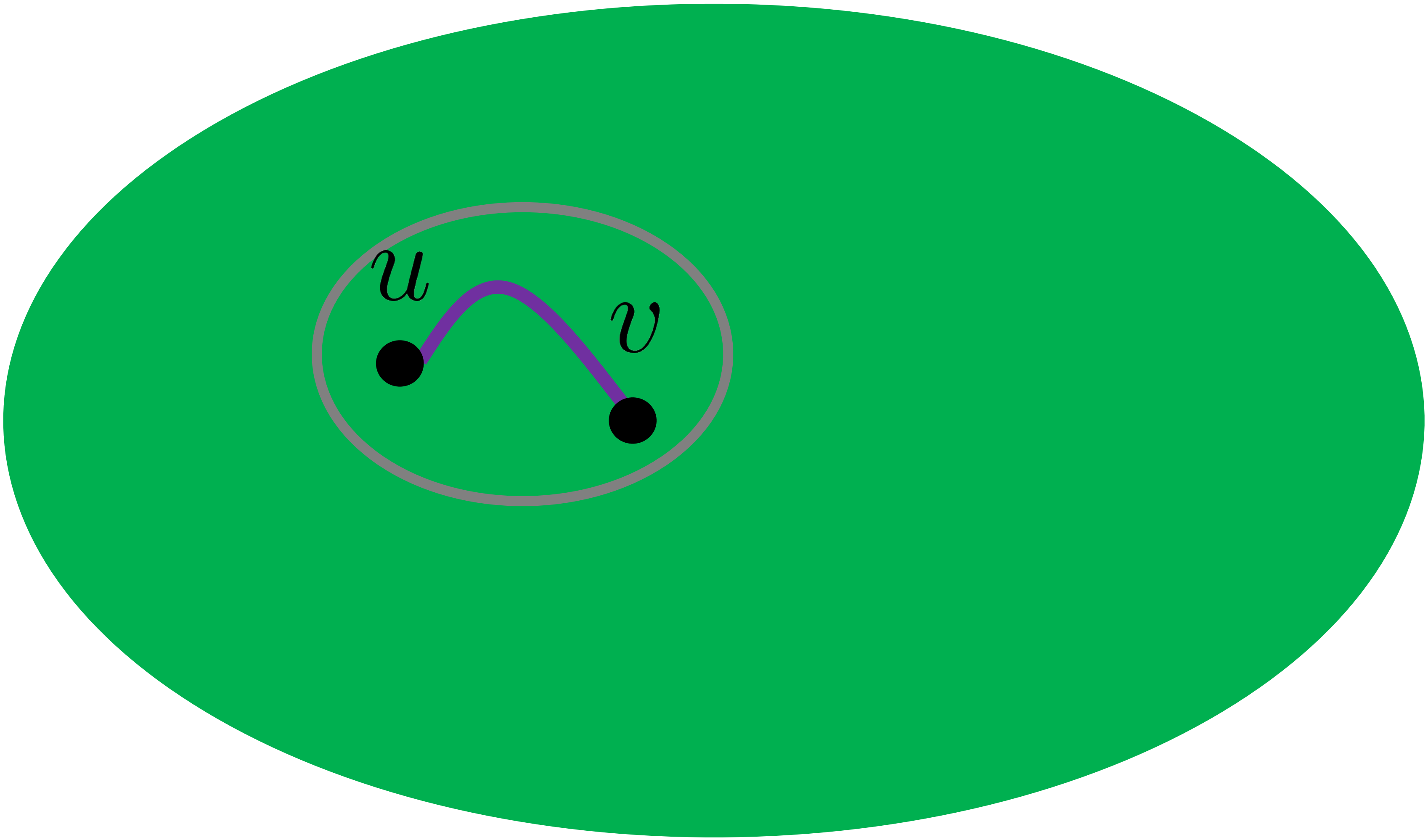
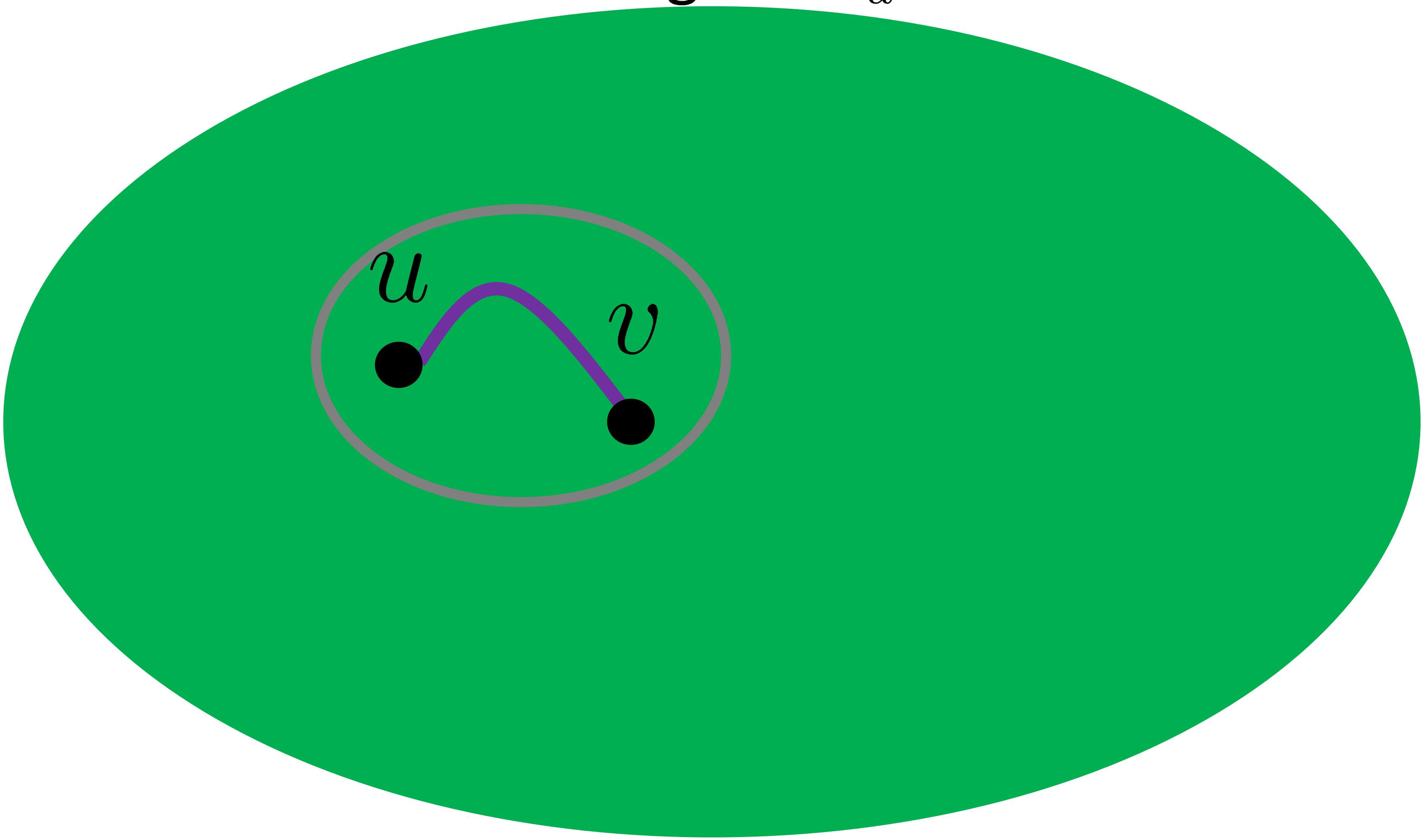


Last case: $R_u = R_v$ and the shortest path doesn't touch ∂R_u

Label stores the entire region R_u



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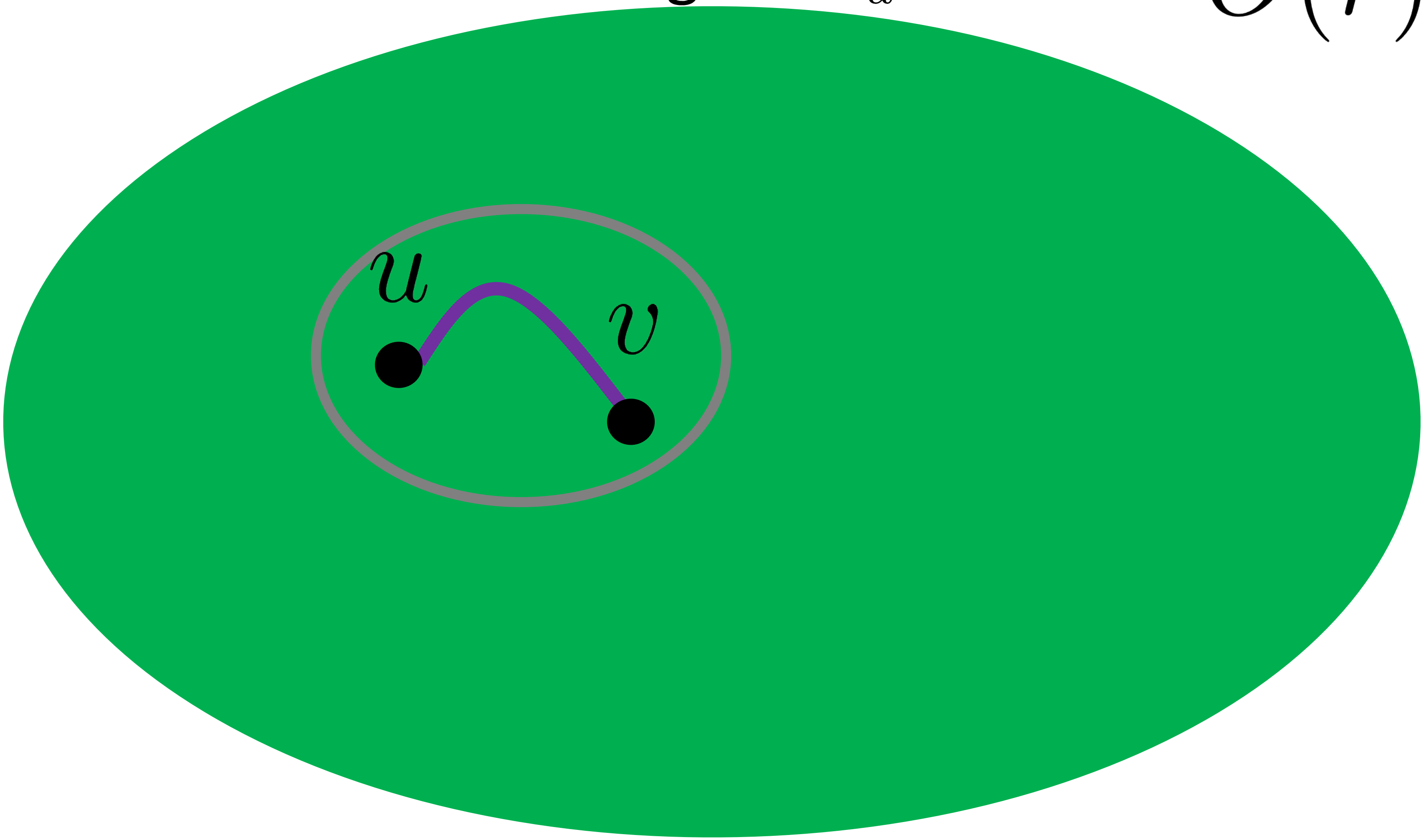
$\tilde{O}(r)$



Choose r:

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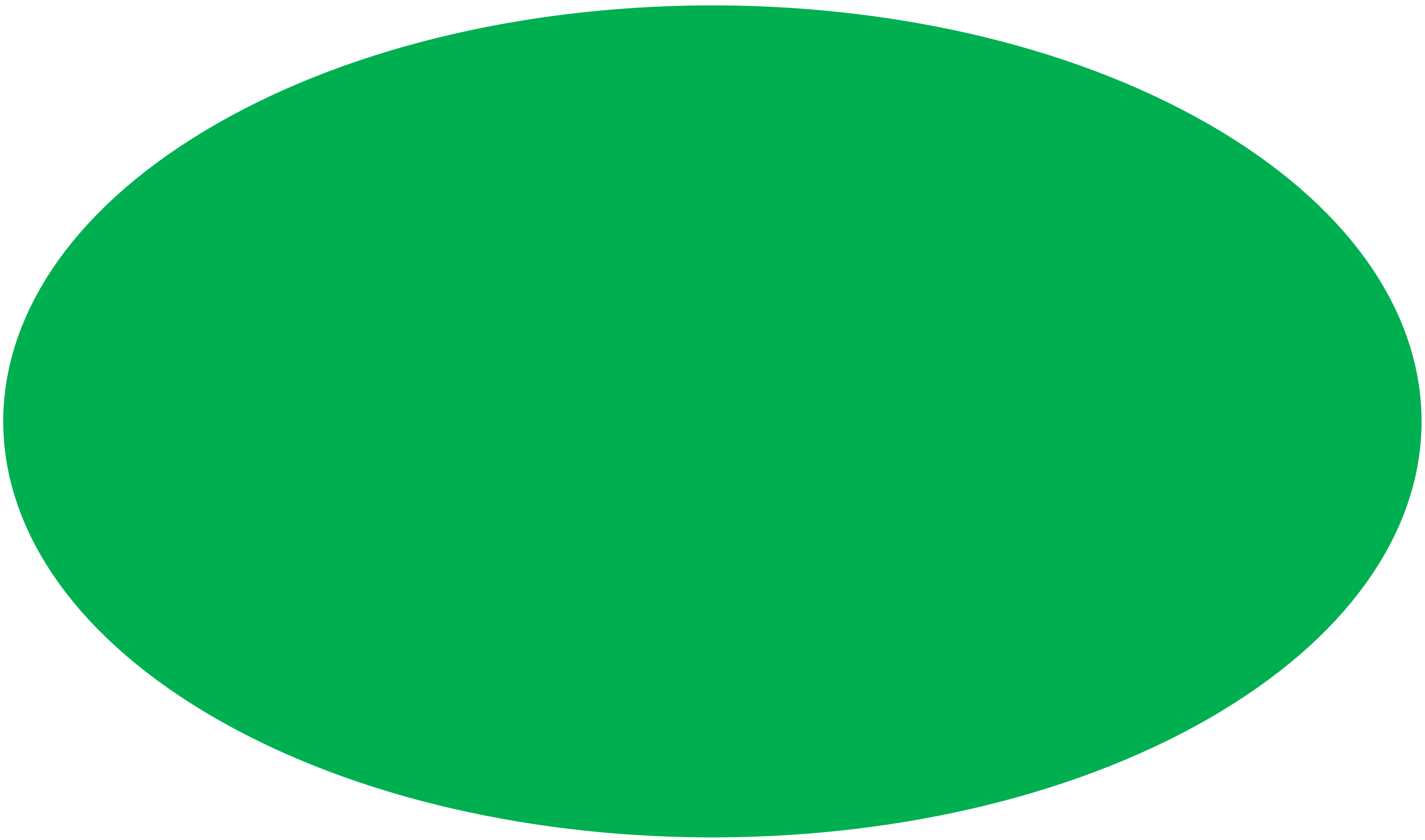
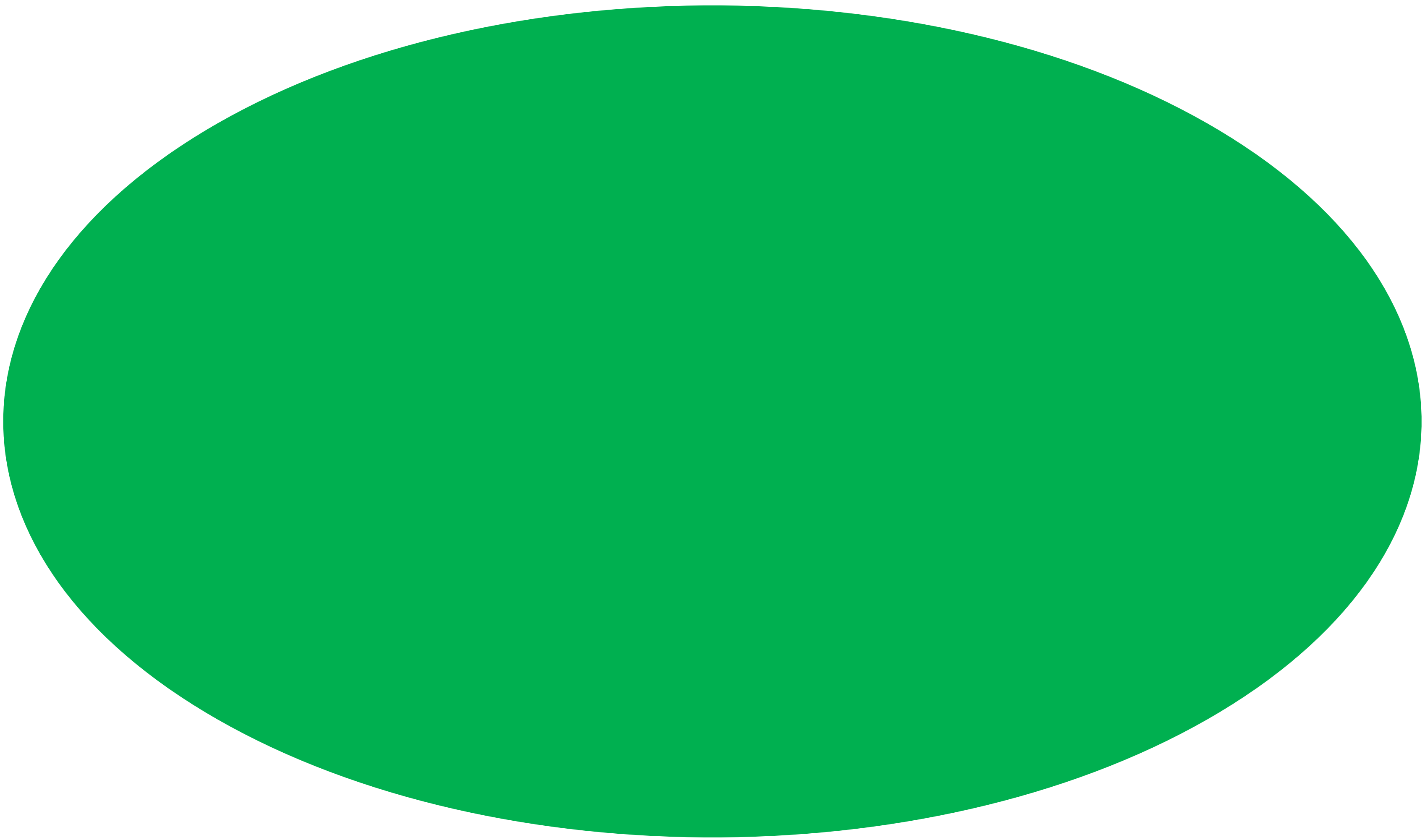
More carefully…



More carefully…

a vertex can belong to many seperators



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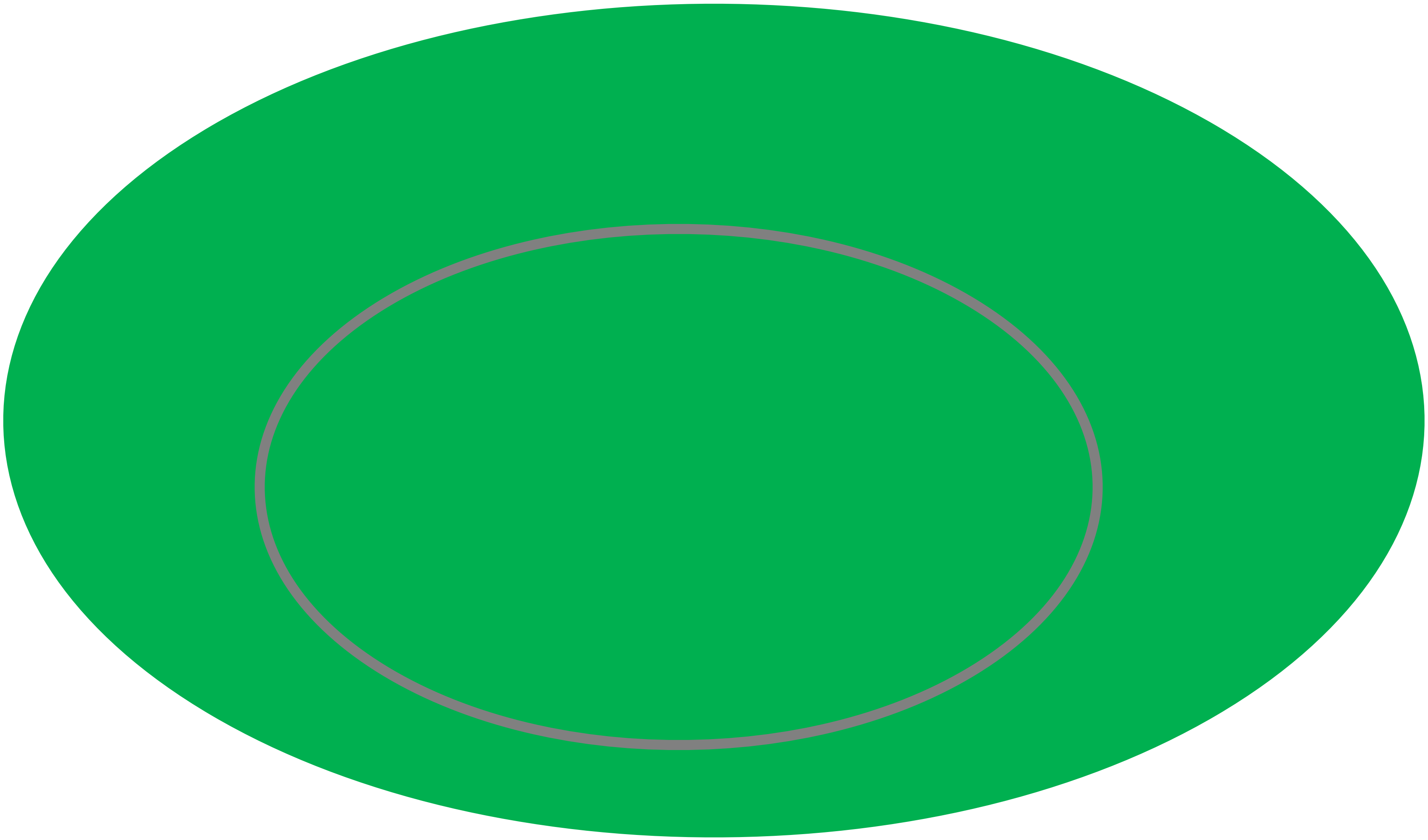

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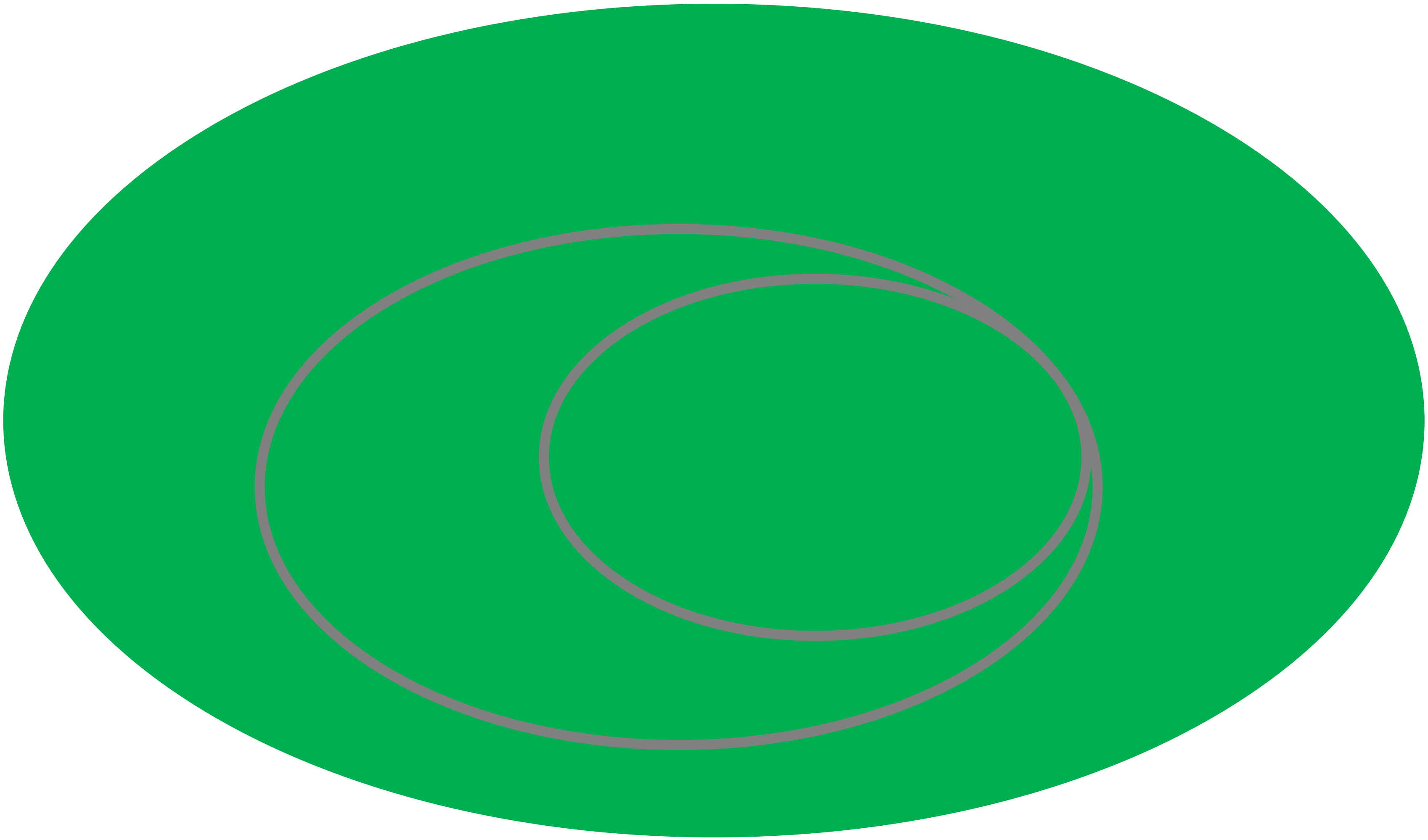

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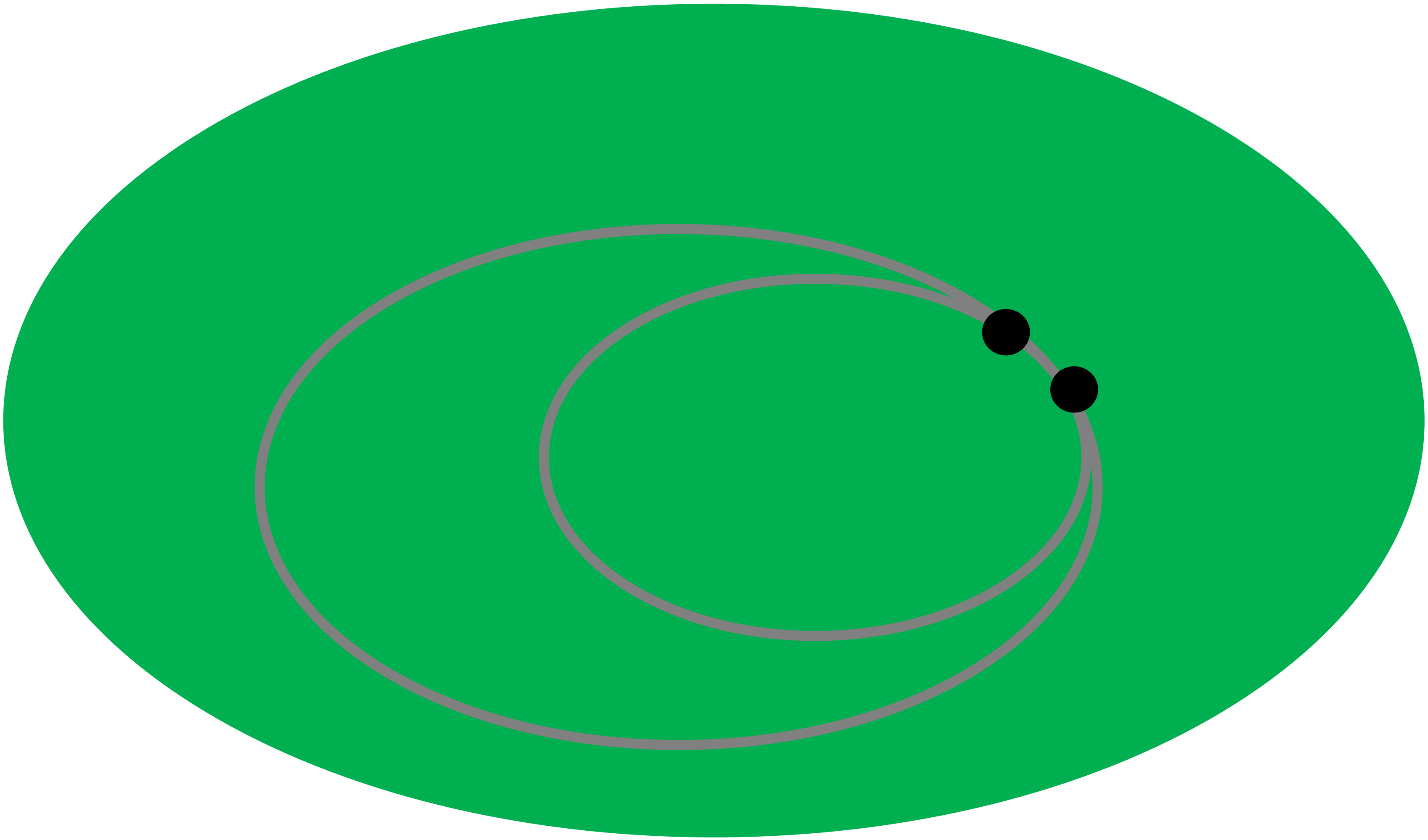

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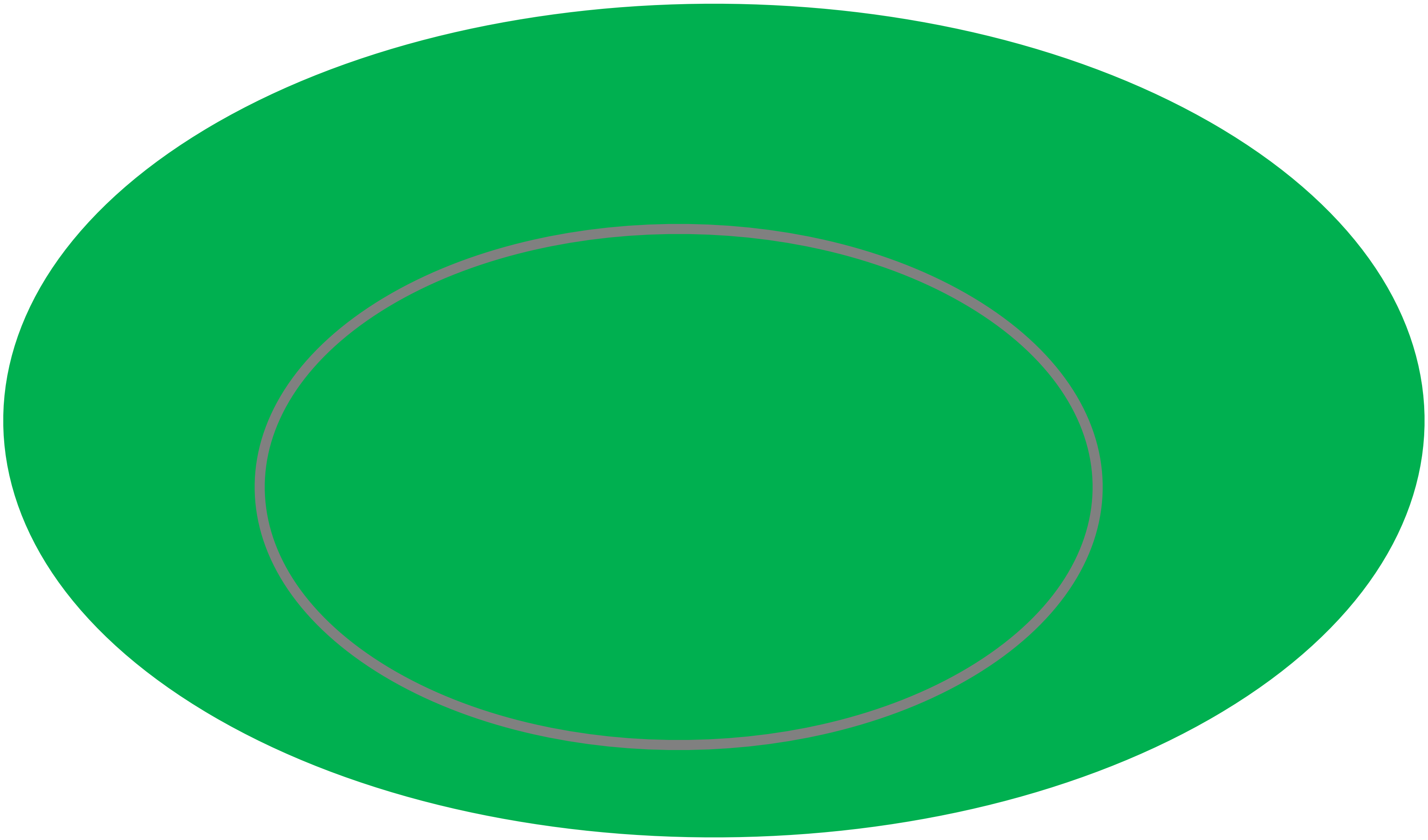

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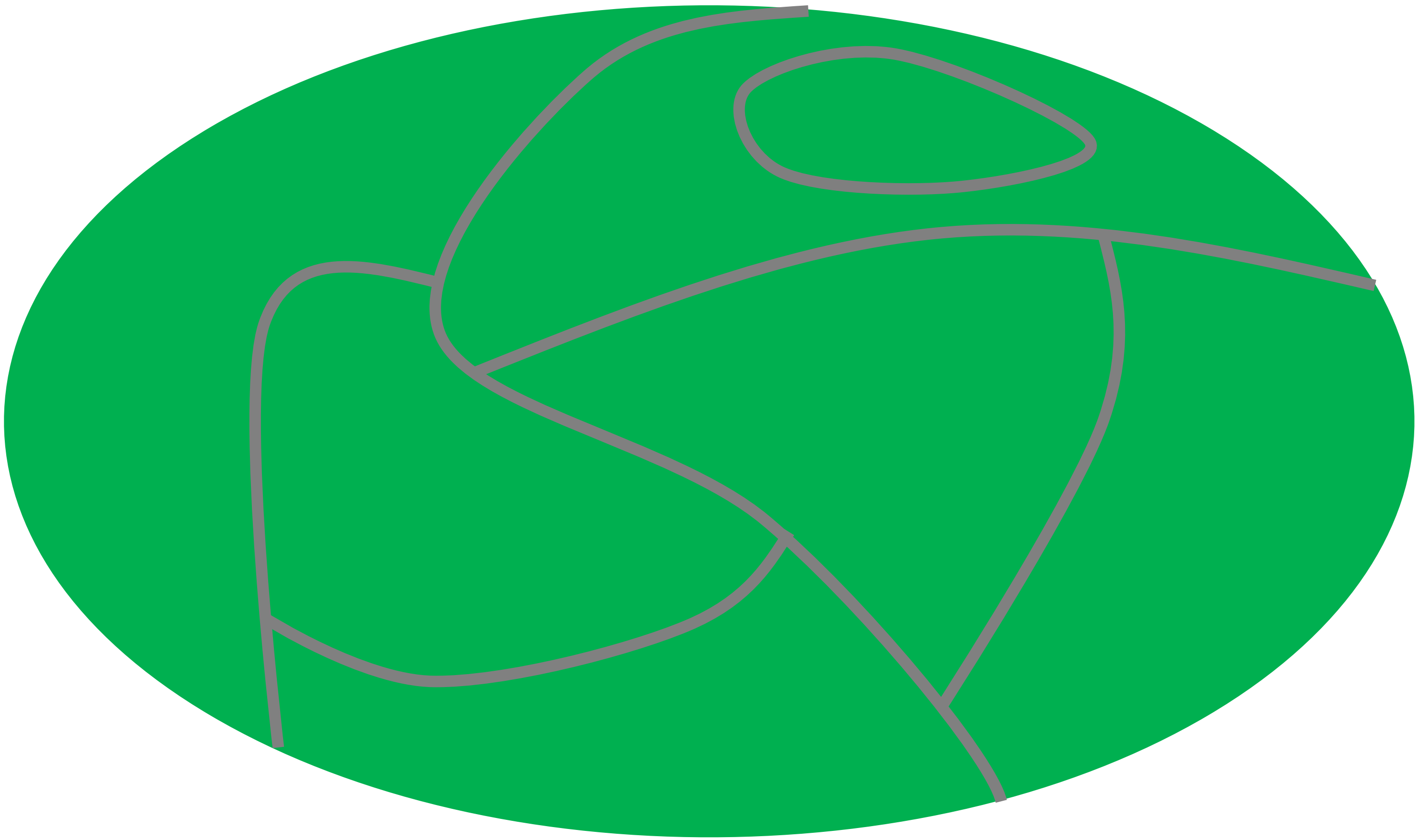

More carefully…

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- Lower bounds for fault tolerant setting?

A note about lower bounds

[Gavoille et al. SODA '01]

general argument:

k labels encode d data bits $\rightarrow \Omega(\frac{d}{k})$ label size

How about $\omega(\sqrt{n})$ lower bound for fault tolerant?

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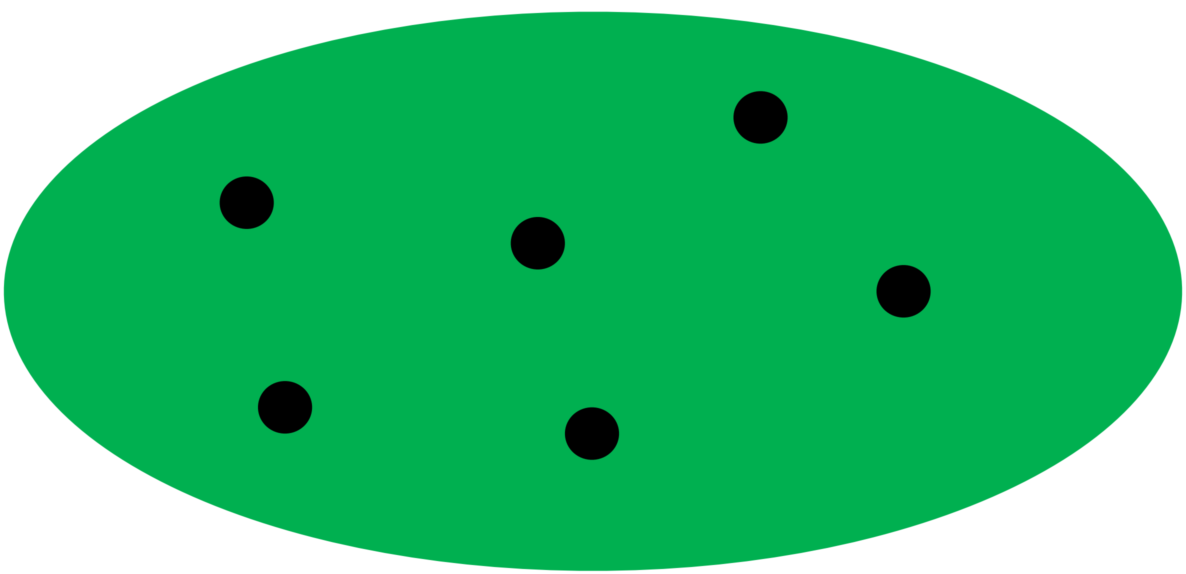

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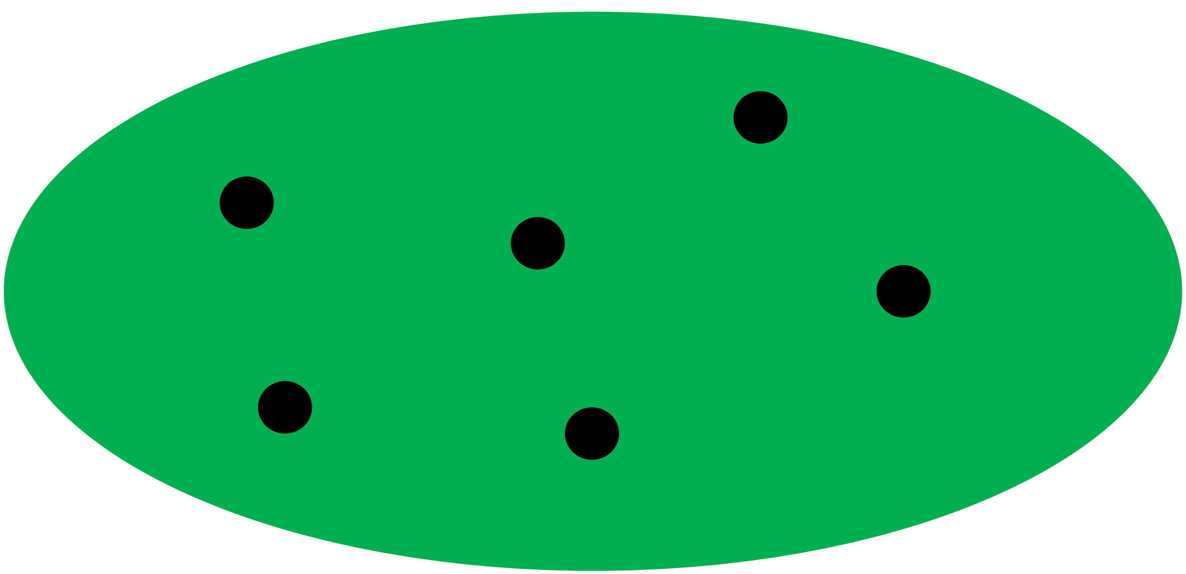
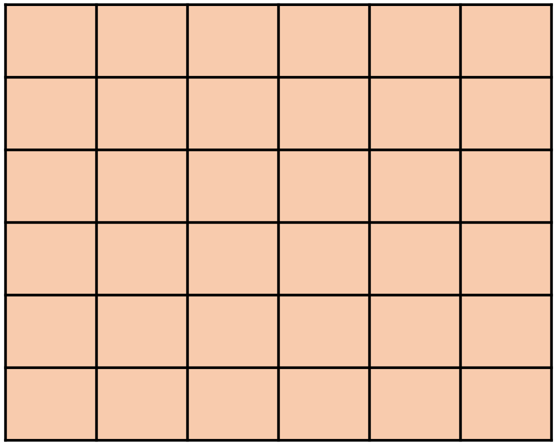


k by k distance matrix



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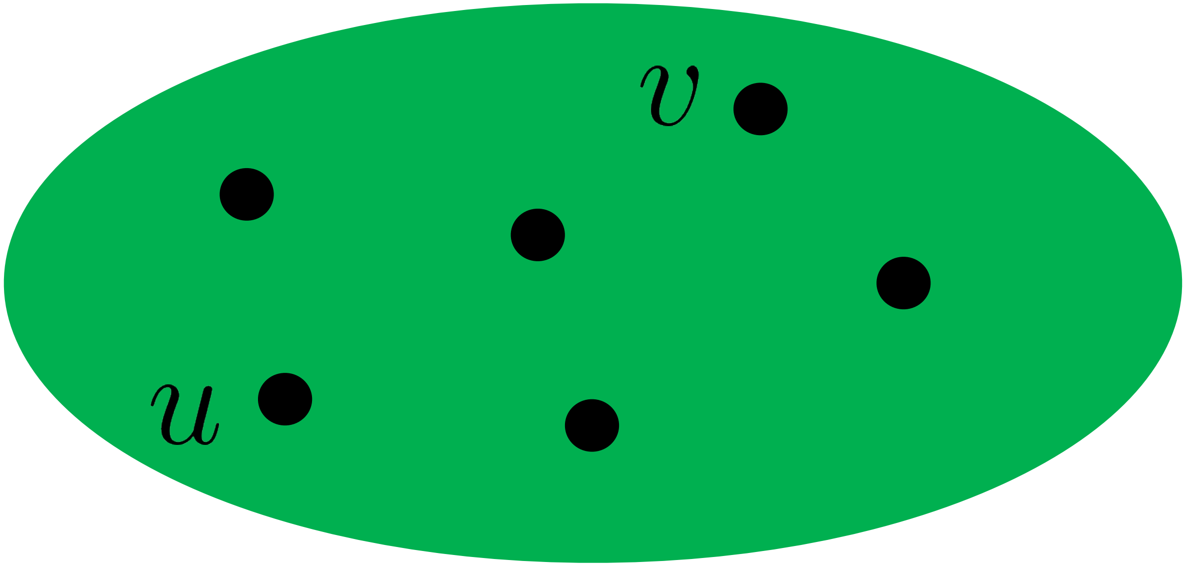

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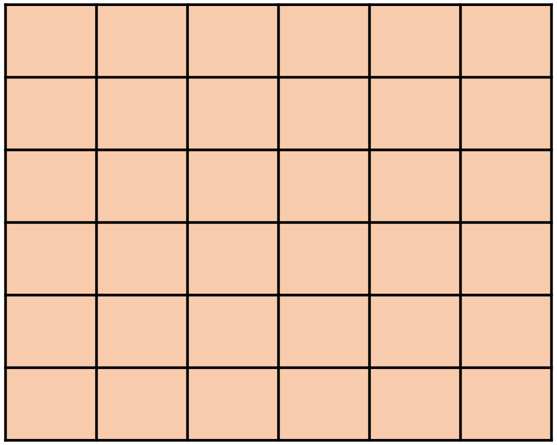

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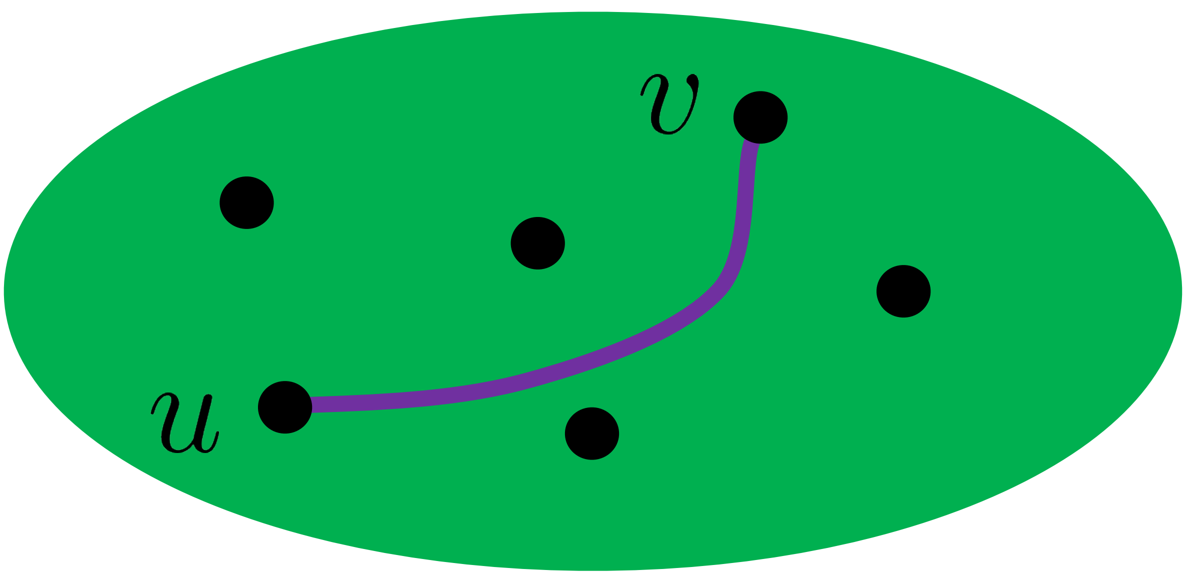

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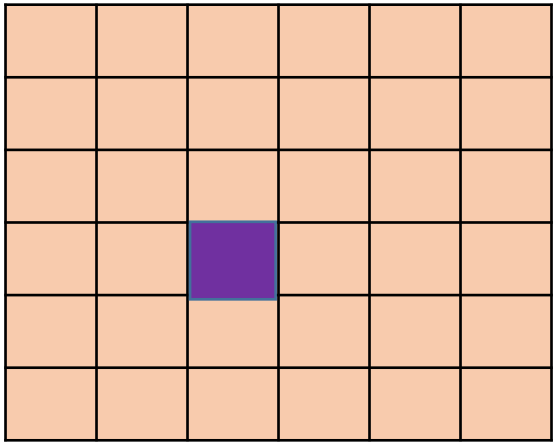

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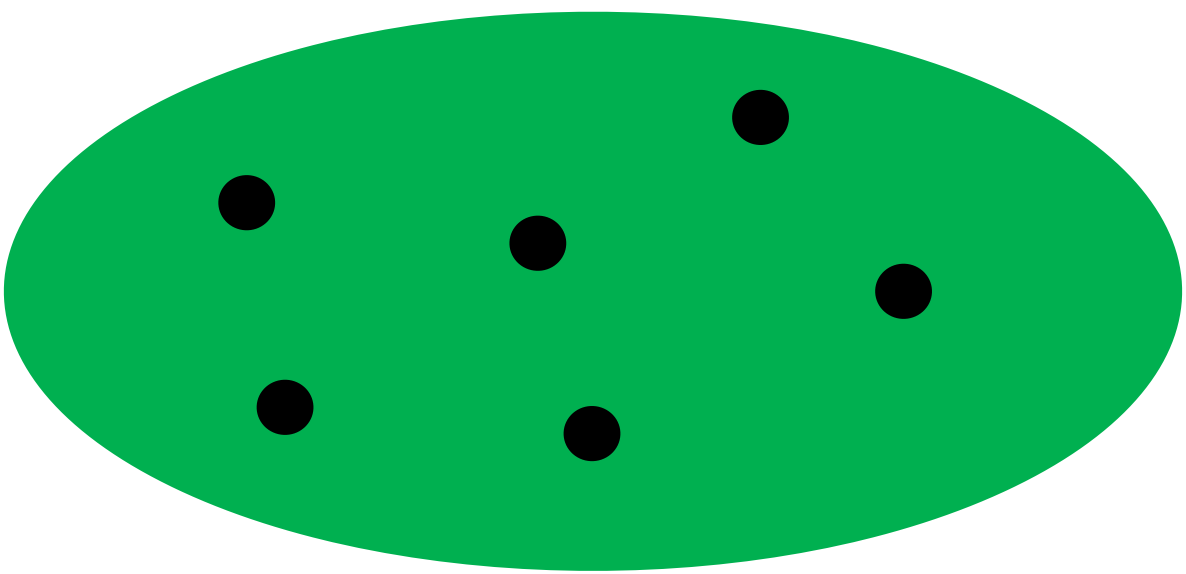
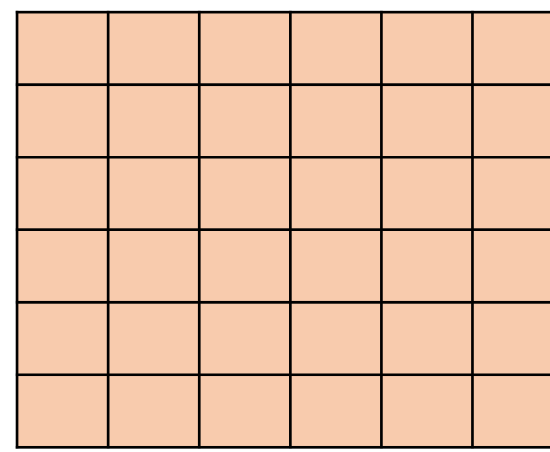


k by k distance matrix



A note about lower bounds

[Gavoille et al. SODA '01]

general argument:

k labels encode d data bits $\rightarrow \Omega(\frac{d}{k})$ label size

How about $\omega(\sqrt{n})$ lower bound for fault tolerant?

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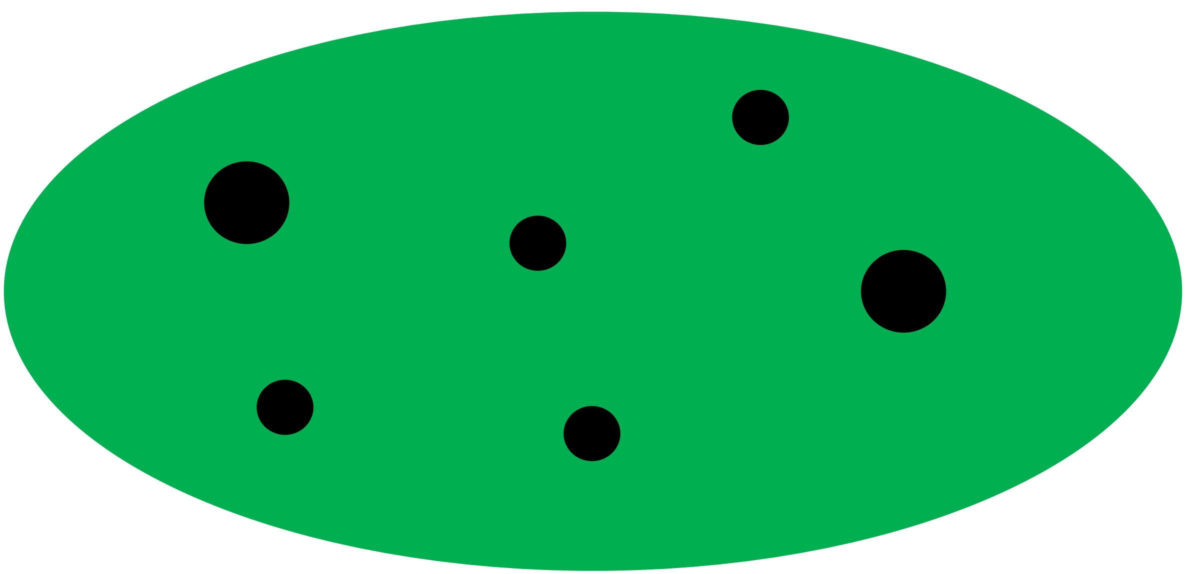

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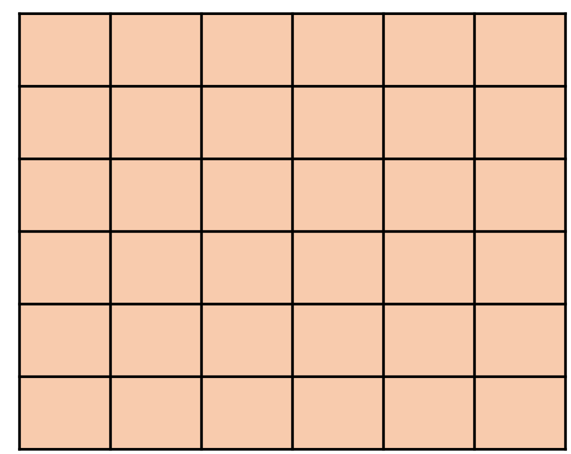

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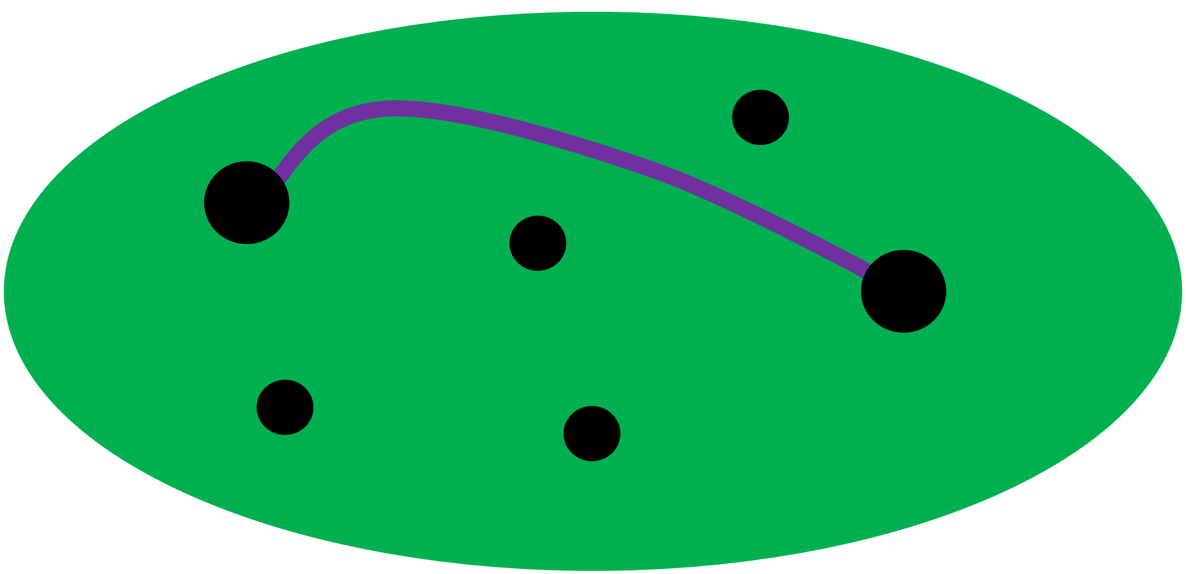

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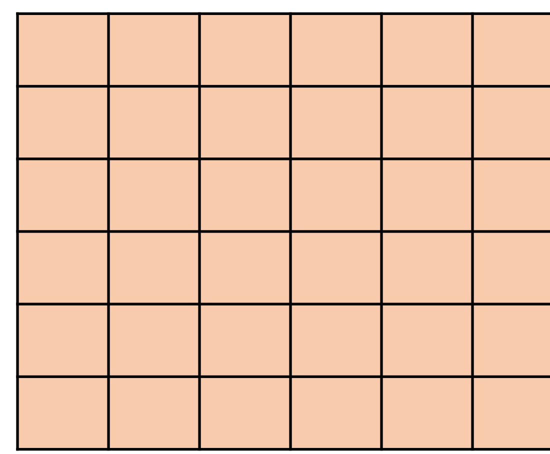

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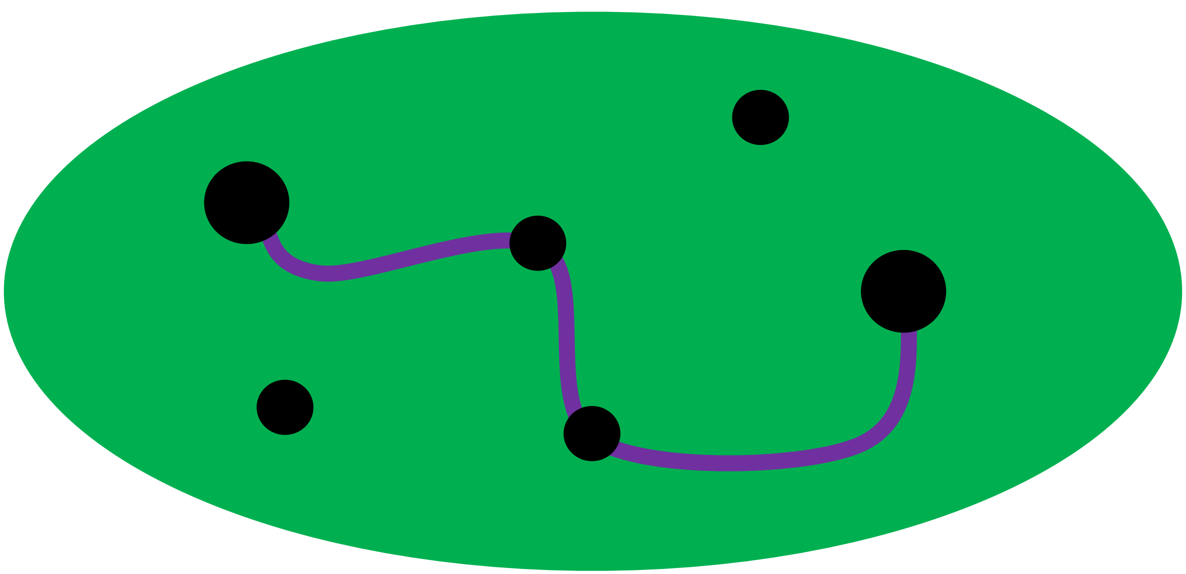

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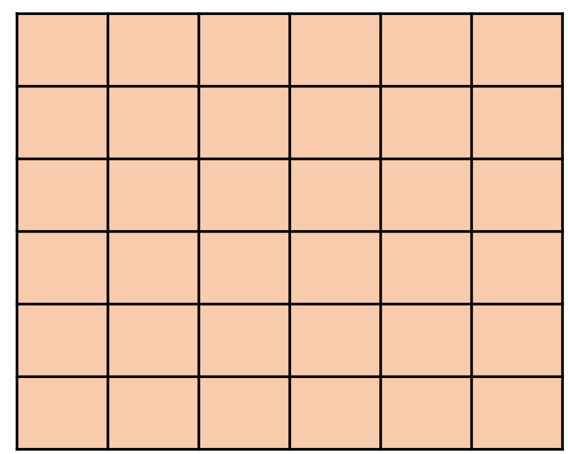
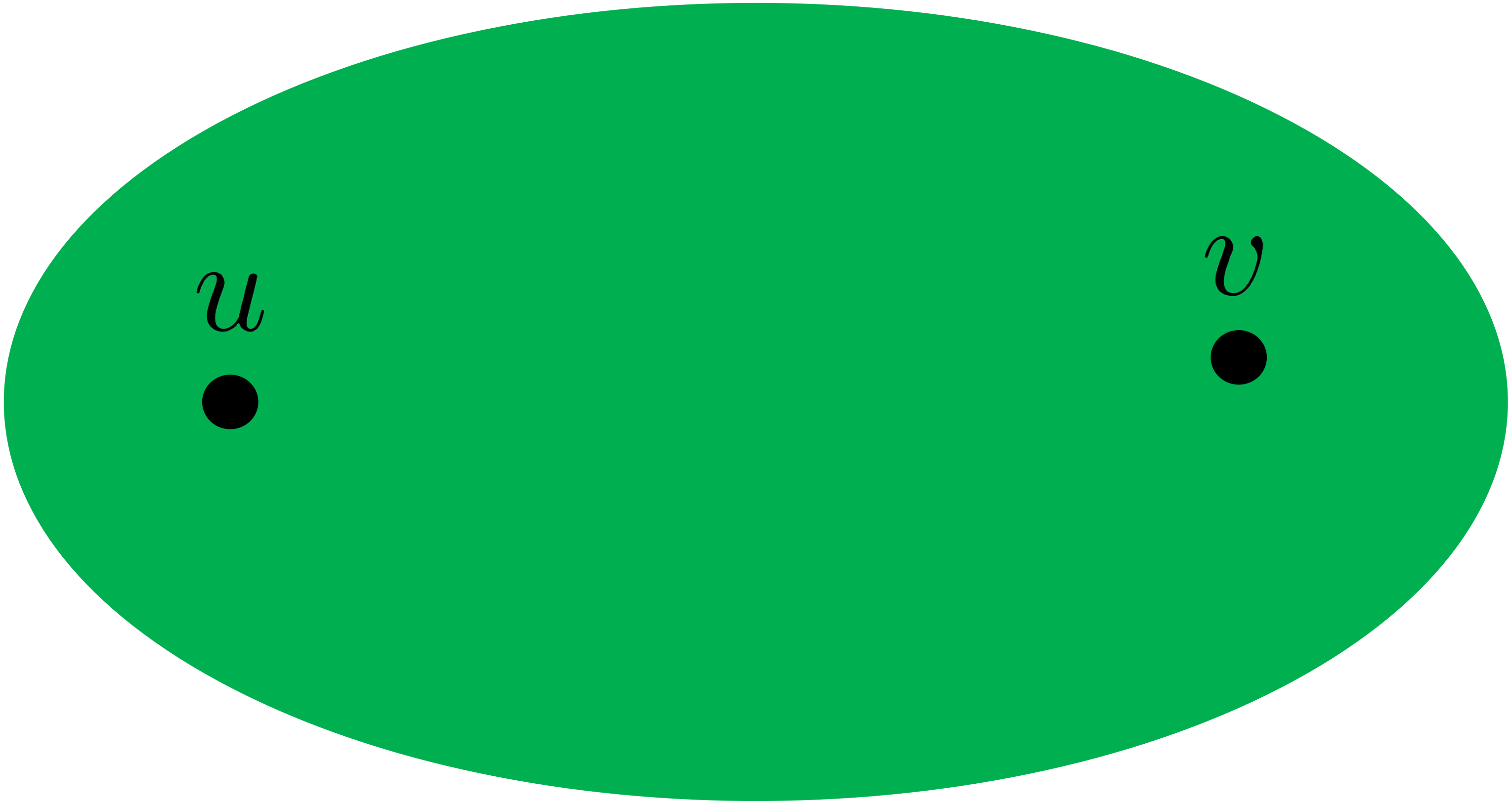


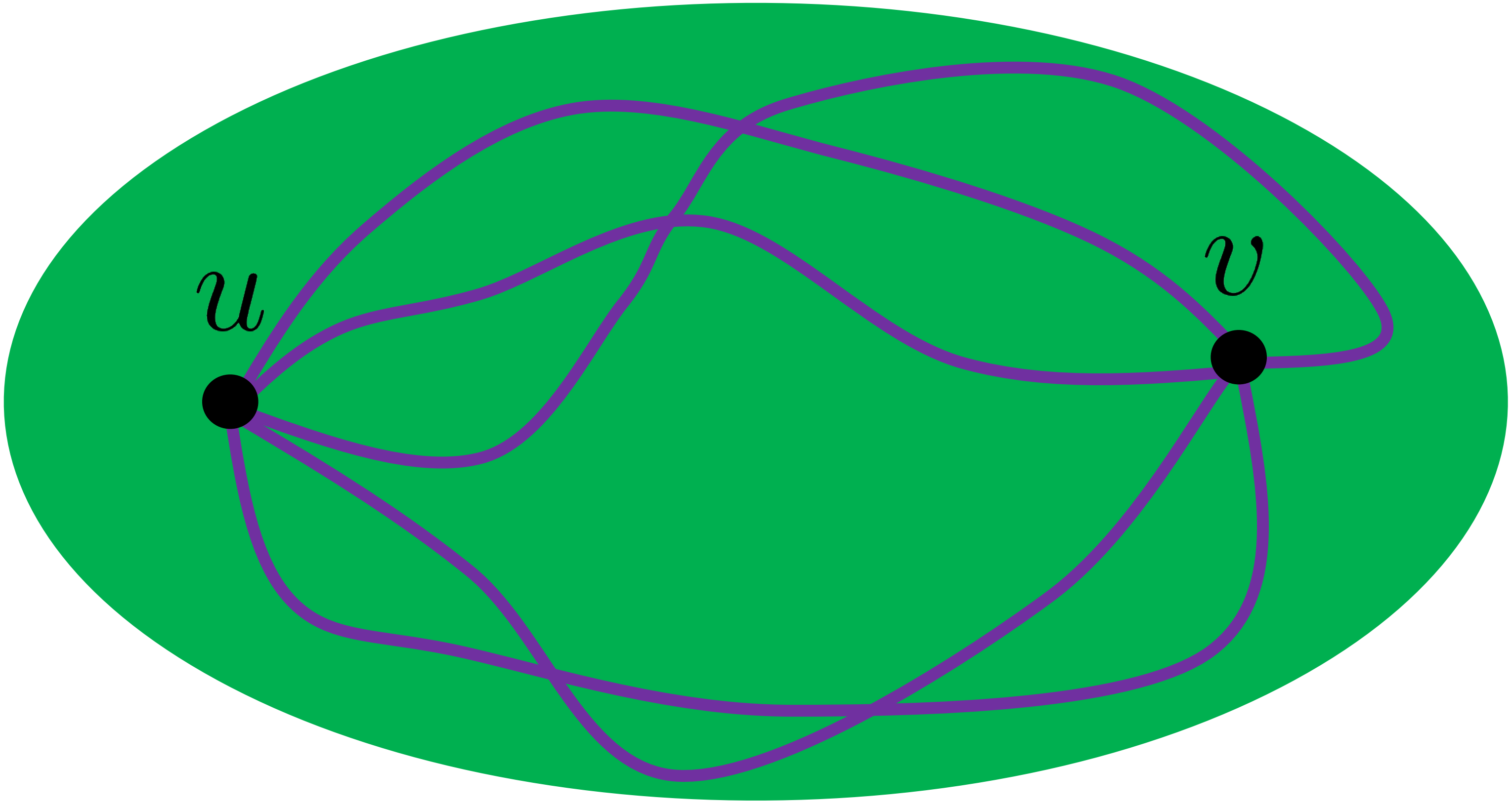
Counting shortest paths

Labels can be extended for counting the number of shortest paths with the same label size up to polylog factors

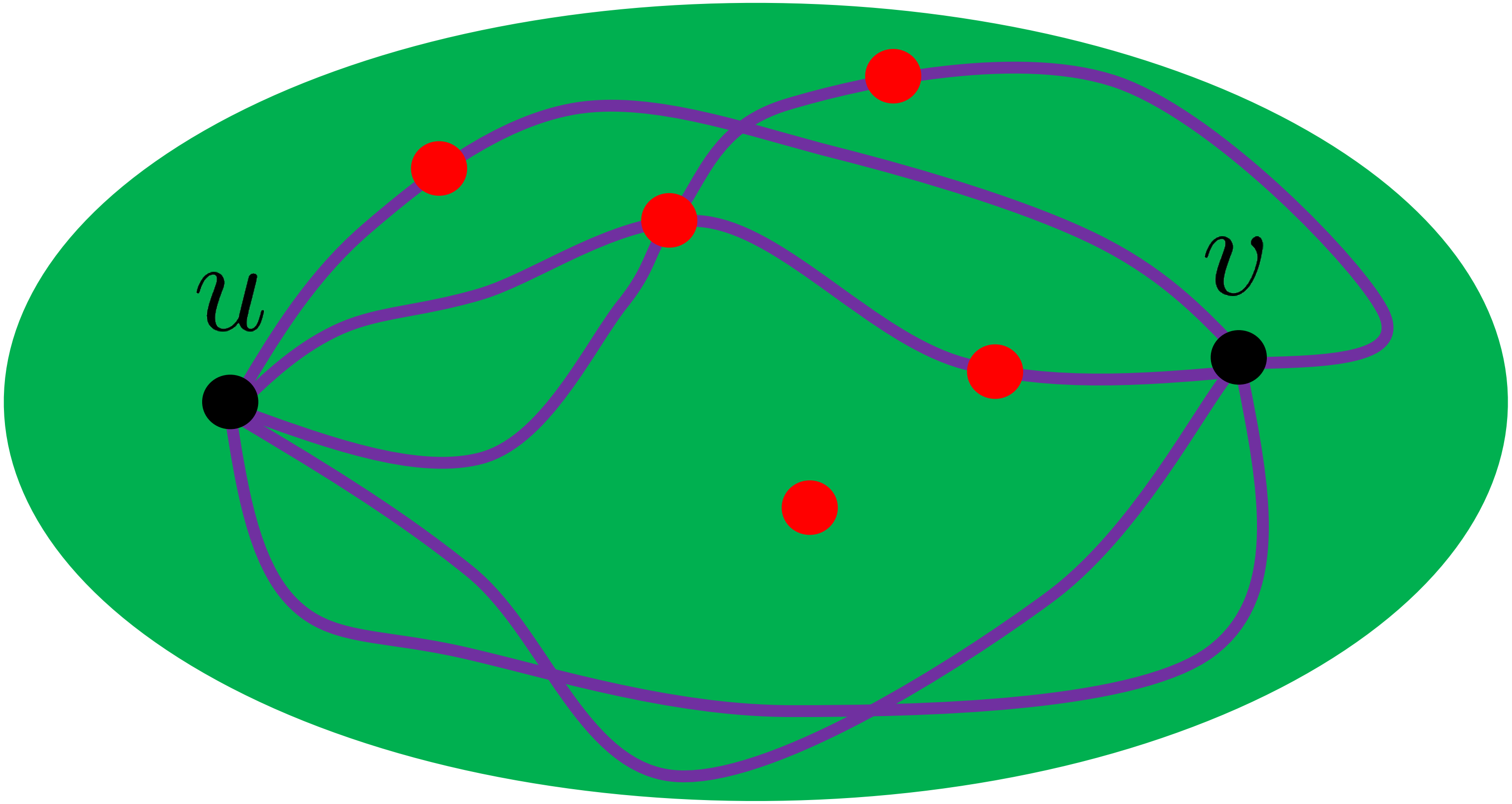
Counting shortest paths



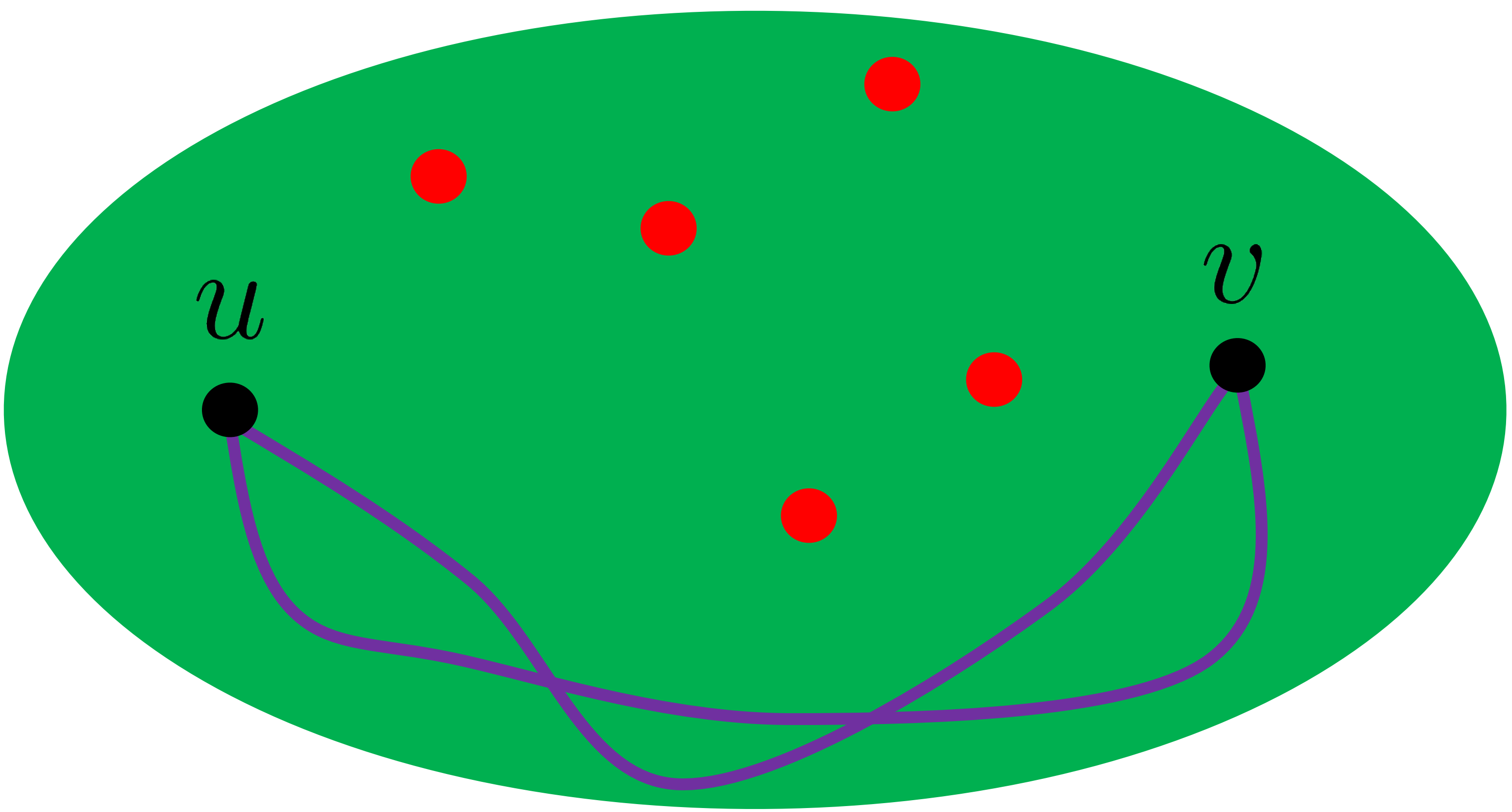
Counting shortest paths



Counting shortest paths

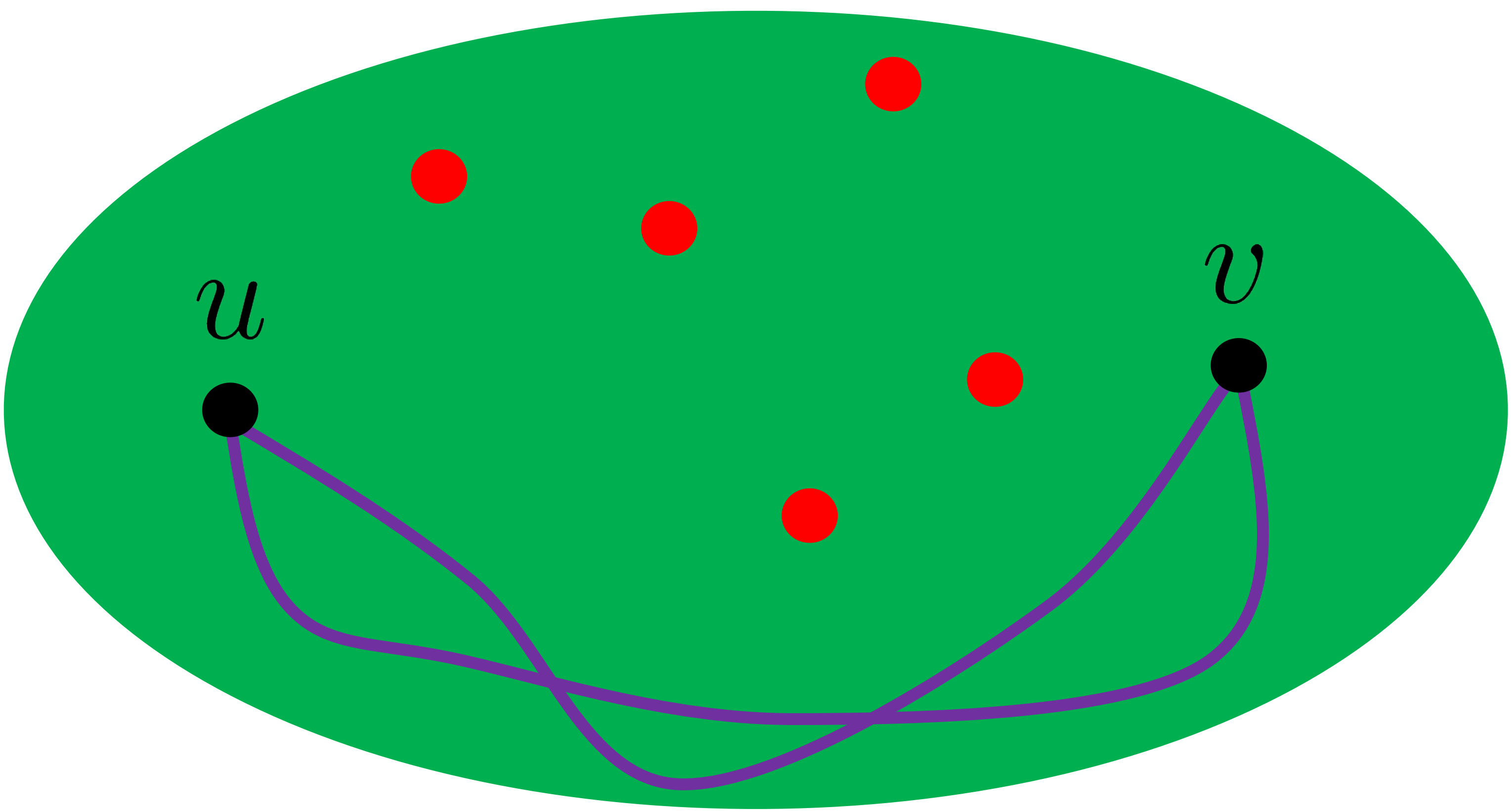


Counting shortest paths



Counting shortest paths

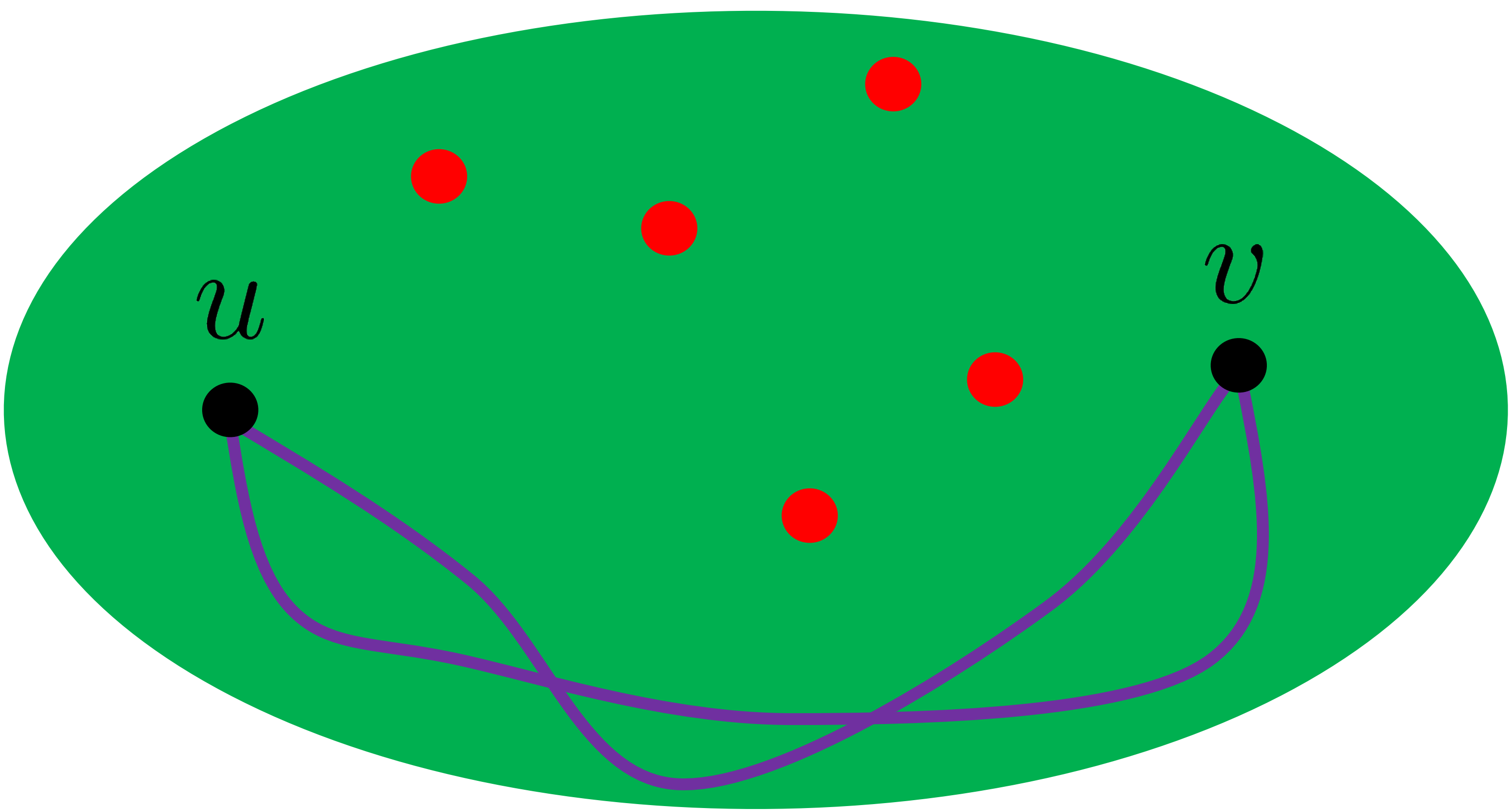
Classic $\tilde{O}(\sqrt{n})$ labels for counting can be used in fault tolerant setting



Counting shortest paths

Classic $\tilde{O}(\sqrt{n})$ labels for counting can be used in fault tolerant setting

Query time $\tilde{O}(\sqrt{n} \cdot k)$



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