Better Tradeoffs for Exact Distance Oracles in Planar Graphs

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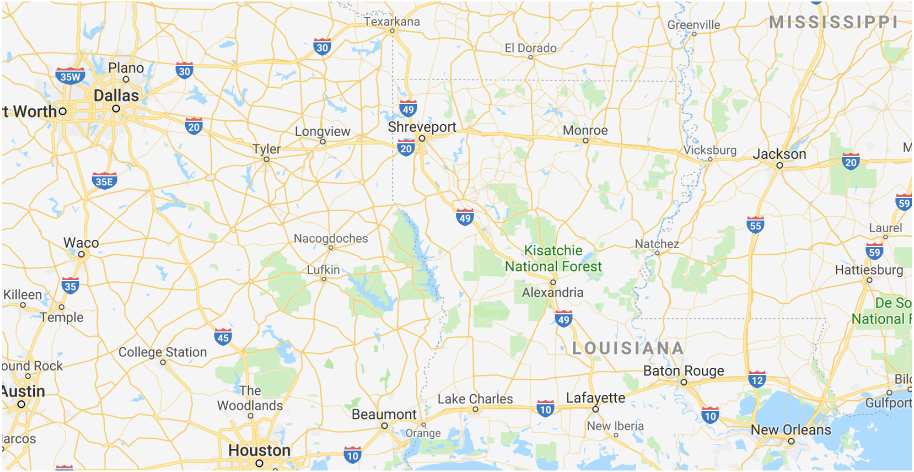
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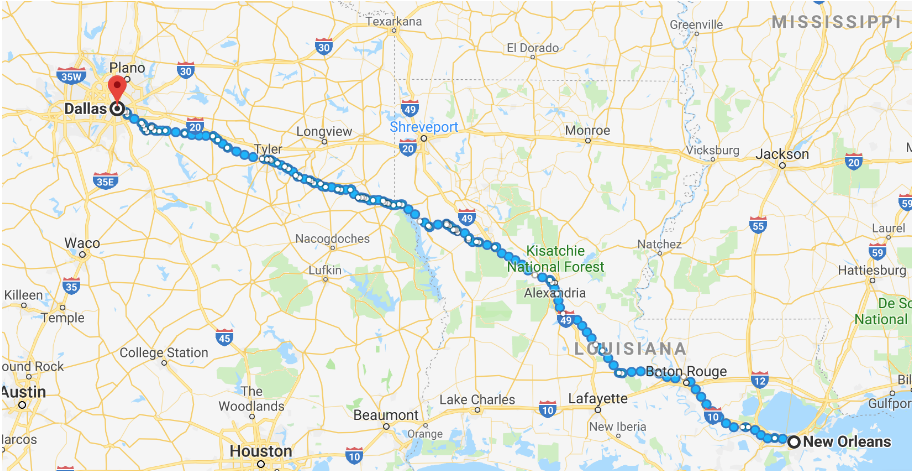

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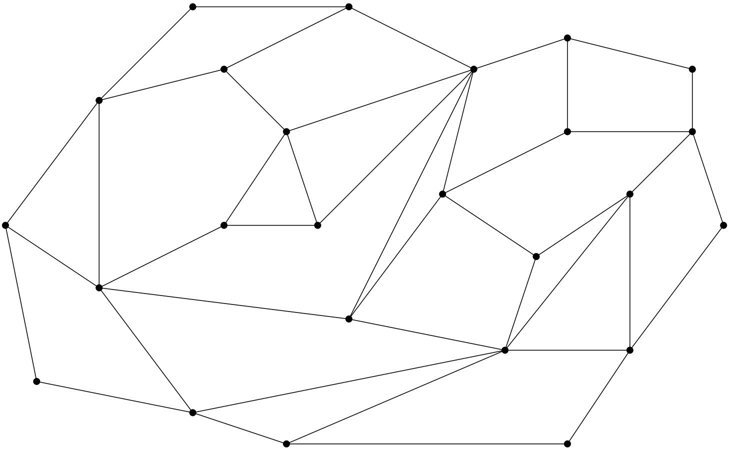
Goal



Preprocess an n-vertex planar graph $G = (V, E)$ with nonnegative arc lengths, so that given any $u, v \in V$ we can compute $d(u, v)$ efficiently.

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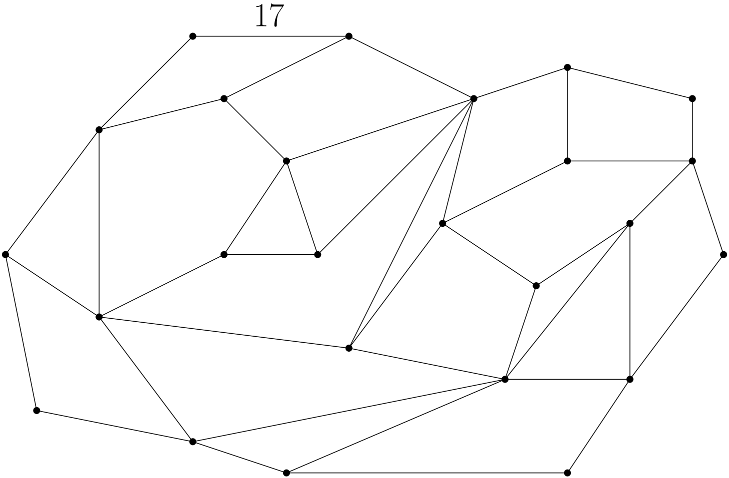

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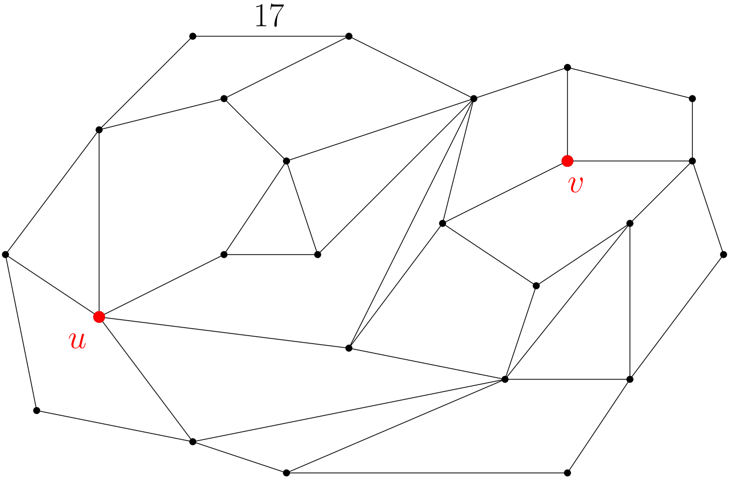

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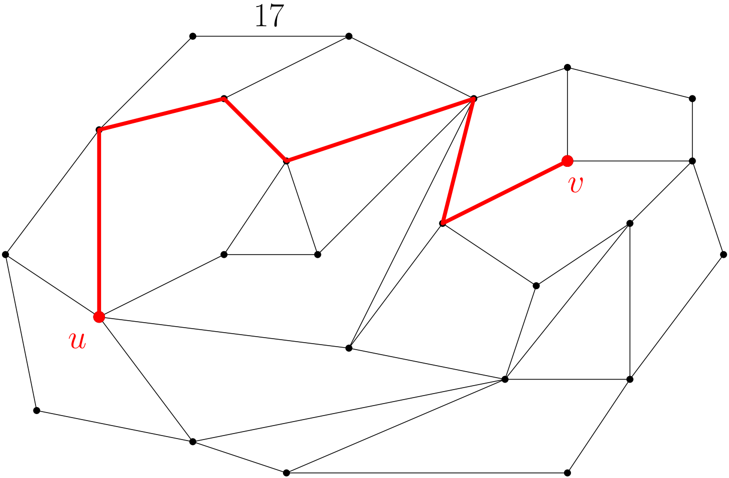
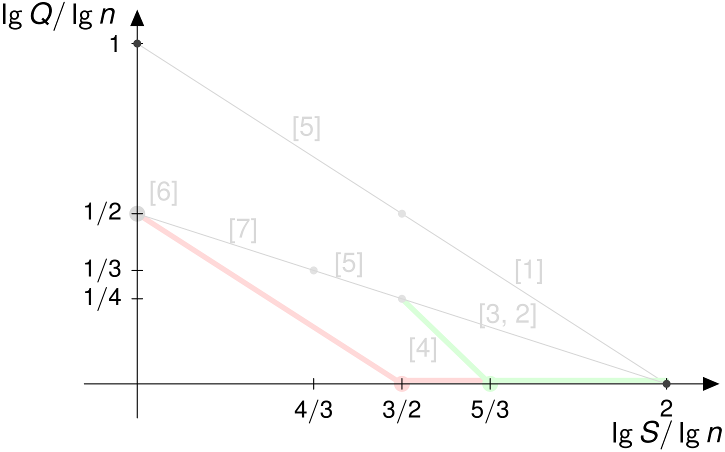


Preprocess an n-vertex planar graph $G = (V, E)$ with nonnegative arc lengths, so that given any $u, v \in V$ we can compute $d(u, v)$ efficiently.

Previous work

The trade-off between the query time Q and the size S of the structure:



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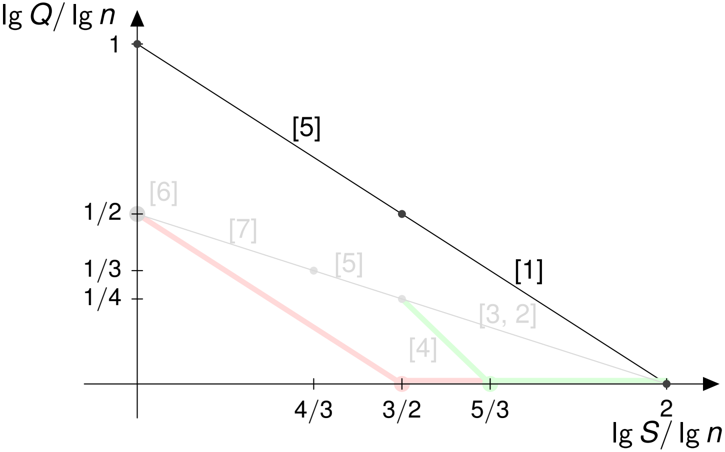

Djidjev [5] and Arikati et al. [1] achieved $Q = O(n^2/S^2)$.

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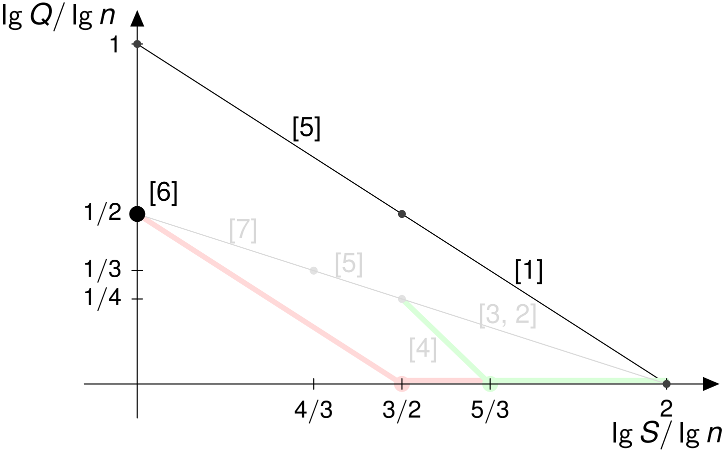

Fakcharoenphol and Rao [6] show that $S = \tilde{O}(n)$ and $Q = \tilde{O}(\sqrt{n})$ is possible.

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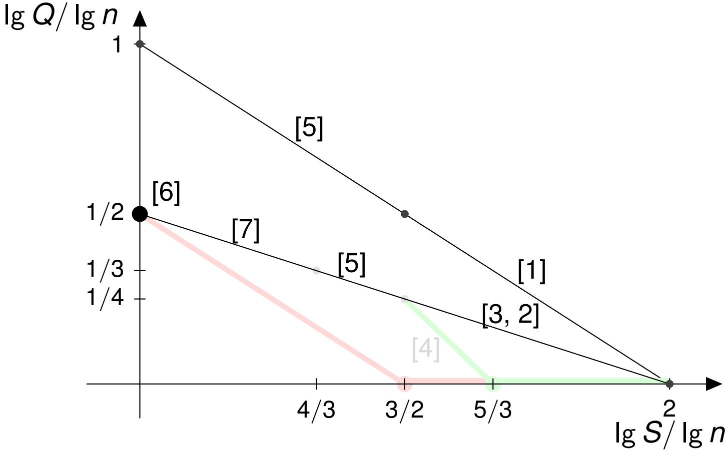

This has been extended to $Q = \tilde{O}(n/\sqrt{S})$ for essentially the whole range of S in a series of papers.

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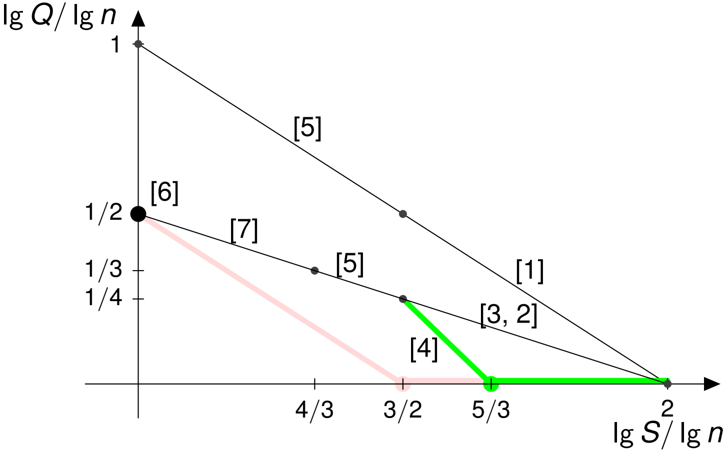

Last year, Cohen-Addad, Dahlgaard, and Wulff-Nilsen [4] showed that this is not optimal, and $S = O(n^{5/3})$ with $Q = O(\log n)$ is possible.

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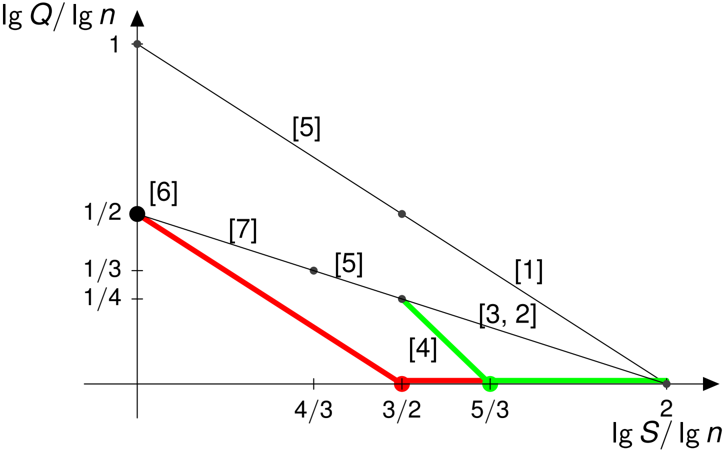

We improve this to $S = O(n^{1.5})$ and $Q = O(\log n)$.

Main result

For any $S \in [n, n^2]$, we construct an oracle of size S that answers an exact distance query in $Q = \tilde{O}(\max\{1, n^{1.5}/S\})$ time.

At the heart of the above construction is a structure with $S = O(n^{1.5})$ and $Q = O(\log n)$ that, similarly to the result of Cohen-Addad et al., uses the Voronoi diagram technique introduced by Cabello.

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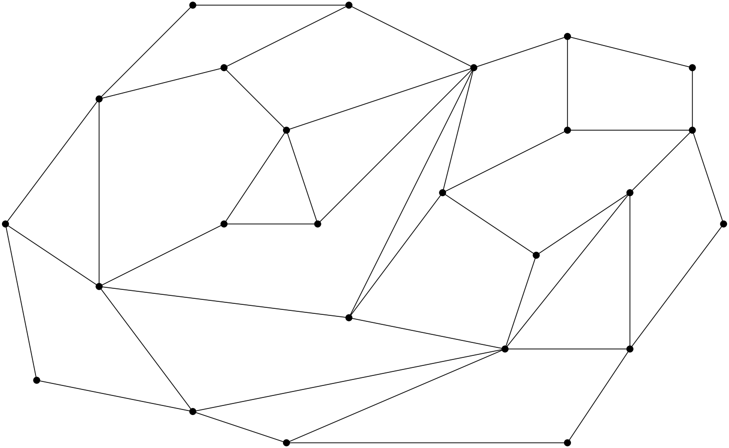
Basic recursion

Miller

There always exists a Jordan curve separator of size $O(\sqrt{n})$ such that there are at most $\frac{2}{3}n$ nodes on its inside/outside.



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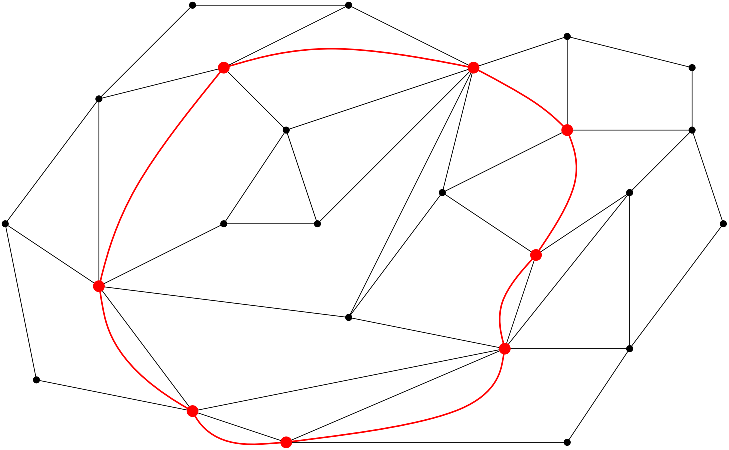

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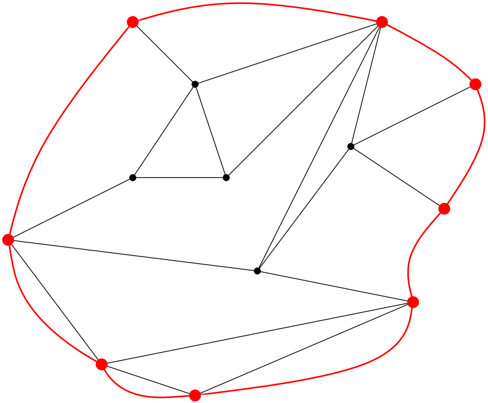

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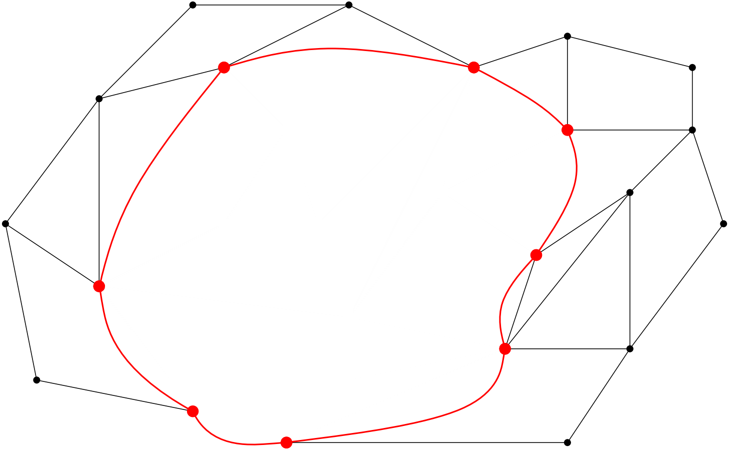

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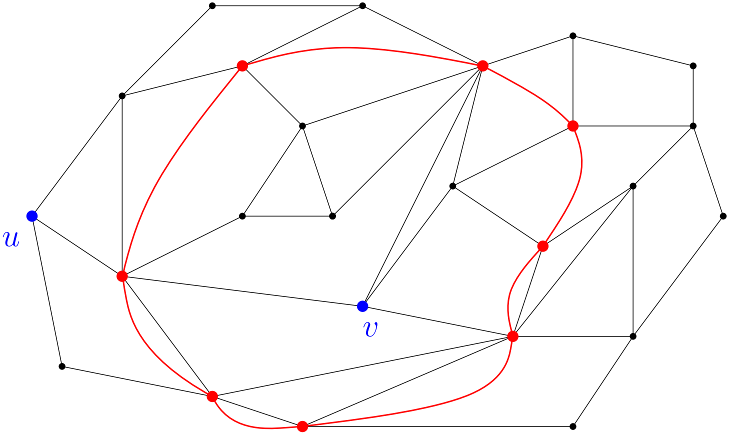

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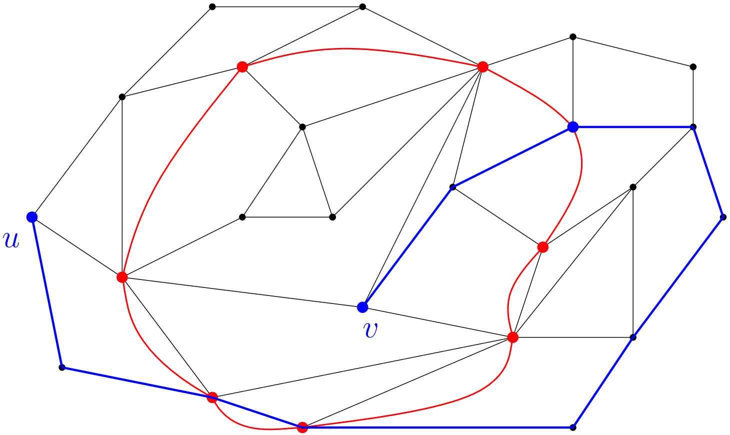

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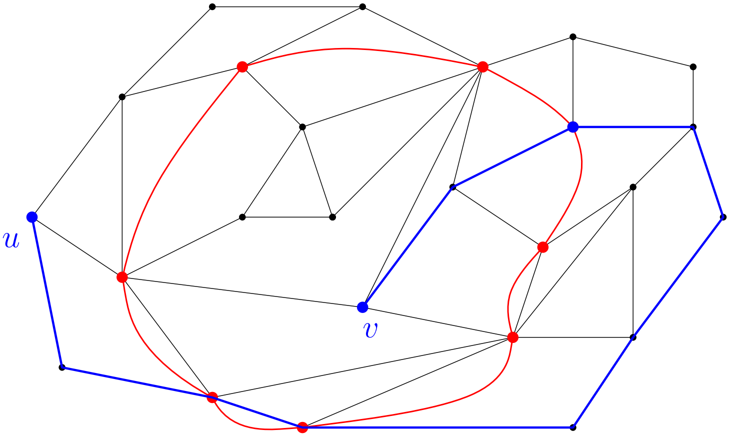
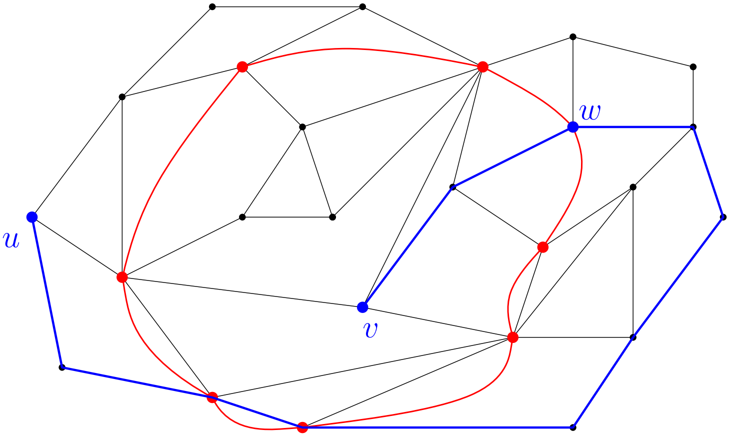


Shortest path can cross the separator multiple times!

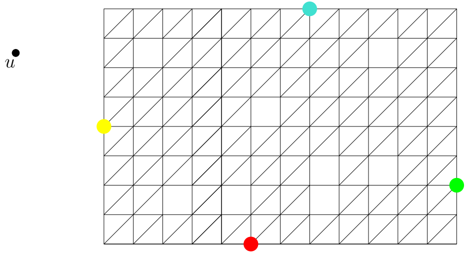
Basic recursion



Find $w \in Sep$ minimising $d_G(u, w) + d_{in}(w, v)$.

Single step

Consider the inside of graph and a fixed node u.



1. For technical reasons, triangulate.
2. For every $w \in Sep$ define $\omega(w) = d_G(u, w)$.
3. Construct the Voronoi diagram.
4. ... finding w reduces to point location!

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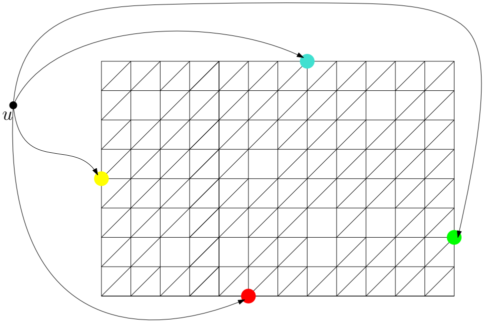

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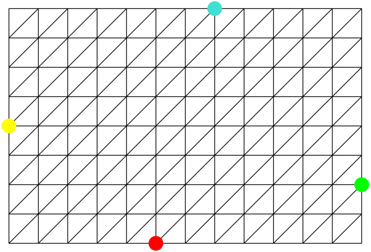

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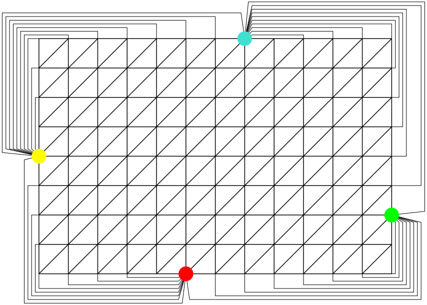

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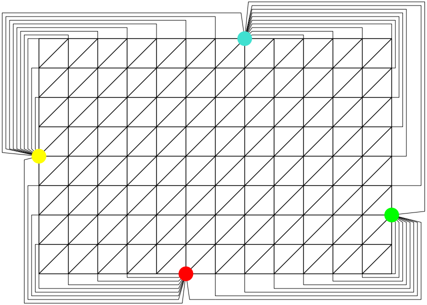

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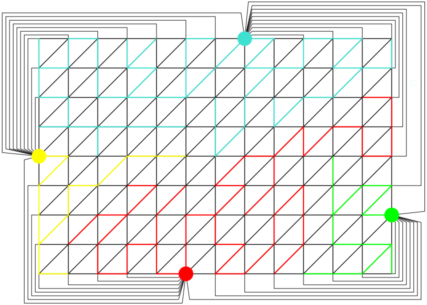

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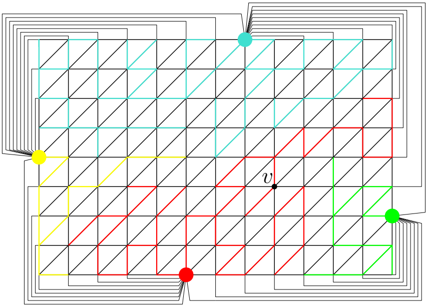

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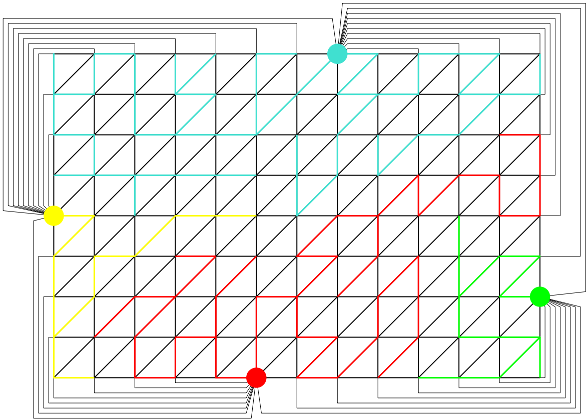
Voronoi diagram



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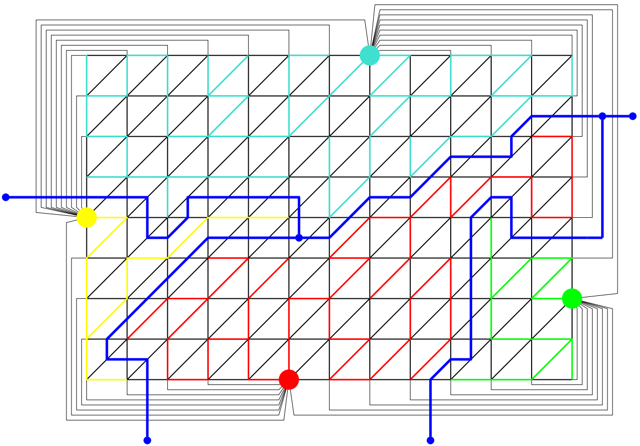
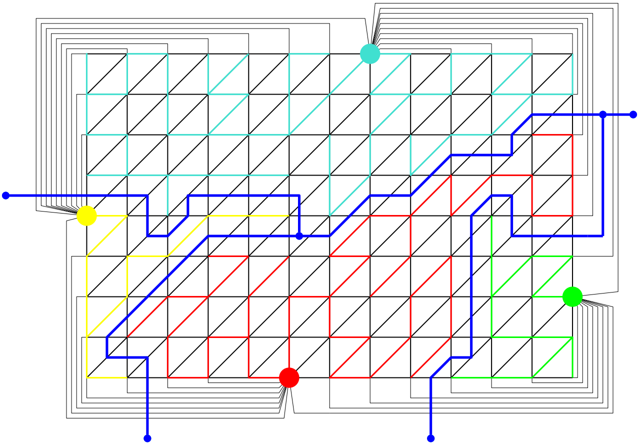


Look at the dual and create many copies of the node corresponding to the external face.

Voronoi diagram



Because all sites are adjacent to the external face, the diagram can be described by a tree on $O(|Sep|)$ nodes.

Point location

Any tree on k nodes contains a centroid node u such that every component of $T \setminus \{u\}$ is of size $\frac{2}{3}k$.

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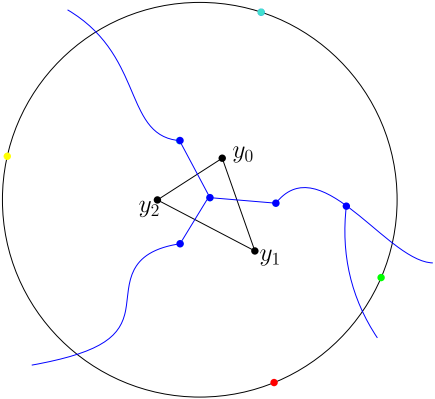

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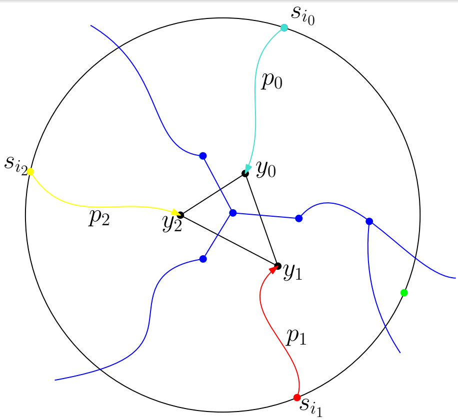

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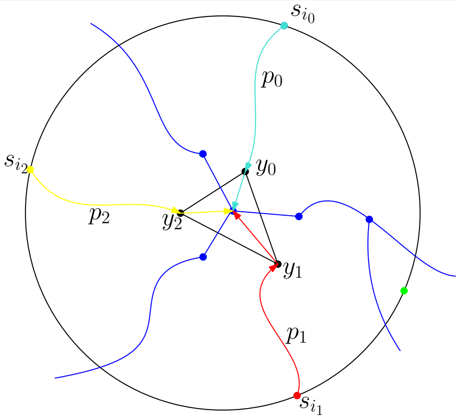

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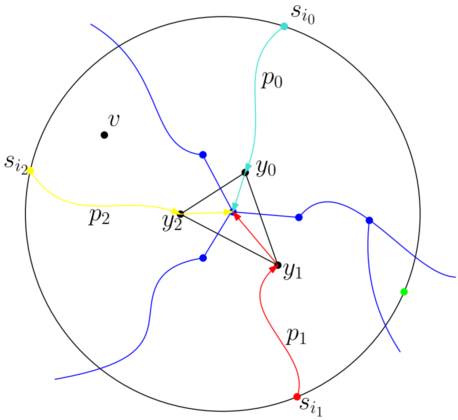
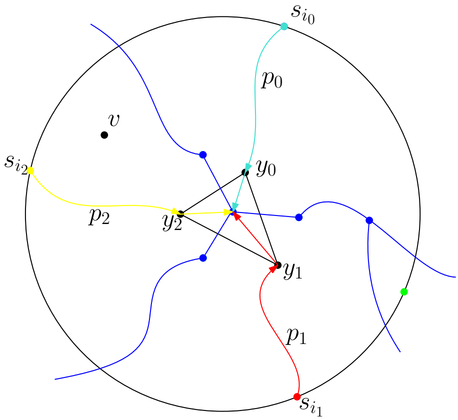


- check that v is on the left of p_2,
- check that v is on the right of p_0,

Point location



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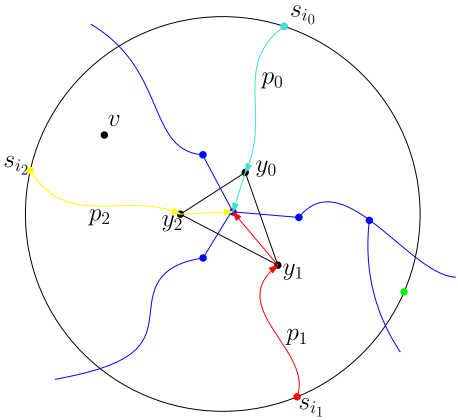

p_j plus the extra edge is a path in the shortest paths tree rooted at s_{i_j} that doesn't depend on ω.

Point location

- By storing preorder numbers for the shortest paths tree rooted at s_{i_j} we can check if v is on the left/right of p_{i_j} in constant time.
- This allows us to detect in constant time the relevant smaller component of the tree representing the Voronoi diagram that needs to be recursively searched for the cell of v (if it is not the cell of any s_{i_j}).

Time: $O(\log n)$

Space:
- $|Sep| \cdot O(n)$ to store the shortest path tree for every site,
- $n \cdot O(\sqrt{n})$ to store the Voronoi diagram and its centroid decomposition for every u,
- $O(n^{1.5})$ in total.

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Basic recursion, again

- Recursively build an exact distance oracle for the inside.
- Recursively build an exact distance oracle for the outside.
- Build a structure that that can be used to find the shortest path from u to v that visits at least one node of the separator.

The overall space is roughly $S(n) = O(n^{1.5}) + 2S(n/2)$, so $O(n^{1.5})$ overall. But the query would take $O(\log^2 n)$...

Can we decrease the number of subproblems where we use the point location structure?

It is enough to query only the structure corresponding to the topmost subproblem where u and v lie on different sides of the separator if we choose the separators more carefully. The query time becomes $O(\log n)$.

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Tradeoff

- Use r-divisions to decrease the number of stored Voronoi diagrams by a factor of $\sqrt{r}$. Then, we need to guess the boundary node u' in the region of u, there are $\sqrt{r}$ possibilities.

Tradeoff

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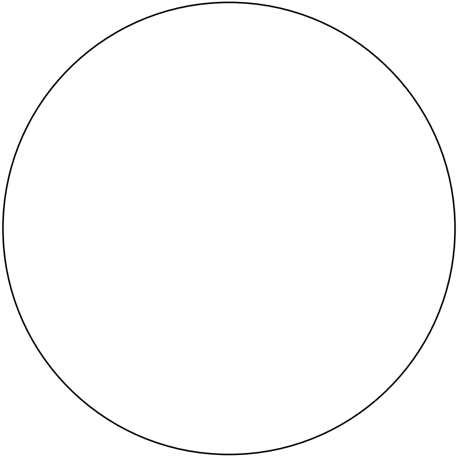

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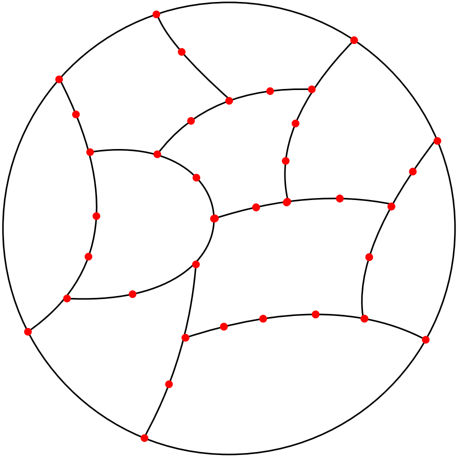

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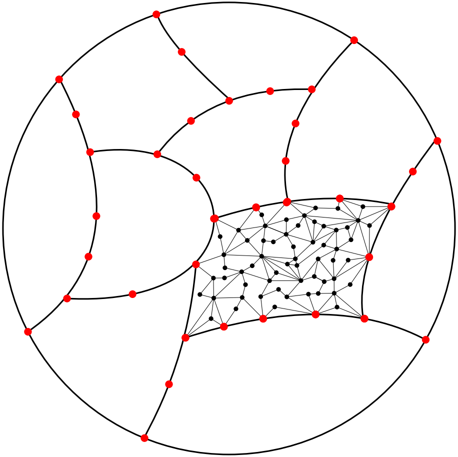

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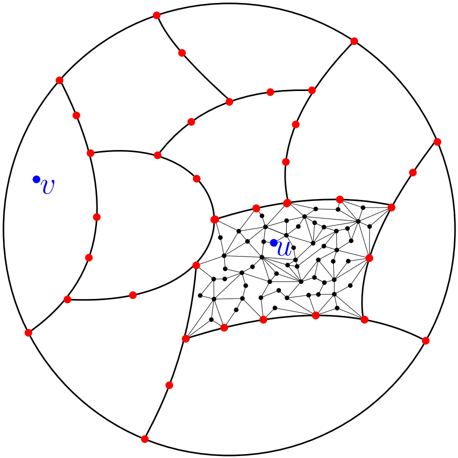

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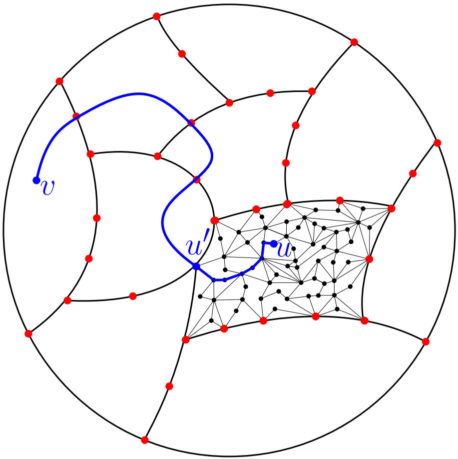

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- Replace the explicitly stored shortest paths trees with the MSSP structure of Klein, except that we need to slightly extend the interface of the link-cut trees used to maintain the current tree.

After some calculations...

Time: $O(\sqrt{r}\log^2 n)$

Space: $O(n^{1.5}/\sqrt{r} + n\log n \log(n/r))$

Time can be decreased to $O(\sqrt{r}\log n \log r)$ with an additional trick.

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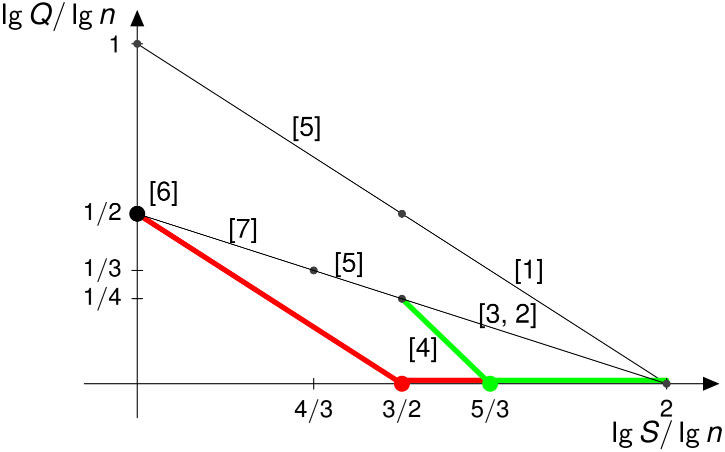

Questions?