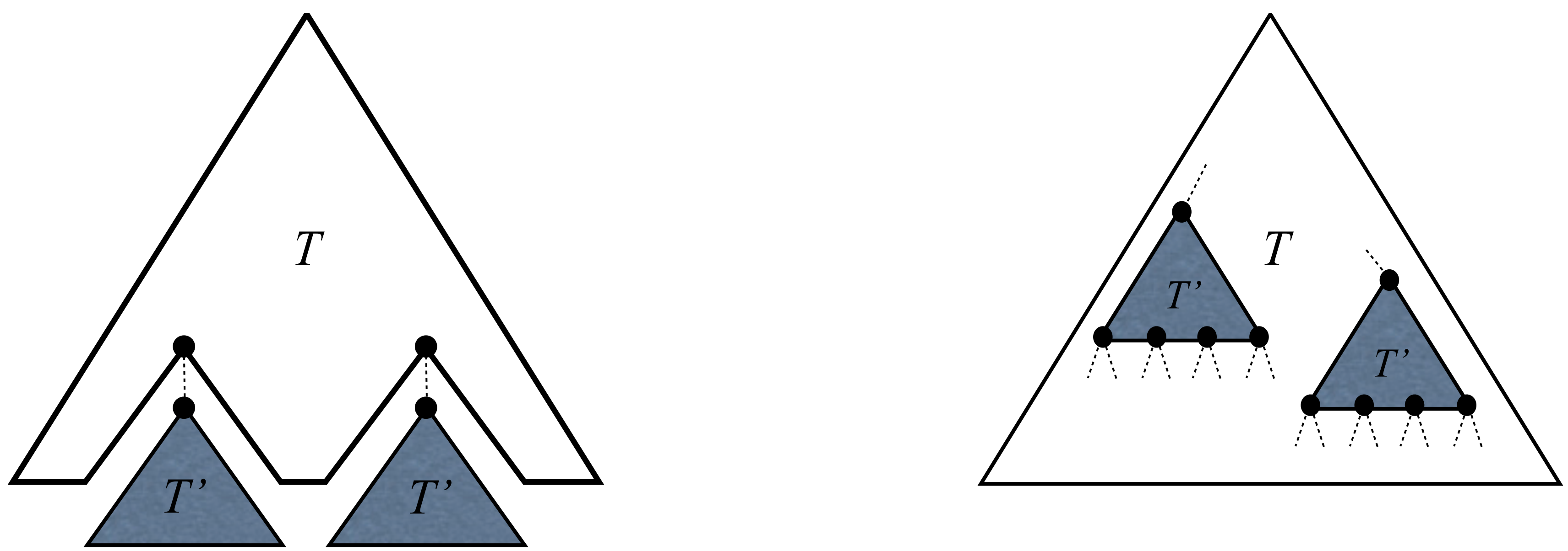
Tree Compression with Top Trees

Philip Bille, Inge Li Gørtz, Gad M. Landau, and Oren Weimann

Outline

- Tree Compression with repetitions
- Previous work
  - DAG compression
  - Tree grammar compression
- Top tree compression
  - Top trees and top tree compression
  - Compression analysis
  - Compressed navigation

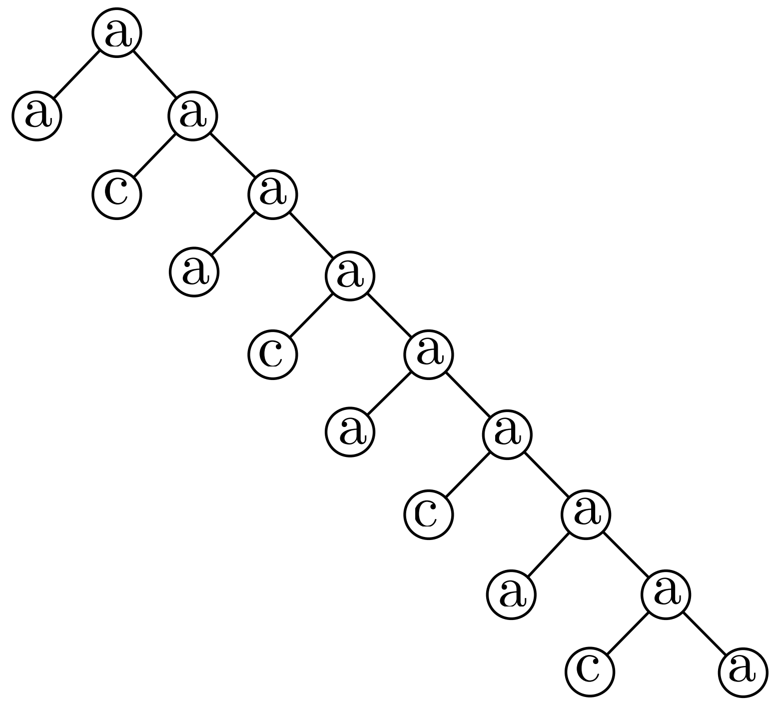
Tree Compression with Repetitions



- Let T be a labeled, rooted tree with N nodes over an alphabet of size σ.
- How to compress T in order to:
    - Take advantage of repetitions (*subtree repeats* or *tree pattern repeats)*
    - Obtain provably good guarantees on compression ratio.
    - Support efficient navigation (access, parent, depth, height, size, NCA, ...)

DAG Compression

- Merge *subtree repeats* into directed acyclic graph (DAG) representing T.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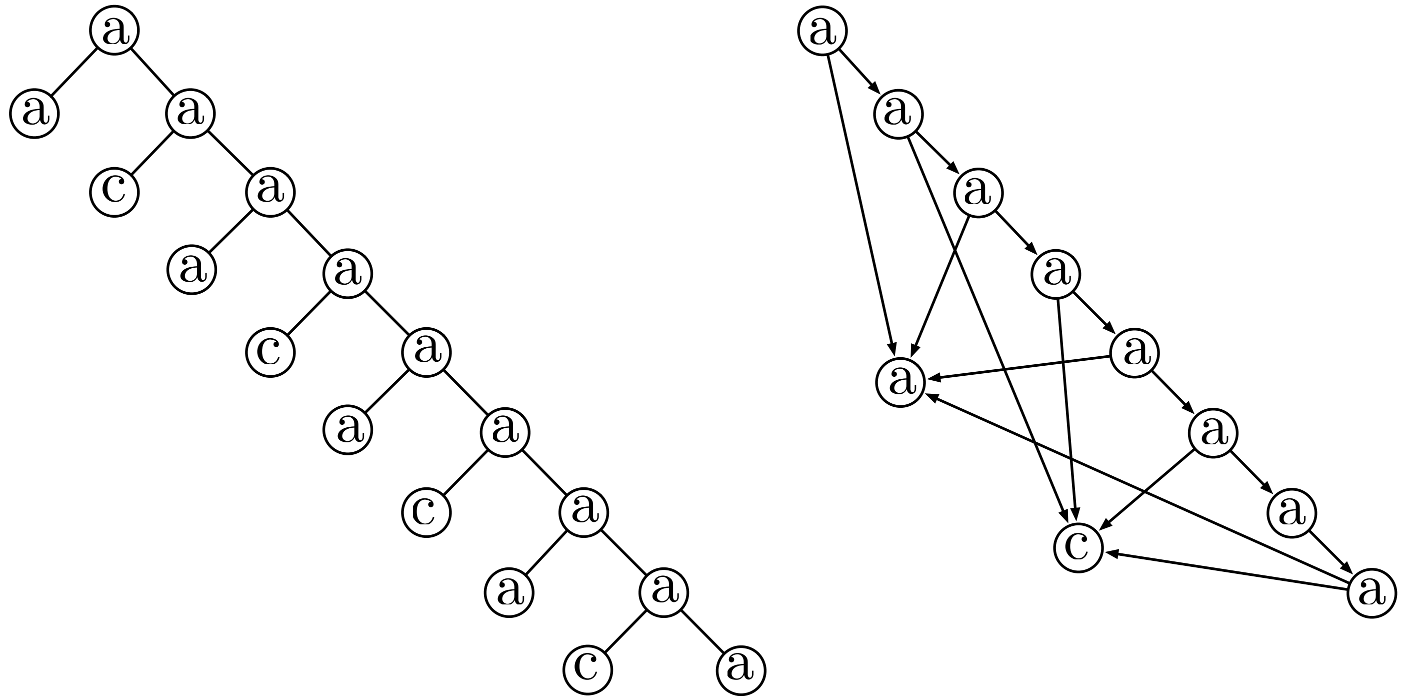

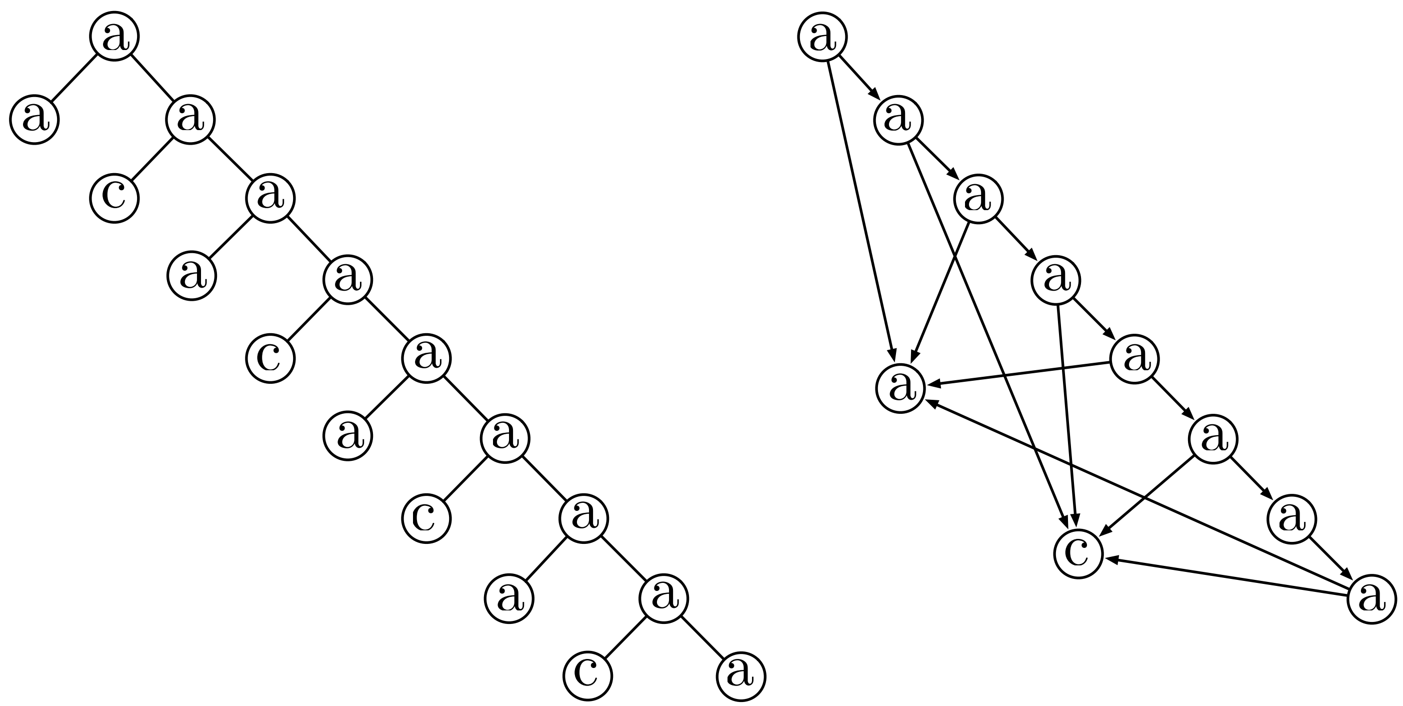

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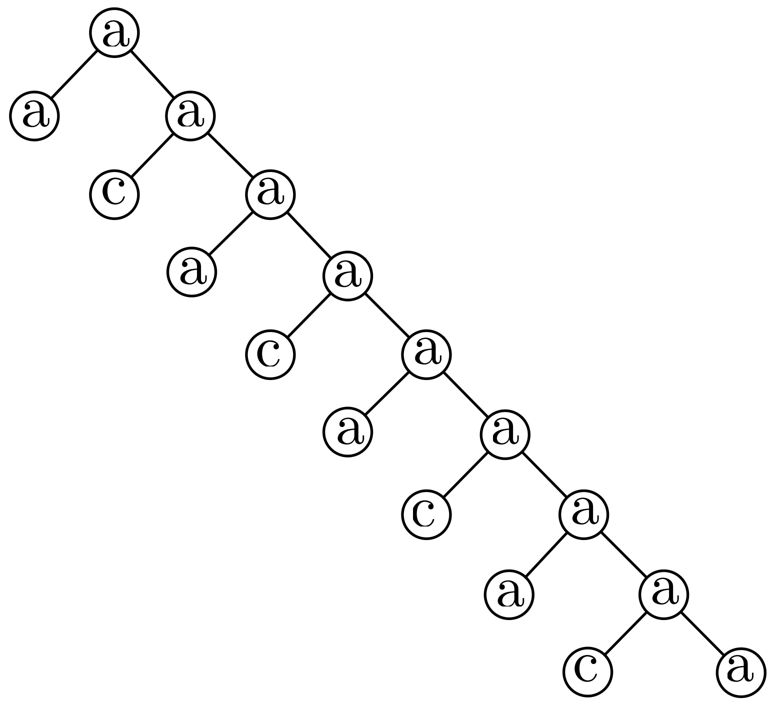

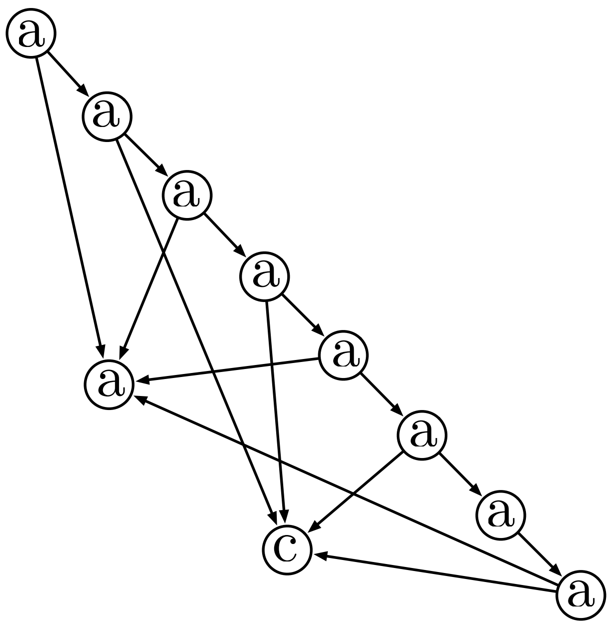

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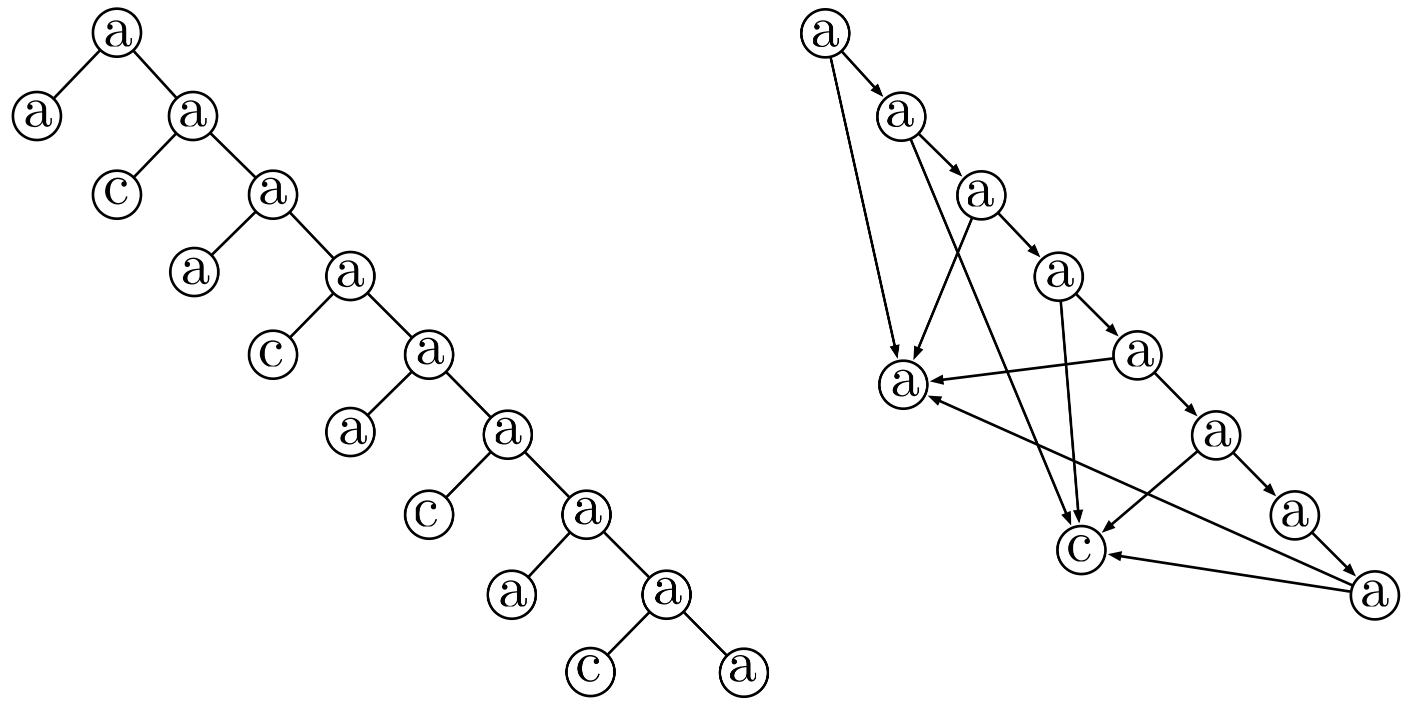

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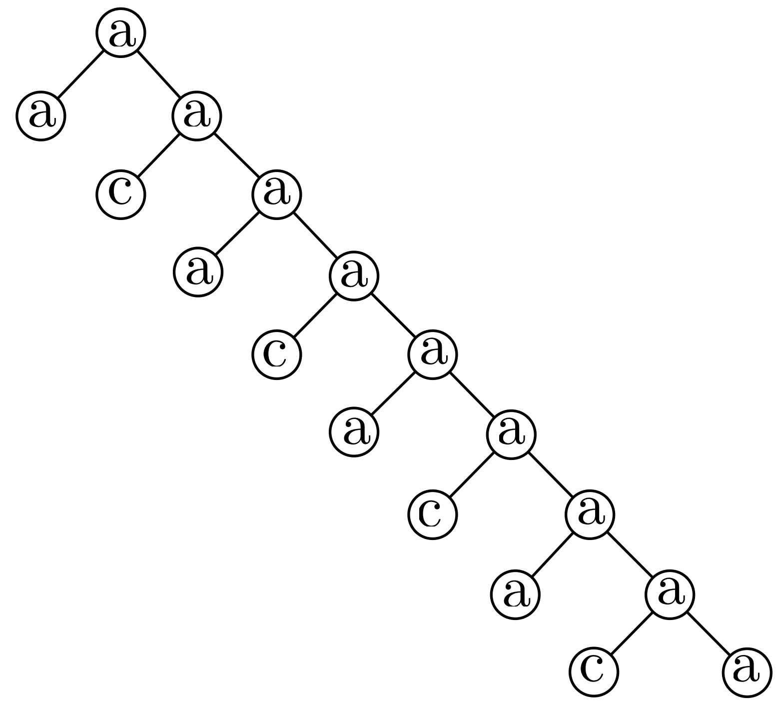

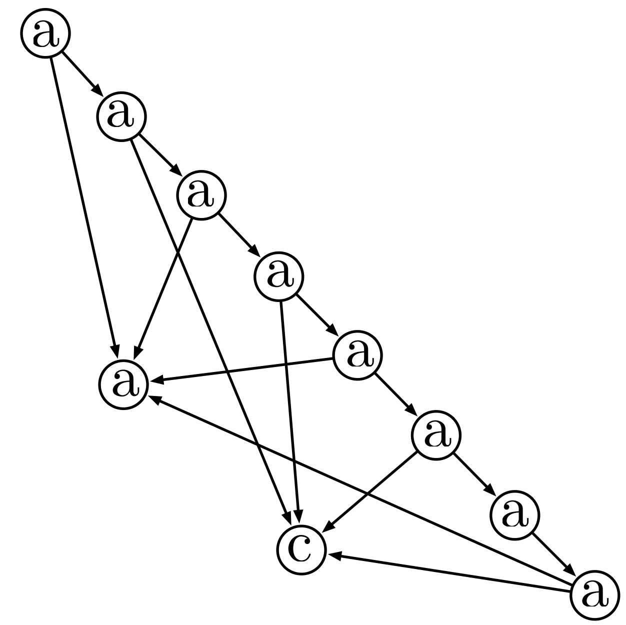

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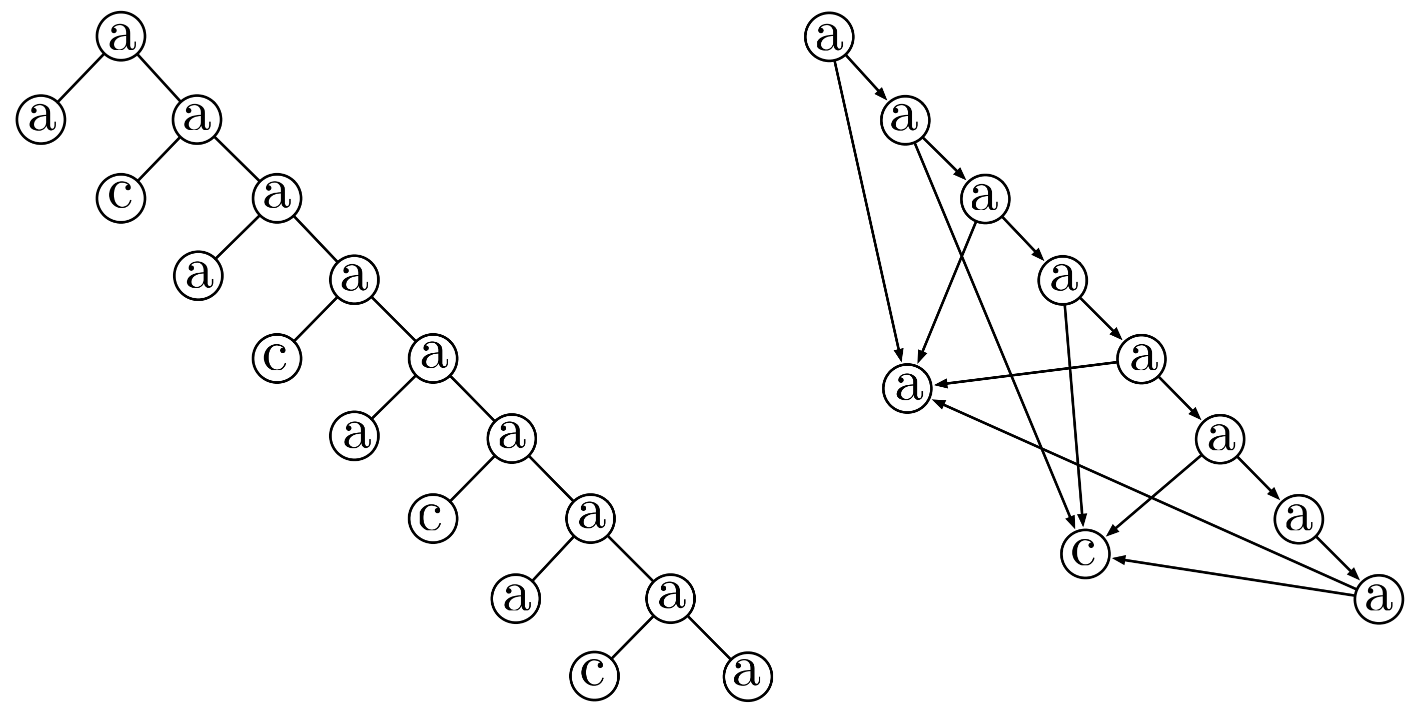

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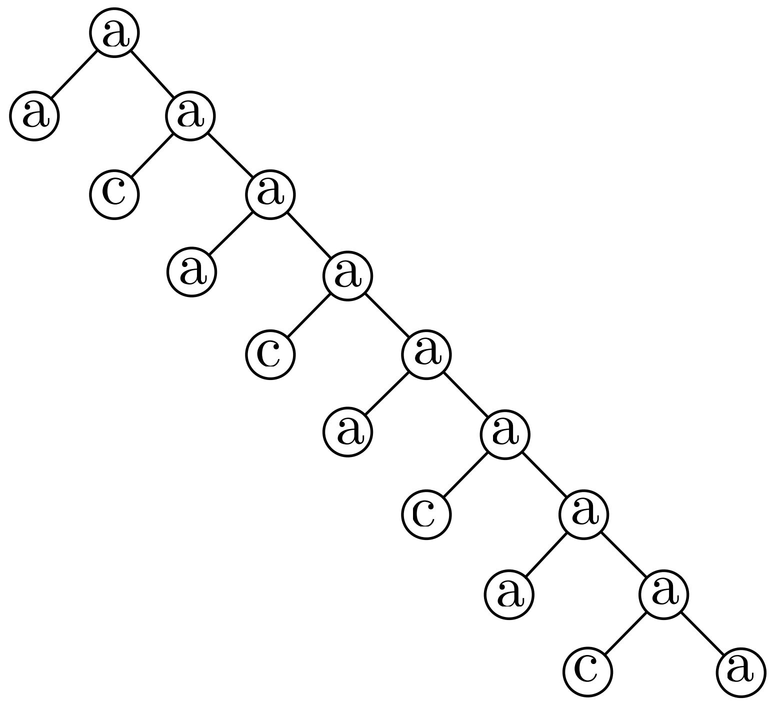

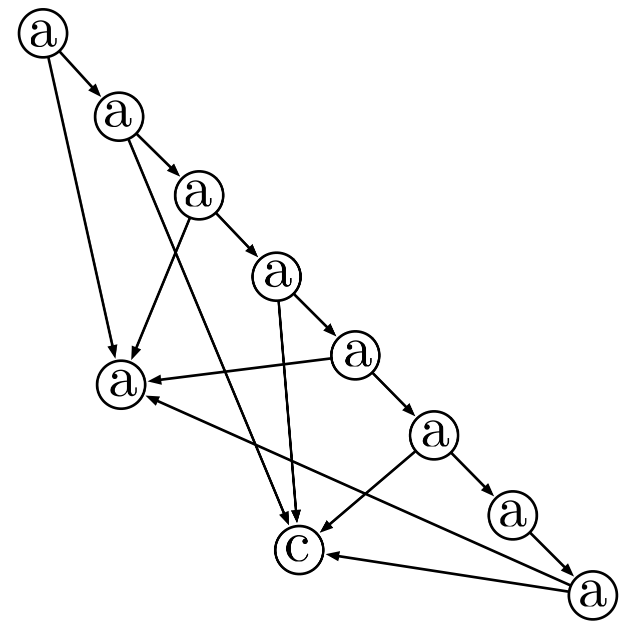

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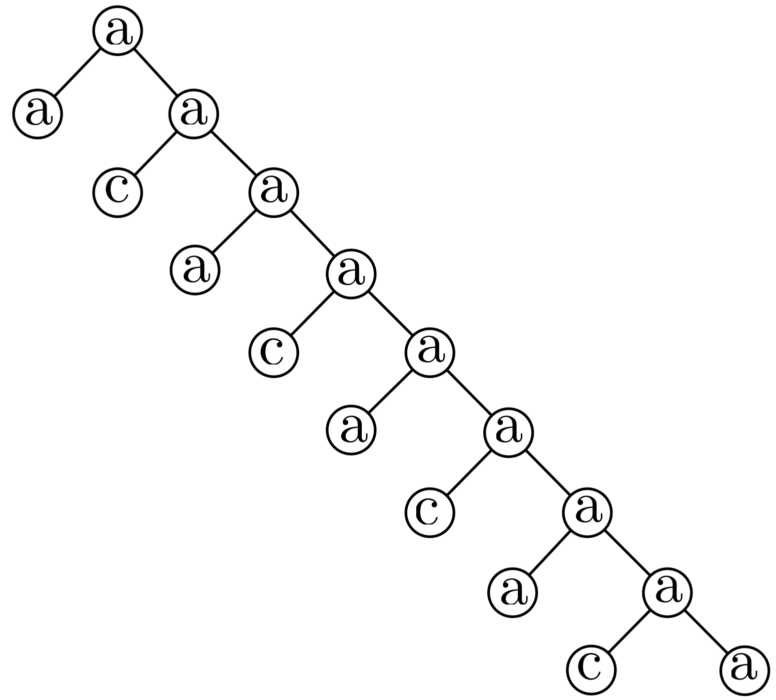
- Popular for XML compression. See e.g. [Buneman, Grohe, Koch 2003] [Frick, Grohe, Koch 2003]

Tree Grammars

- Encode *tree pattern repeats* using a grammar that generates T.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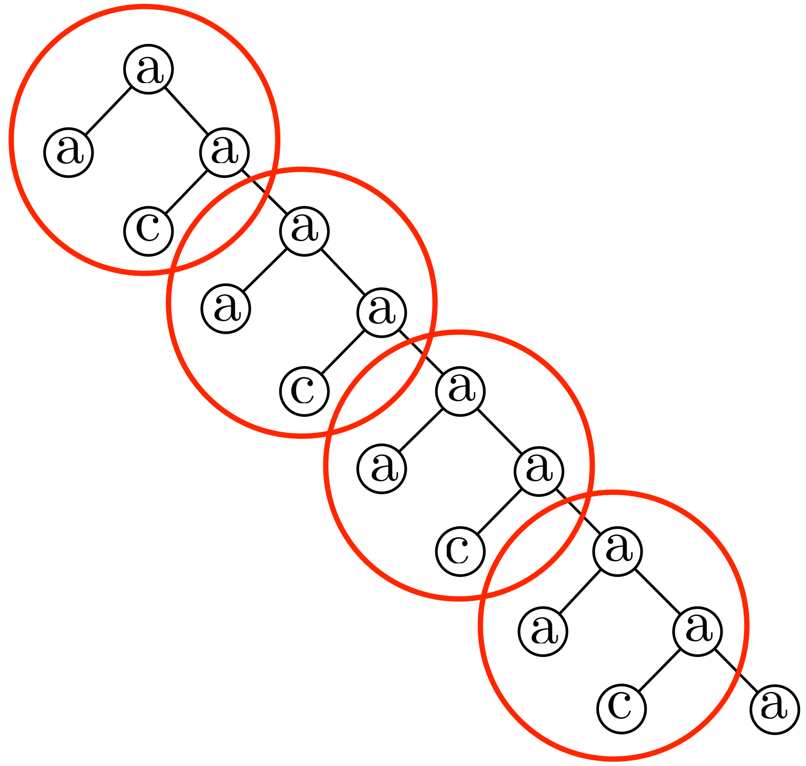
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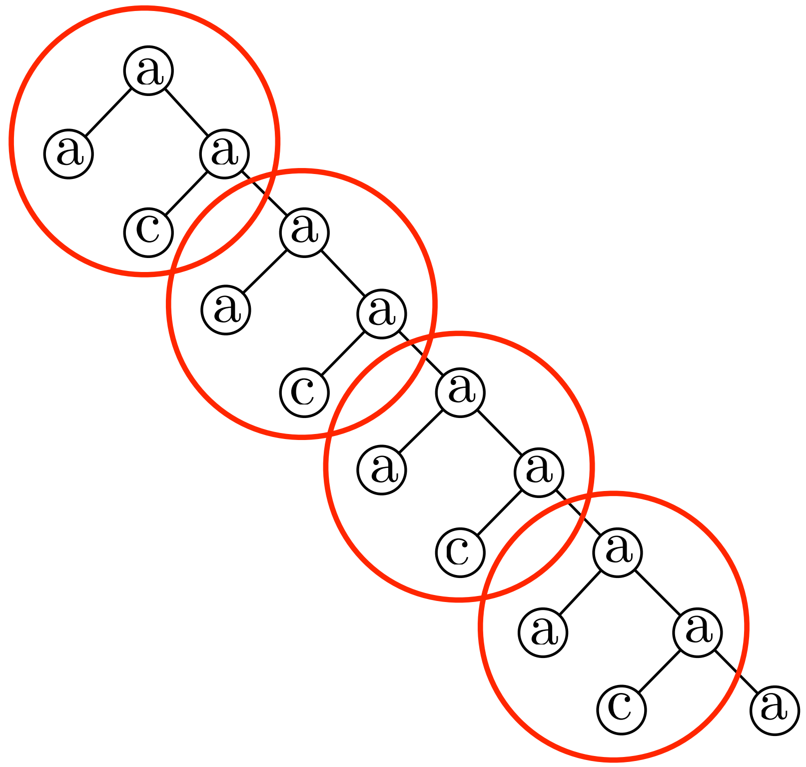
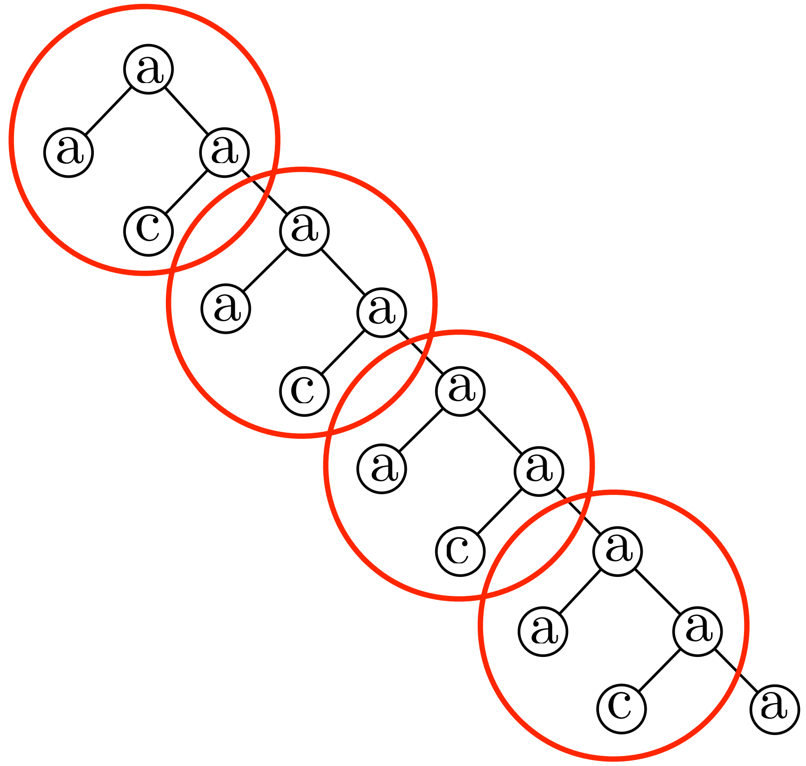


S → A(A(A(A(a))))
A(x) → a(a,a(c,x)))



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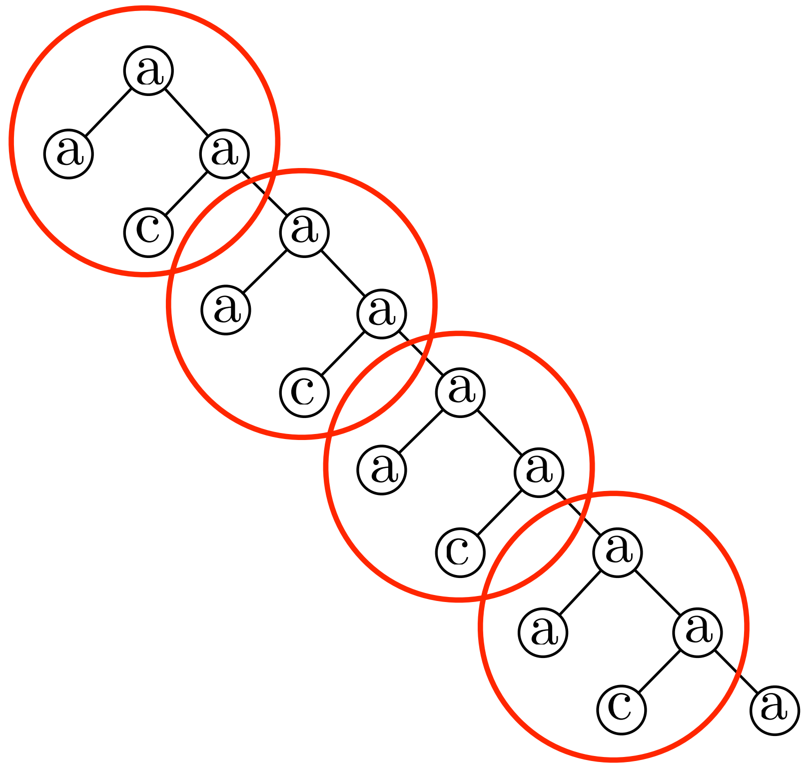

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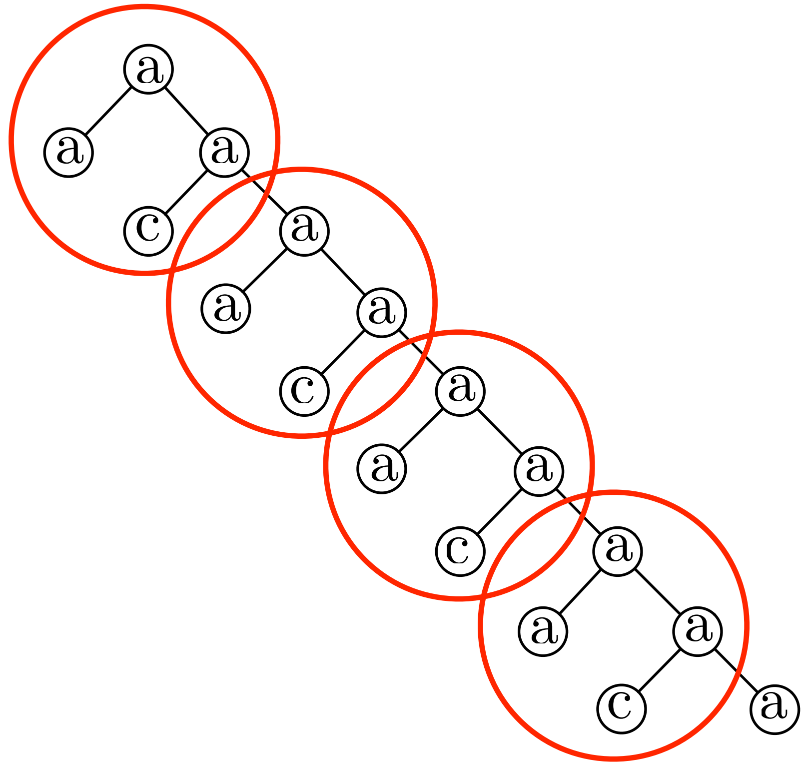

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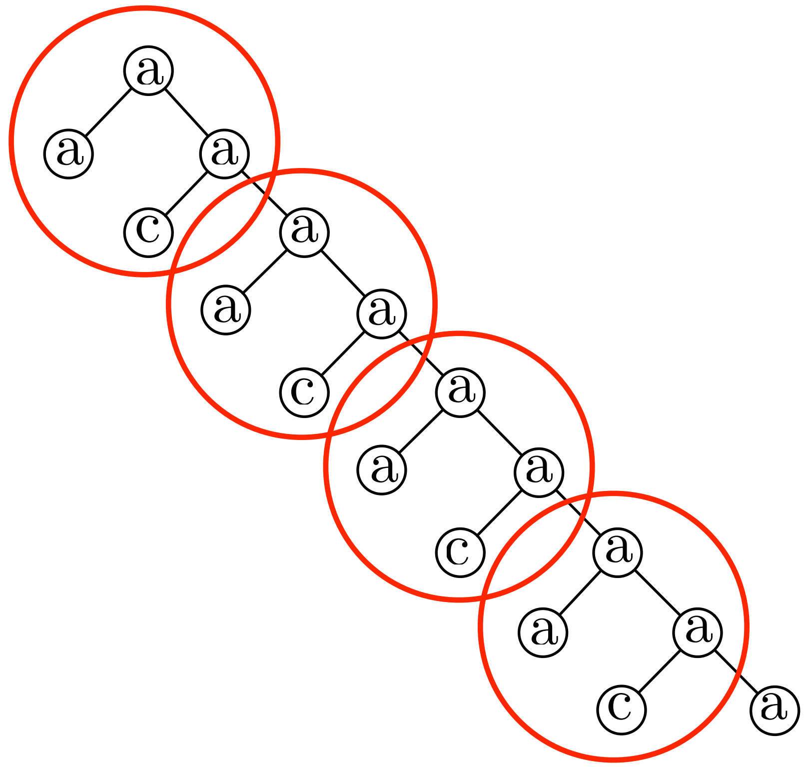

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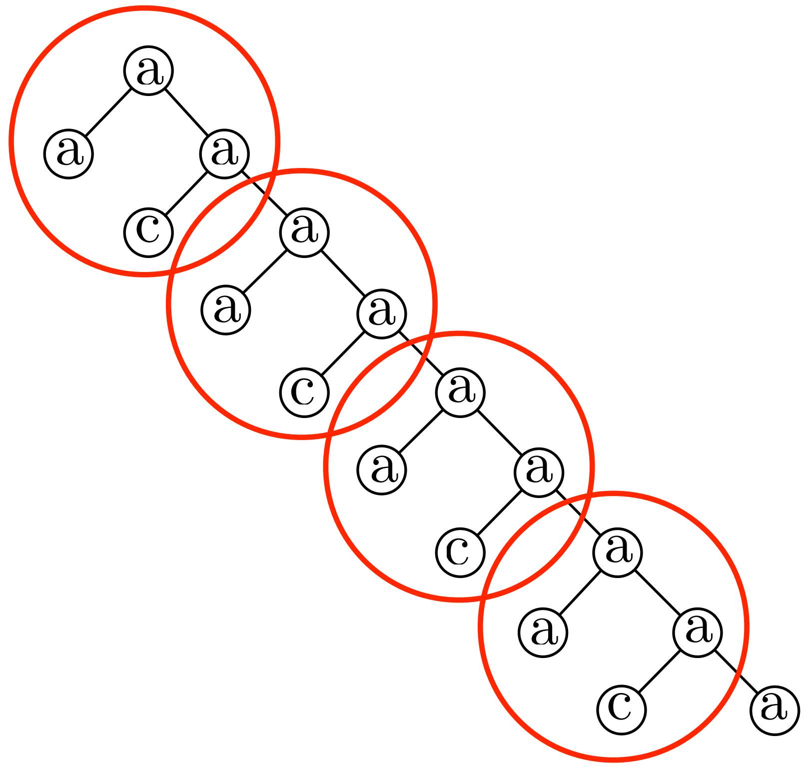

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- Popular for XML compression. See e.g. [Busatto, Lohrey, Maneth 2004] [Maneth, Busatto 2004] [Lohrey, Maneth 2006] [Busatto, Lohrey, Maneth 2008] [Lohrey, Maneth, Mennicke 2010]

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  - Navigational operations in O(log N) time.

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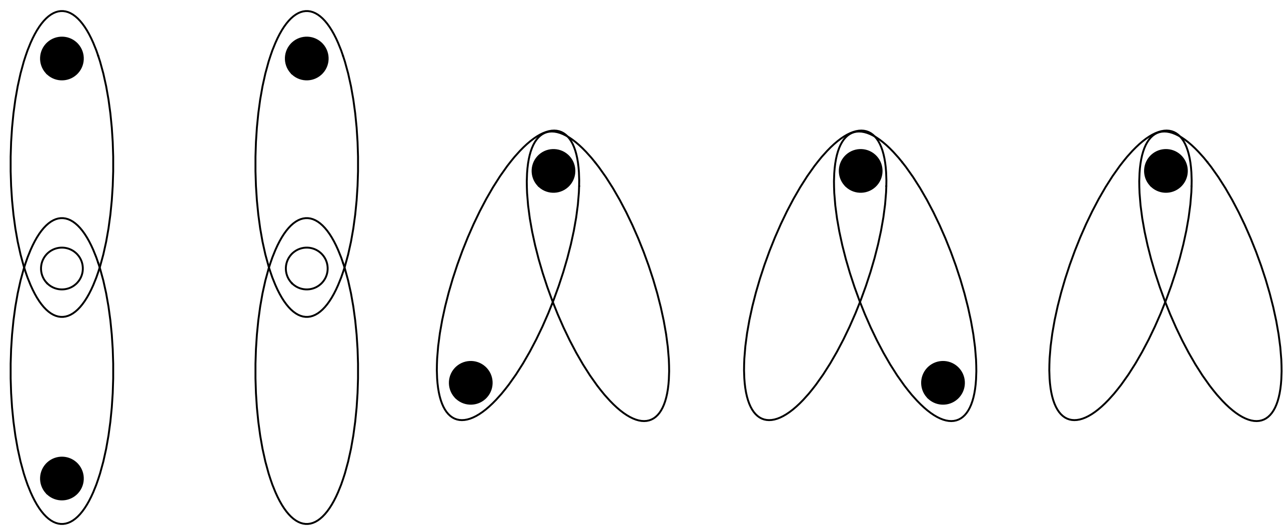
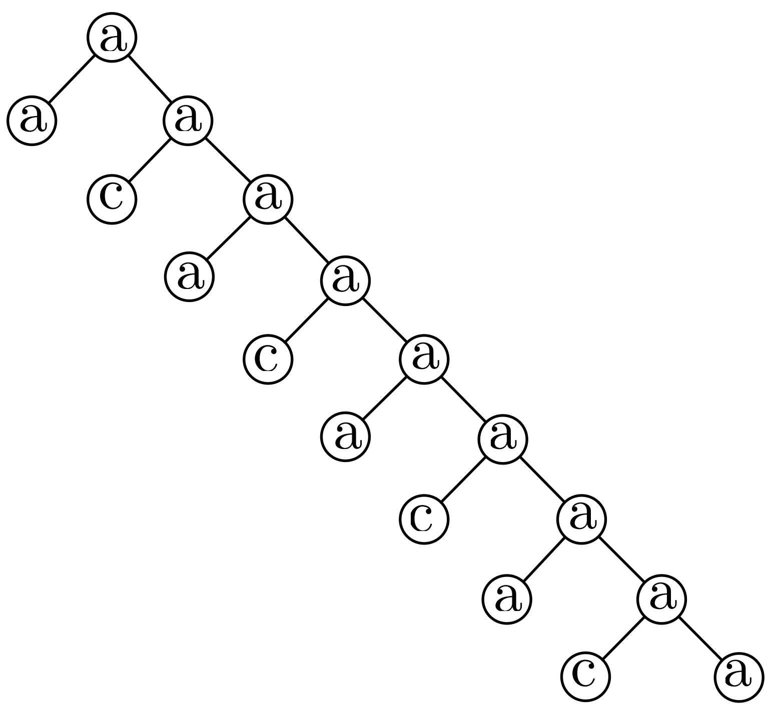
- Each cluster overlaps with adjacent clusters in 1 or 2 *boundary* nodes.

Top Tree Construction



- Start with edges of T as the bottom of cluster hiearchy (leaves of the top tree)
- Merge pairs of clusters greedily to form new clusters.
- Contract each clusters into an edge.
- Repeat until left with a single edge.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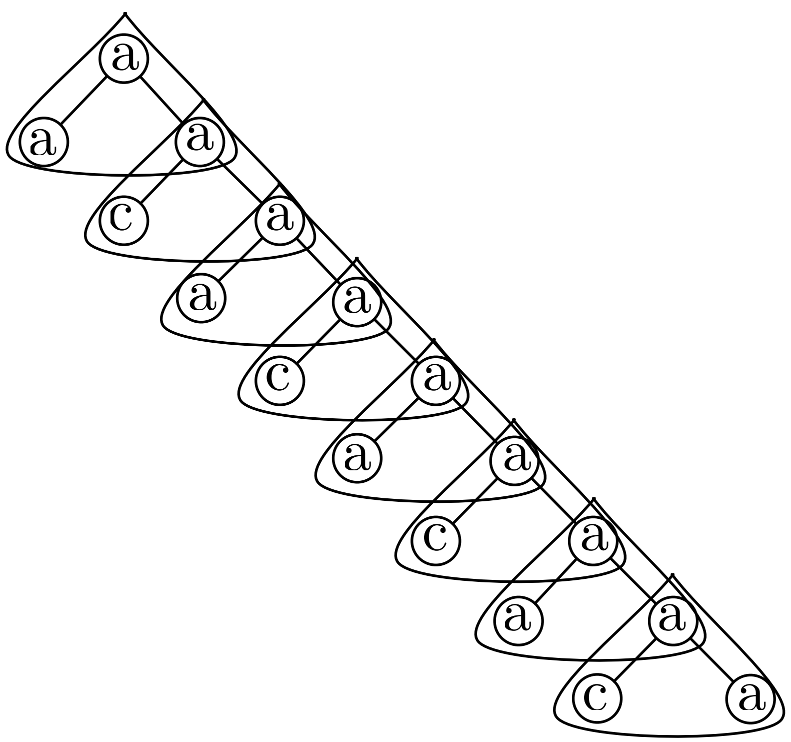
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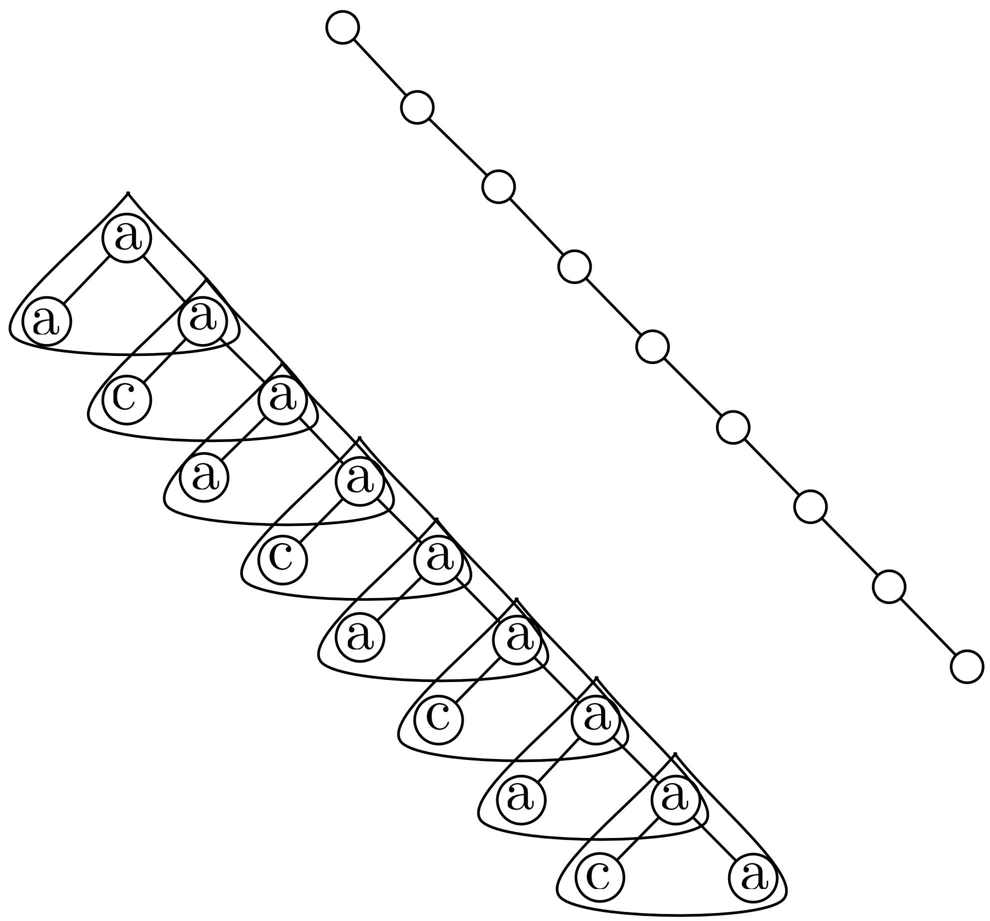
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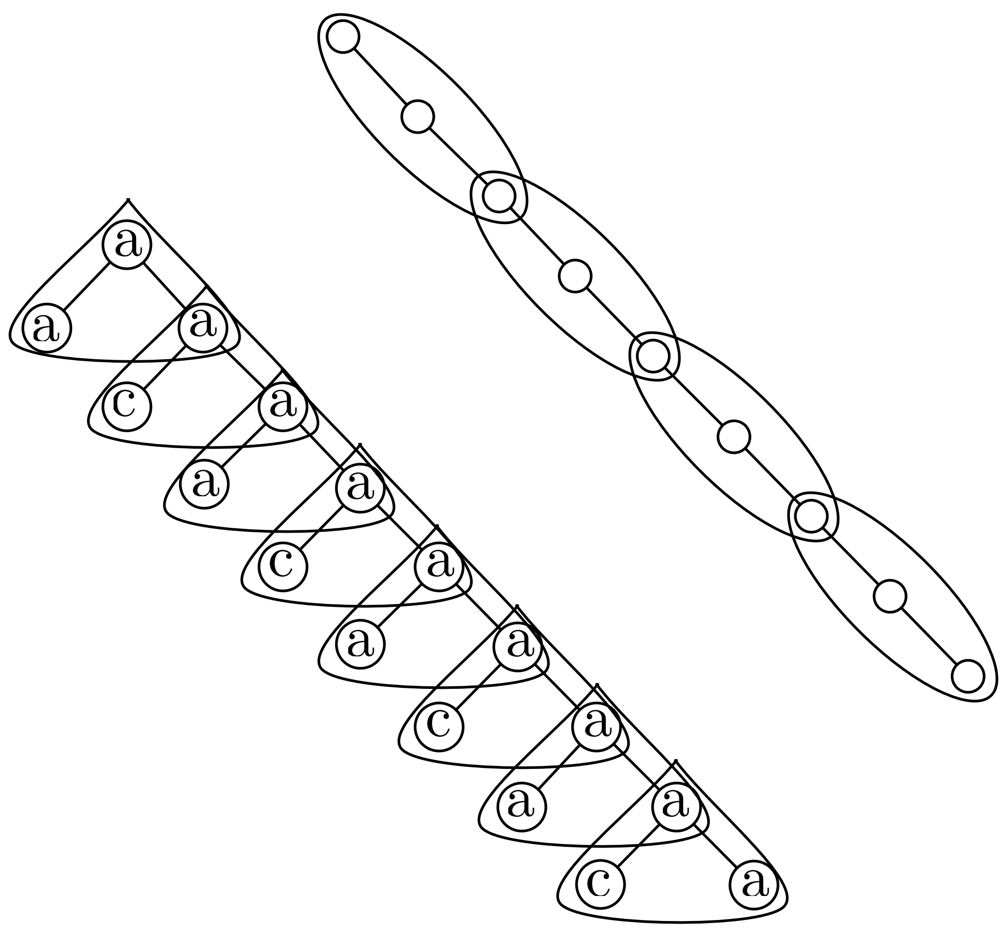
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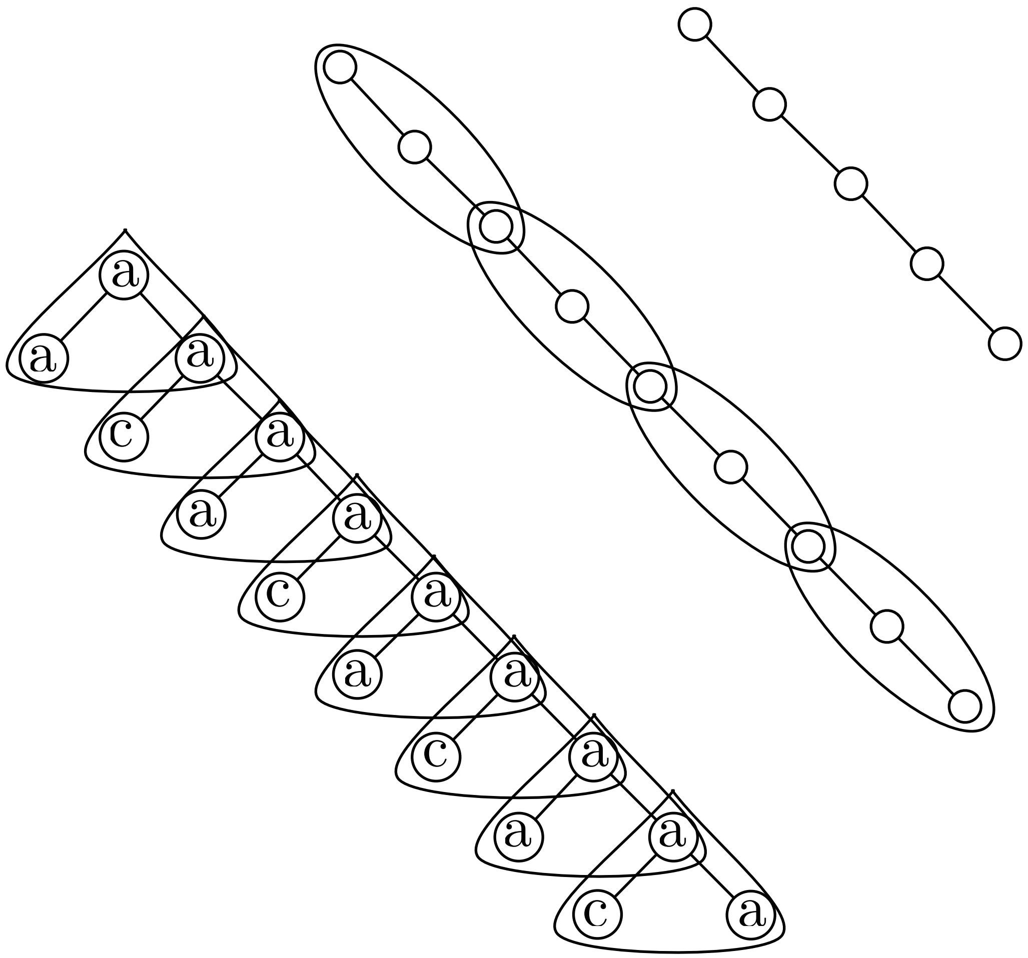
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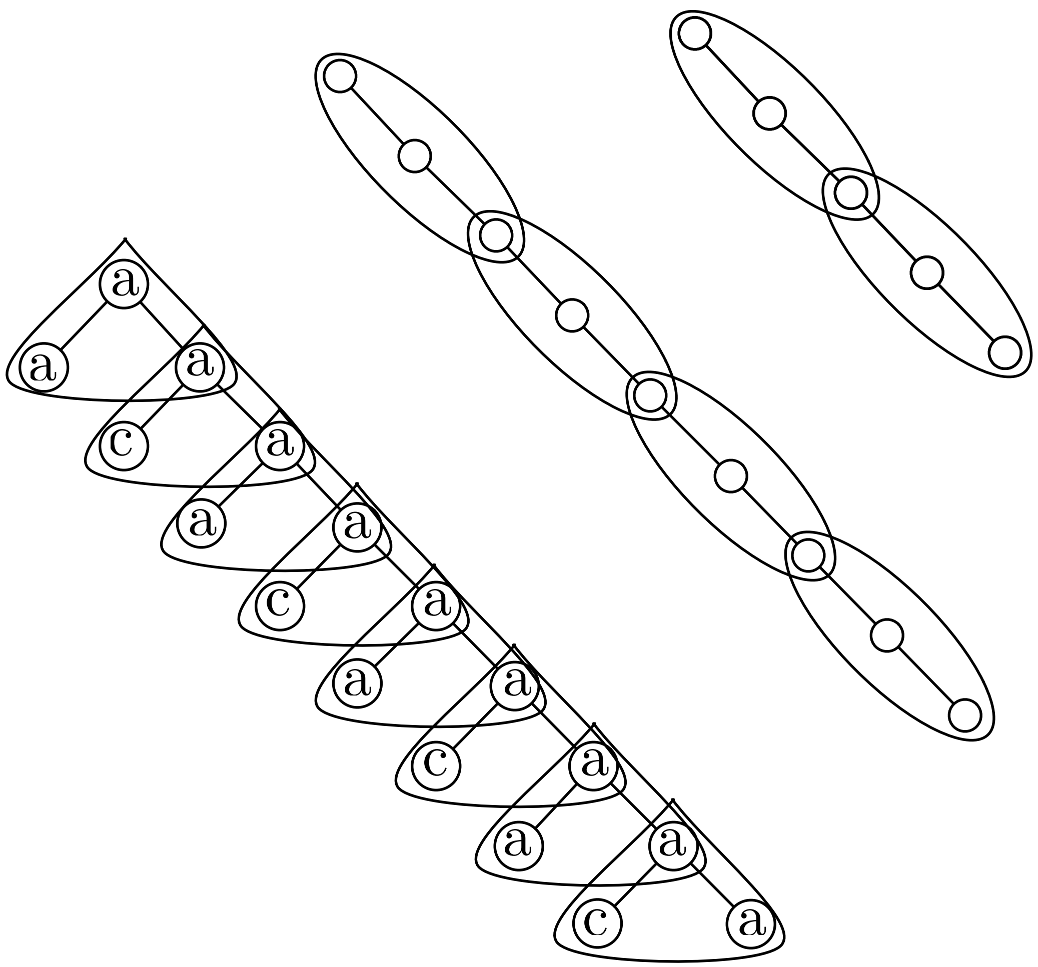
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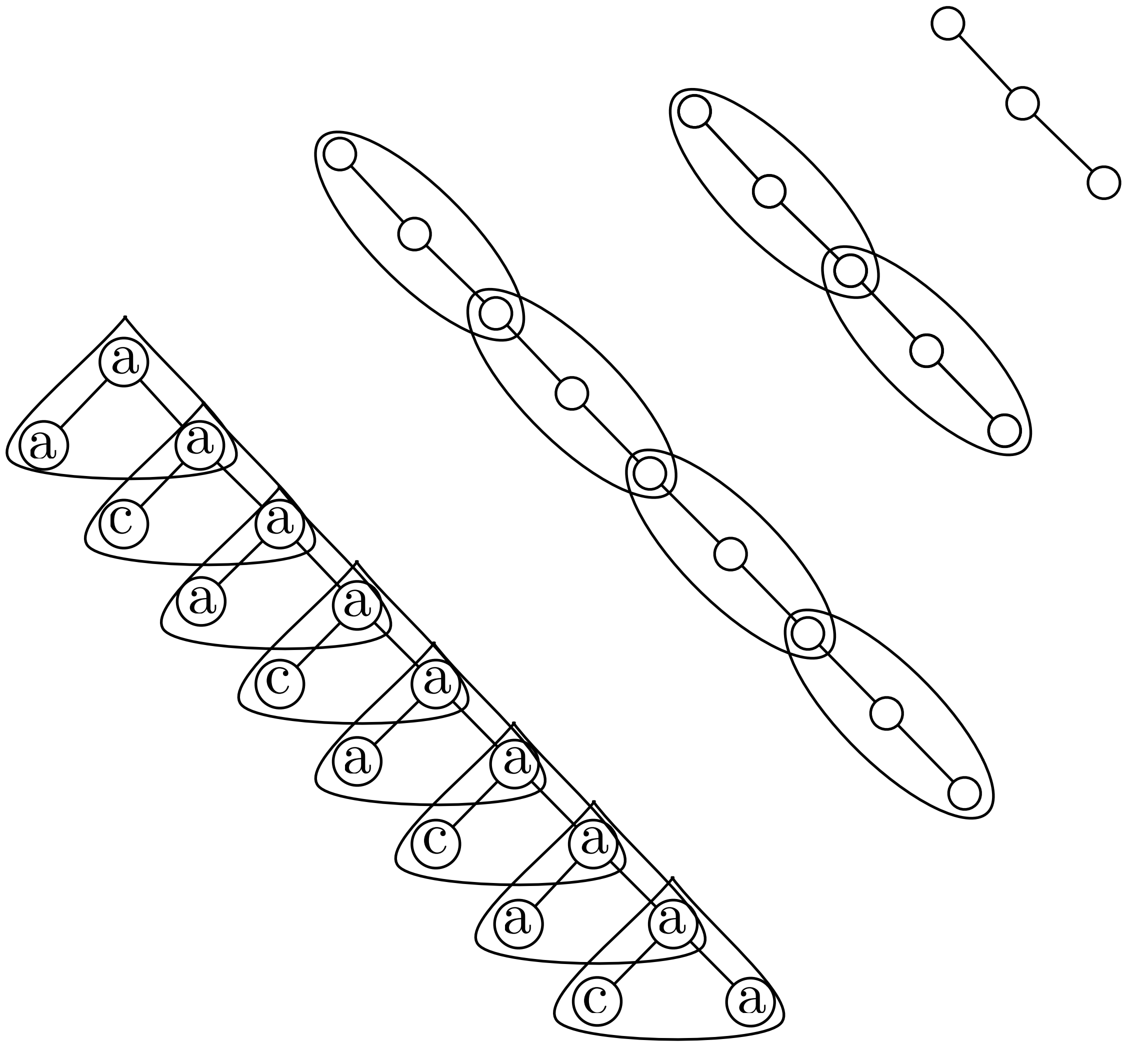
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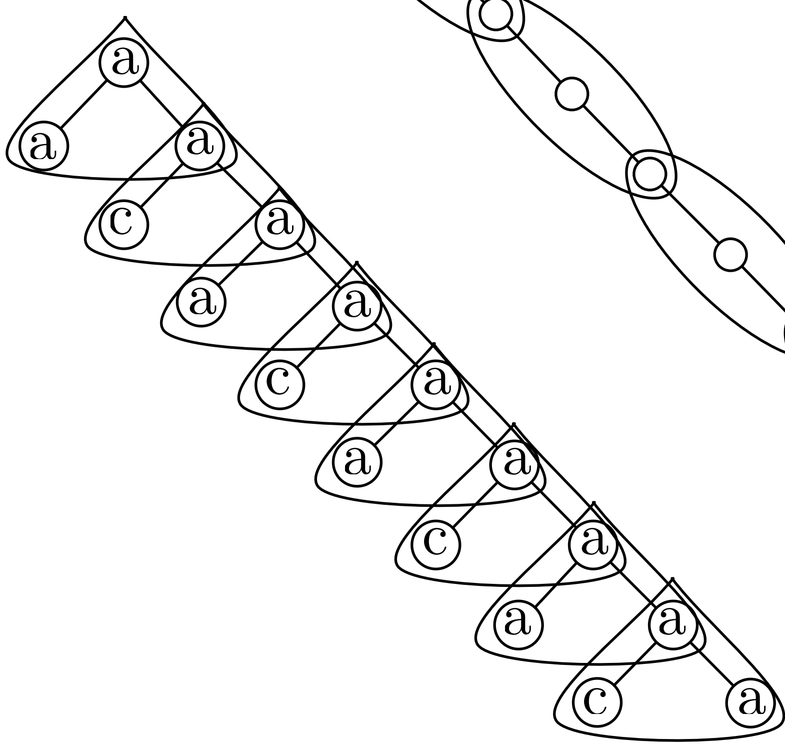
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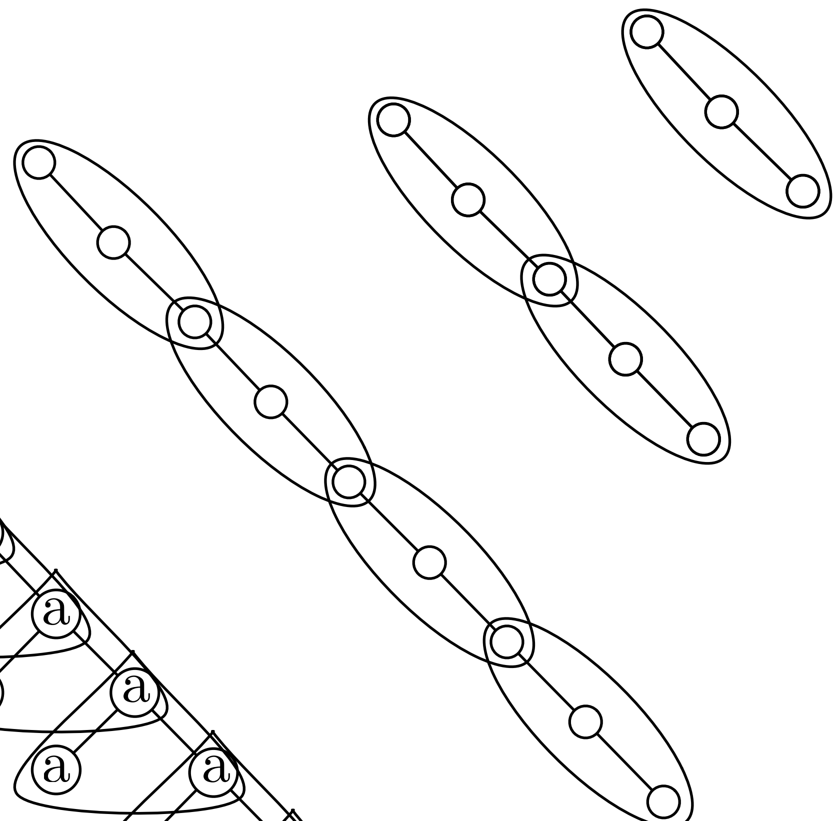

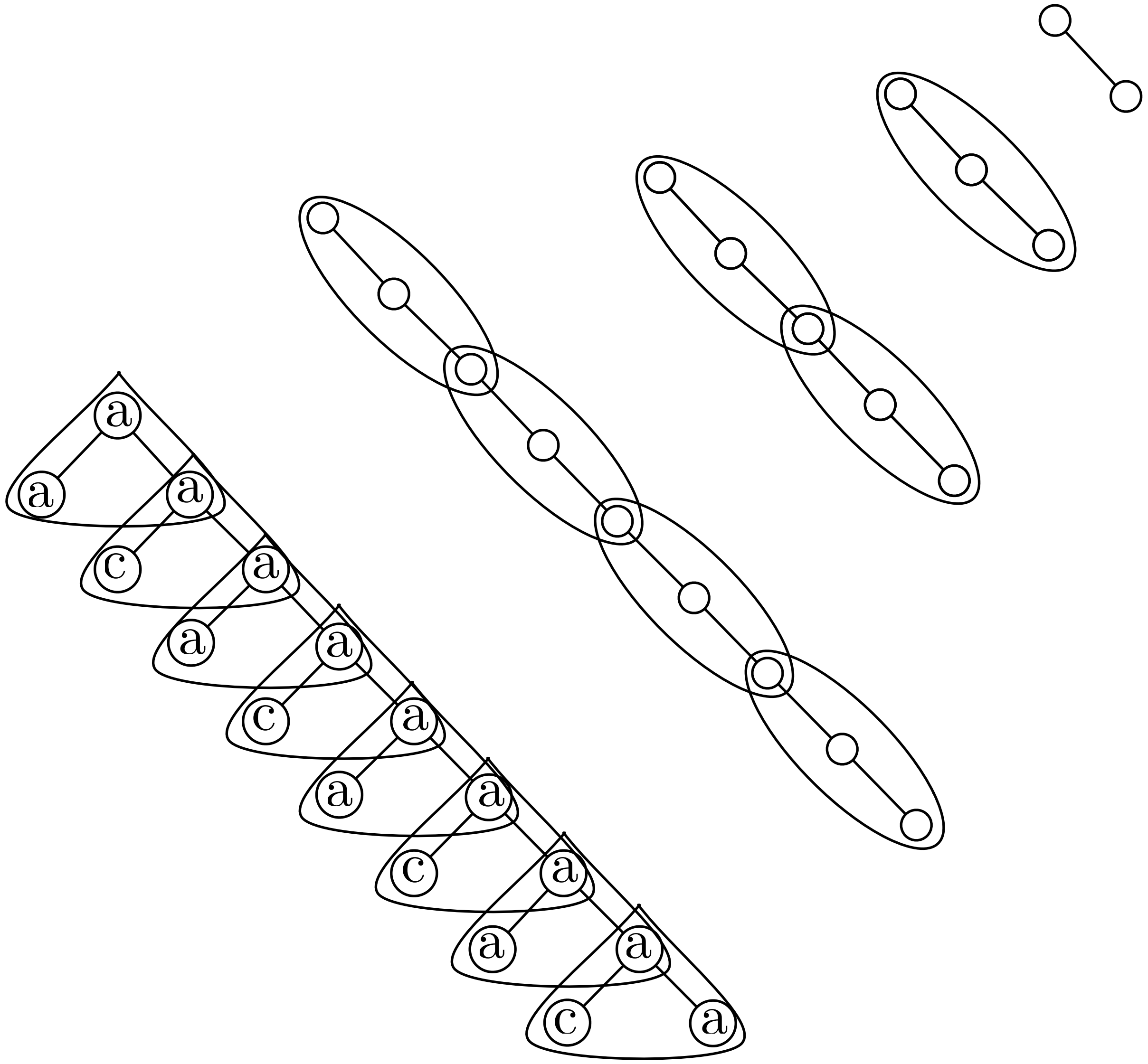
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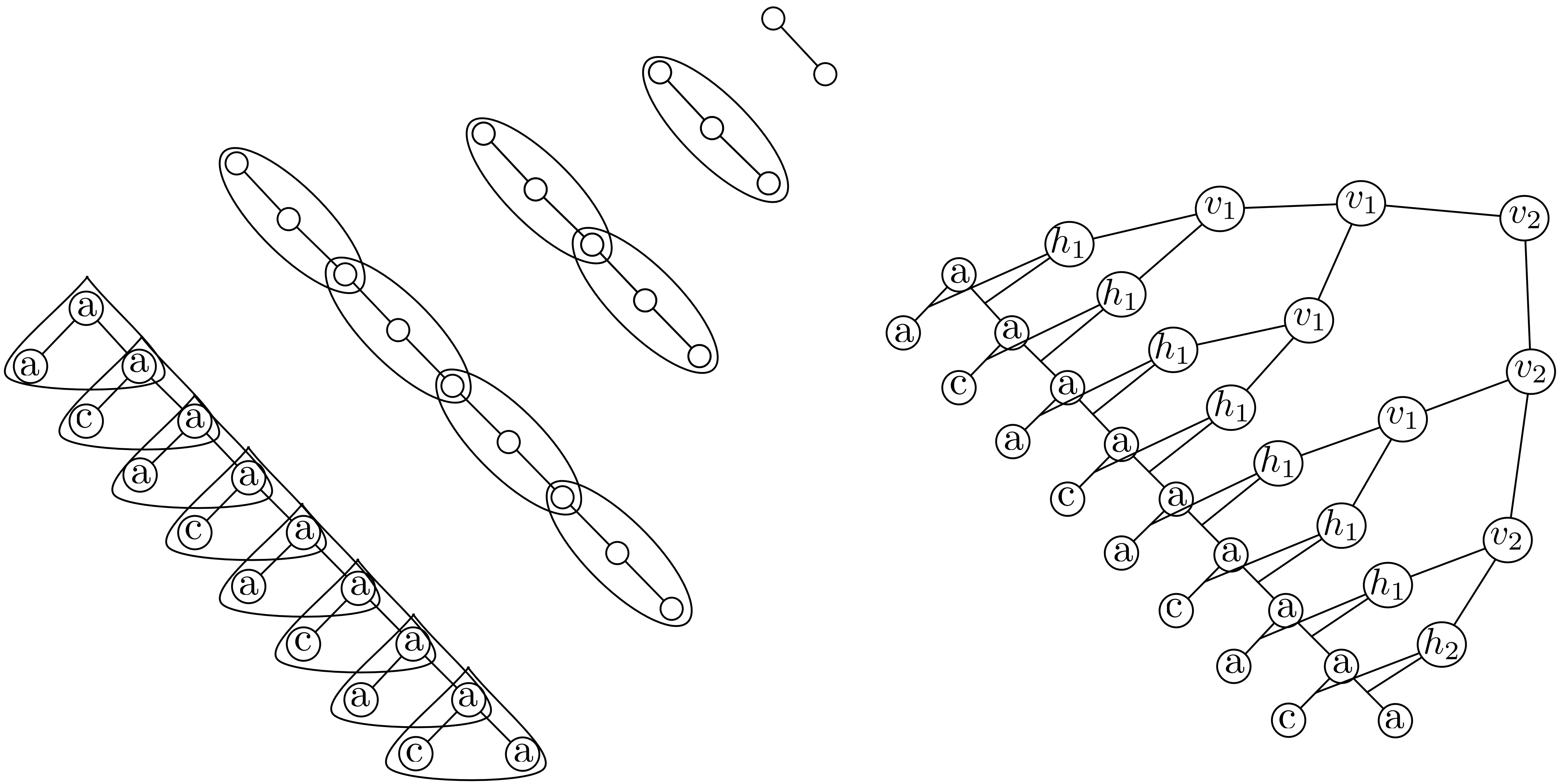

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- Size of top tree is O(N)
- Height of top tree is O(log N)

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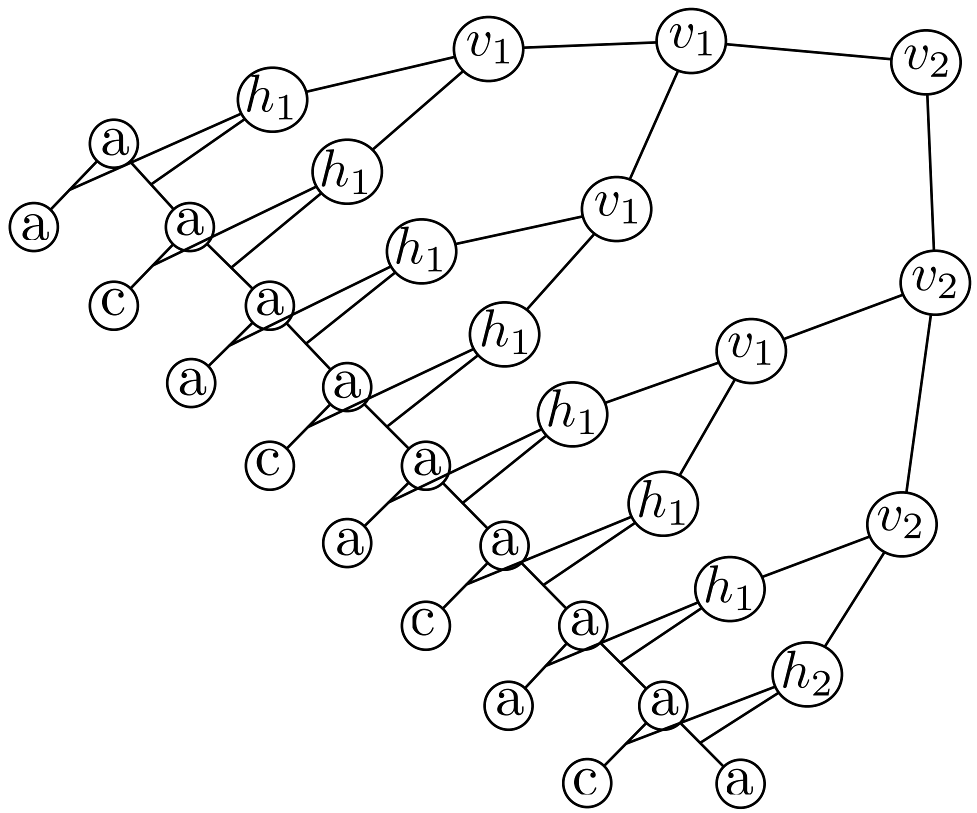
Top Tree Compression

- DAG compress top tree (!)
- Top tree compression may be viewed as *transformation* of input tree into another tree (which compresses well and a supports fast navigation).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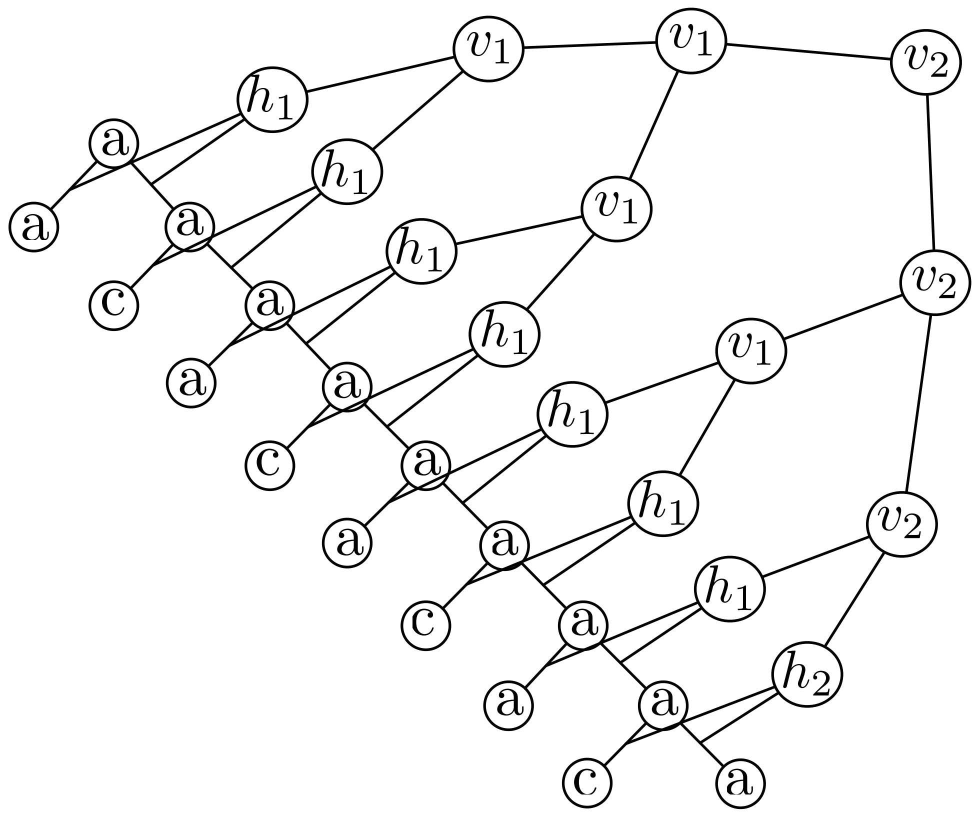
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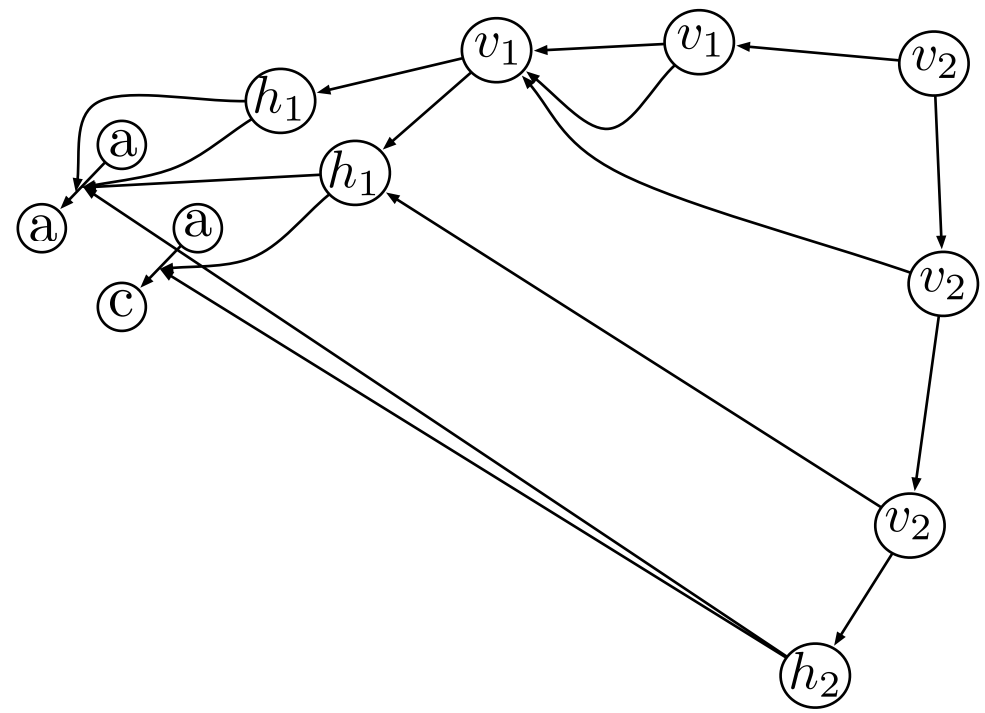
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Top Tree Compression Analysis

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- How good is top tree compression?

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- How good is top tree compression?
- Worst case compression ratio.

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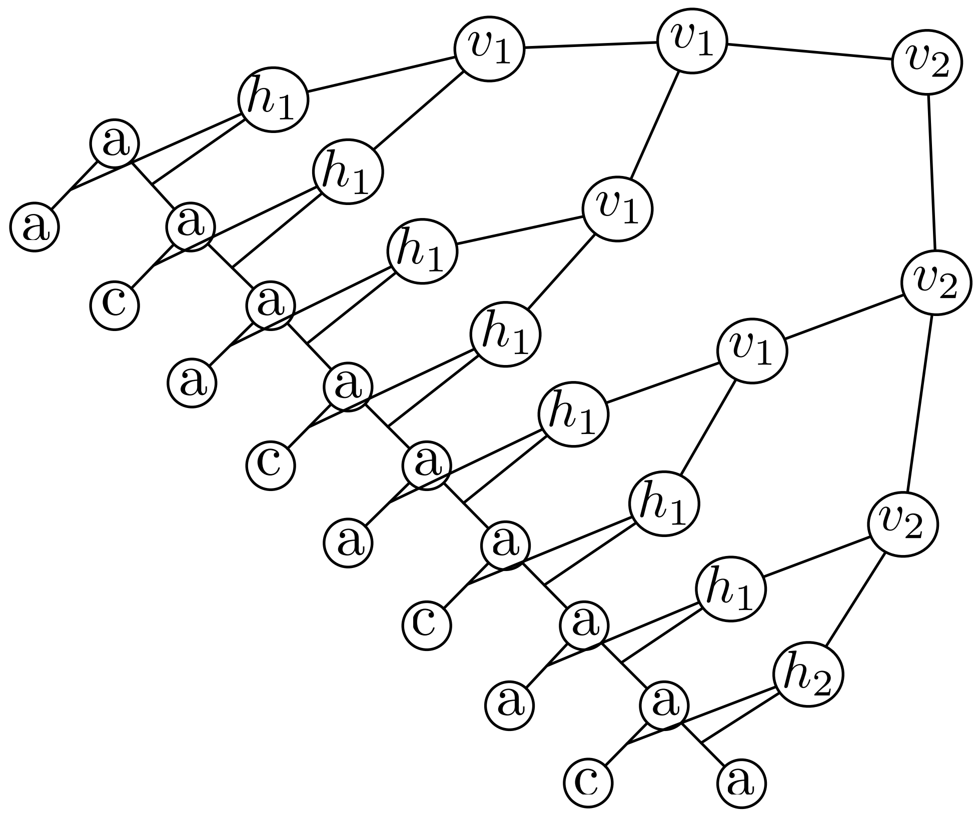
- How good is top tree compression?
- Worst case compression ratio.
- DAG vs. top tree compression.



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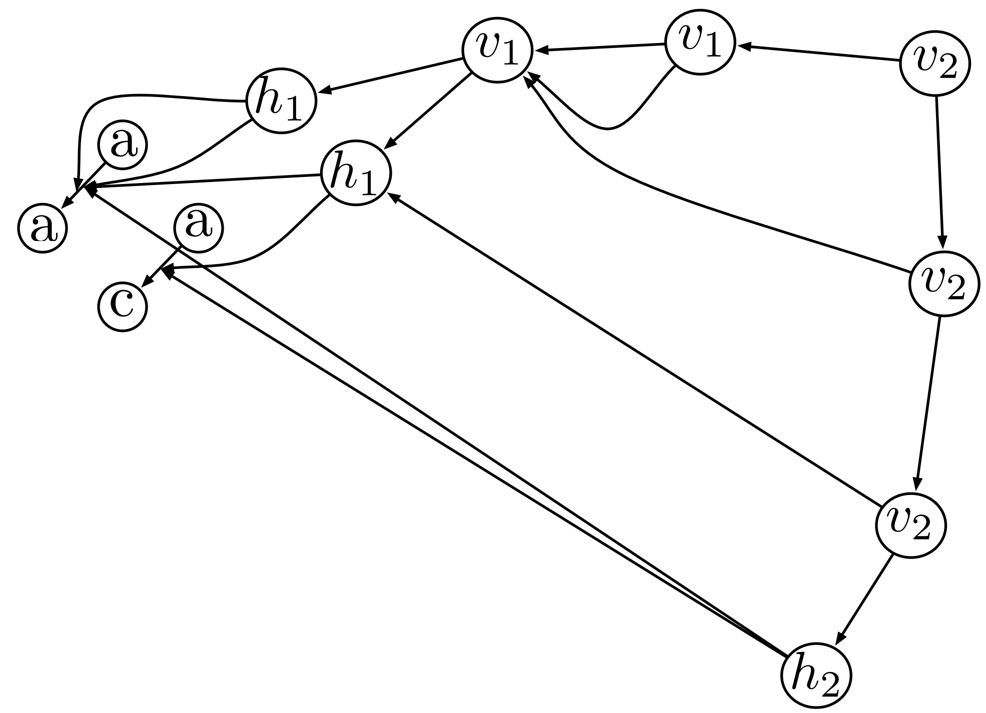

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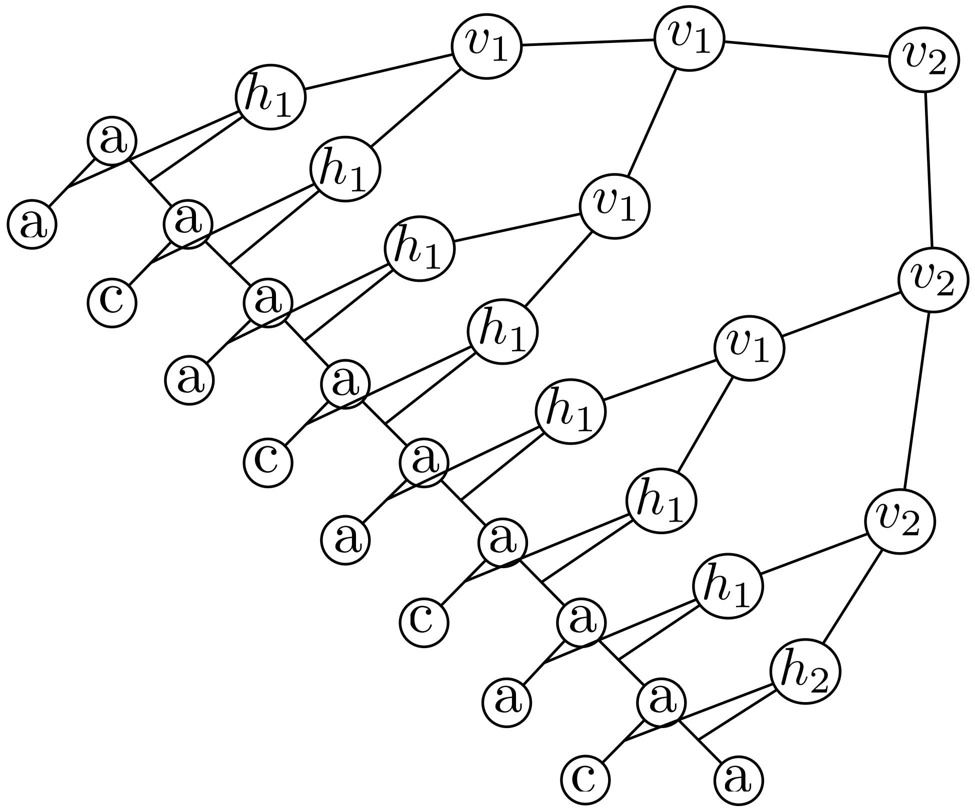
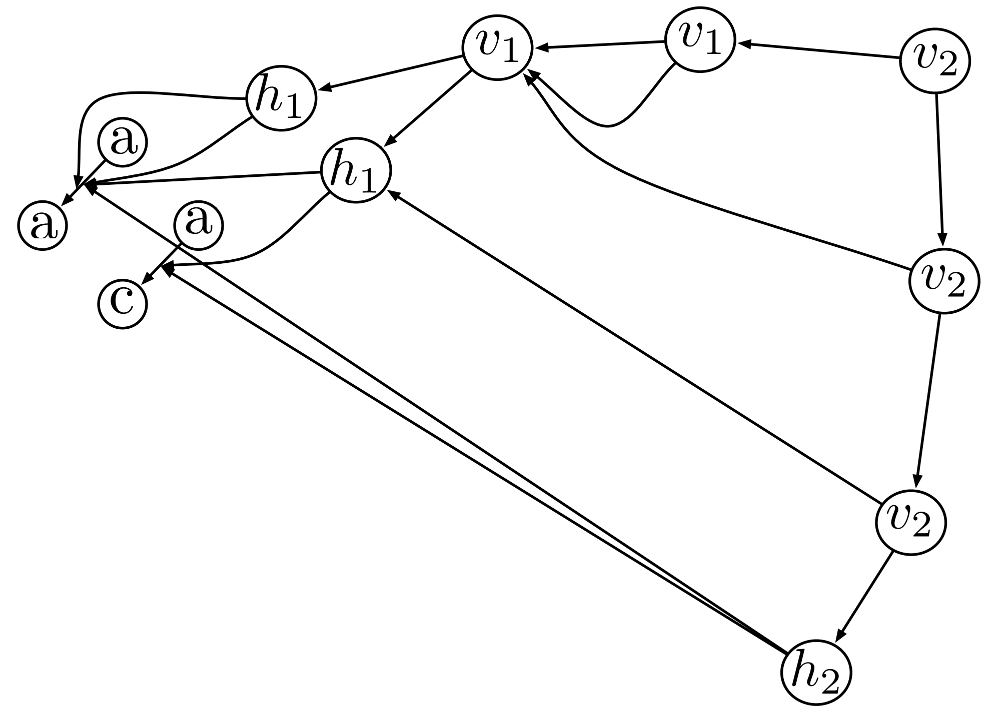
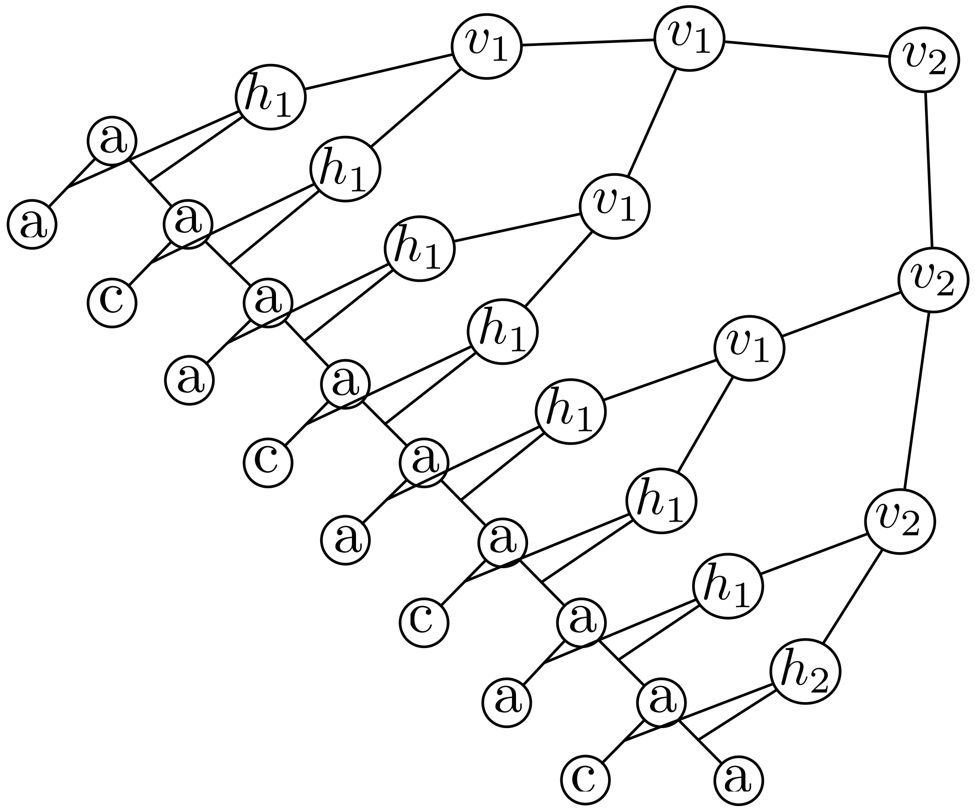
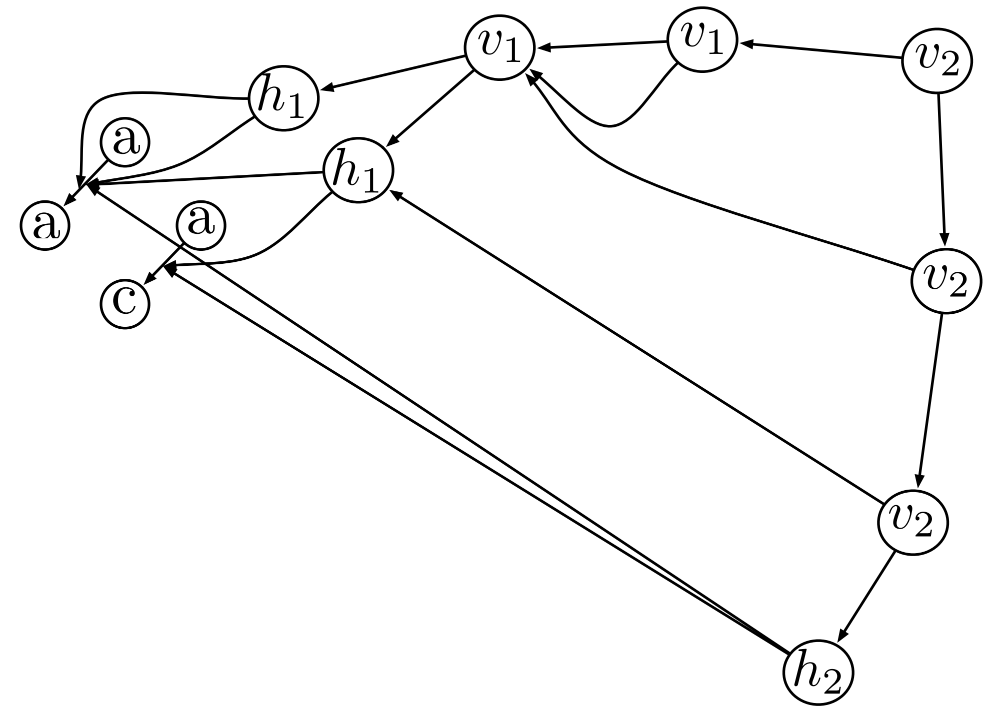




- Identical clusters in T are represented by identical subtrees in top tree.





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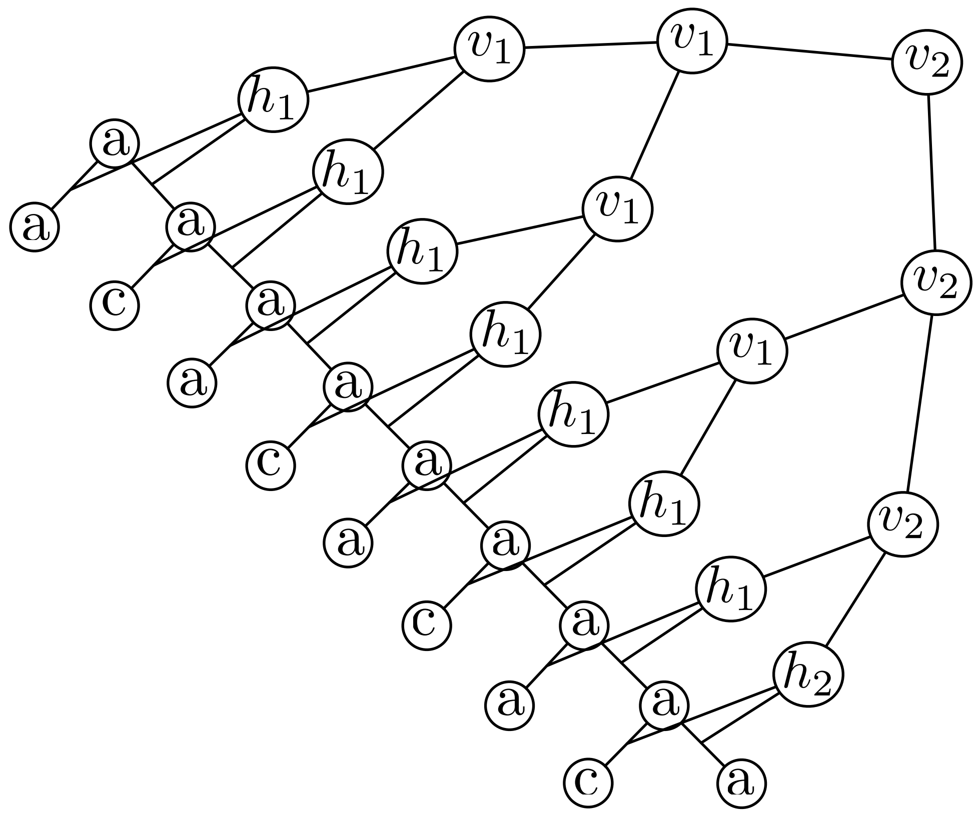

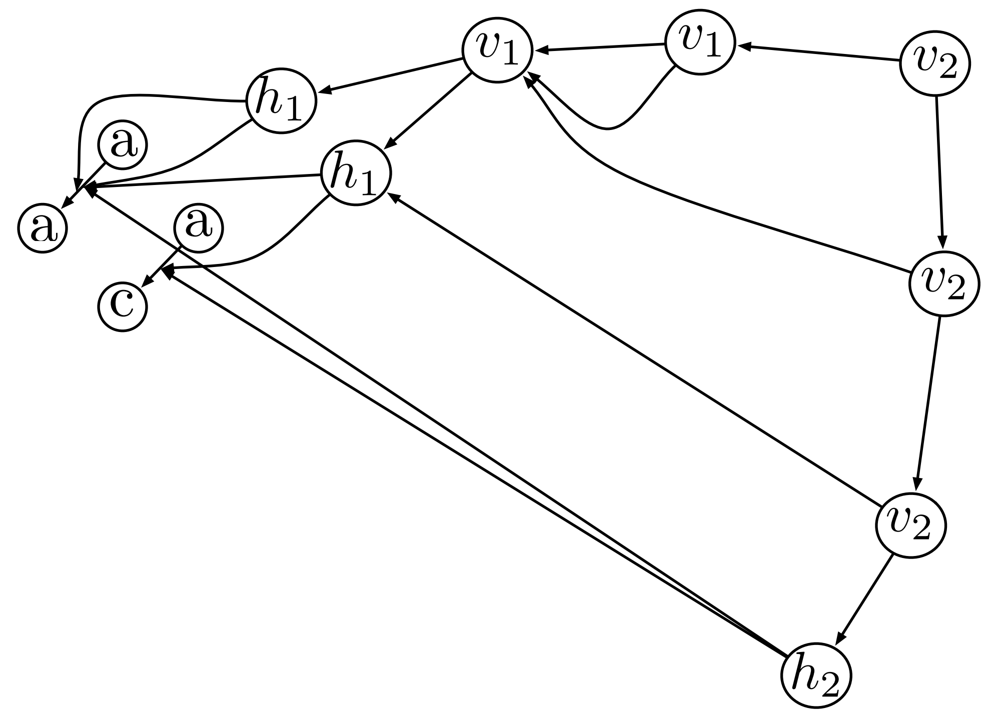

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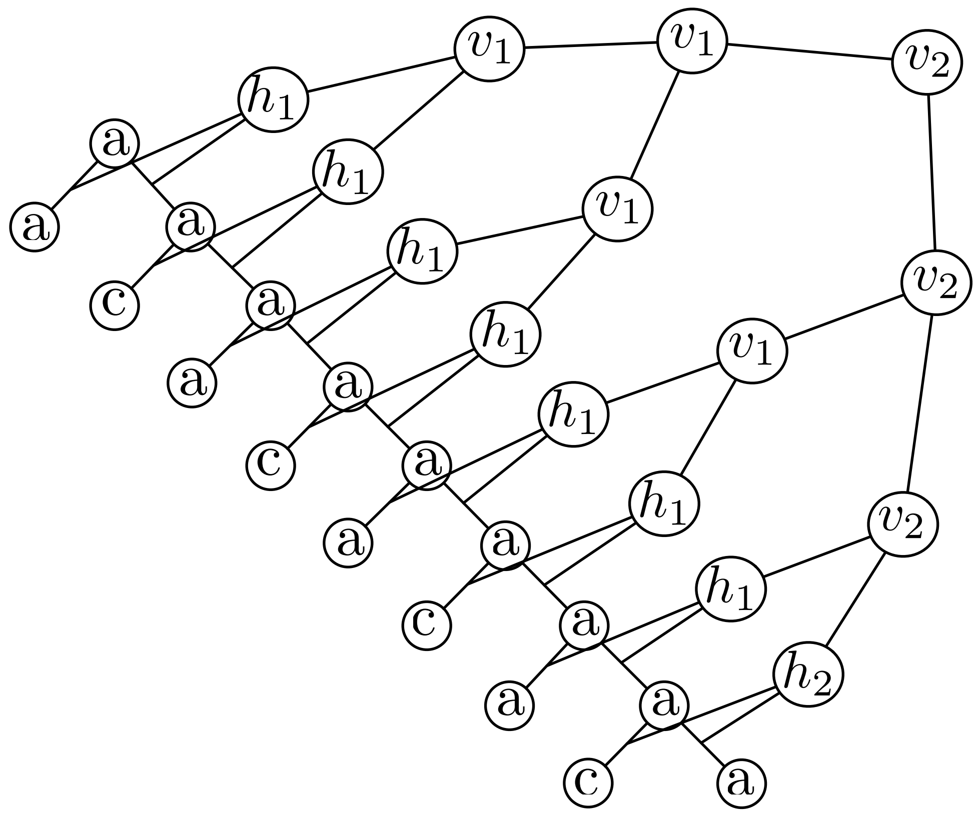
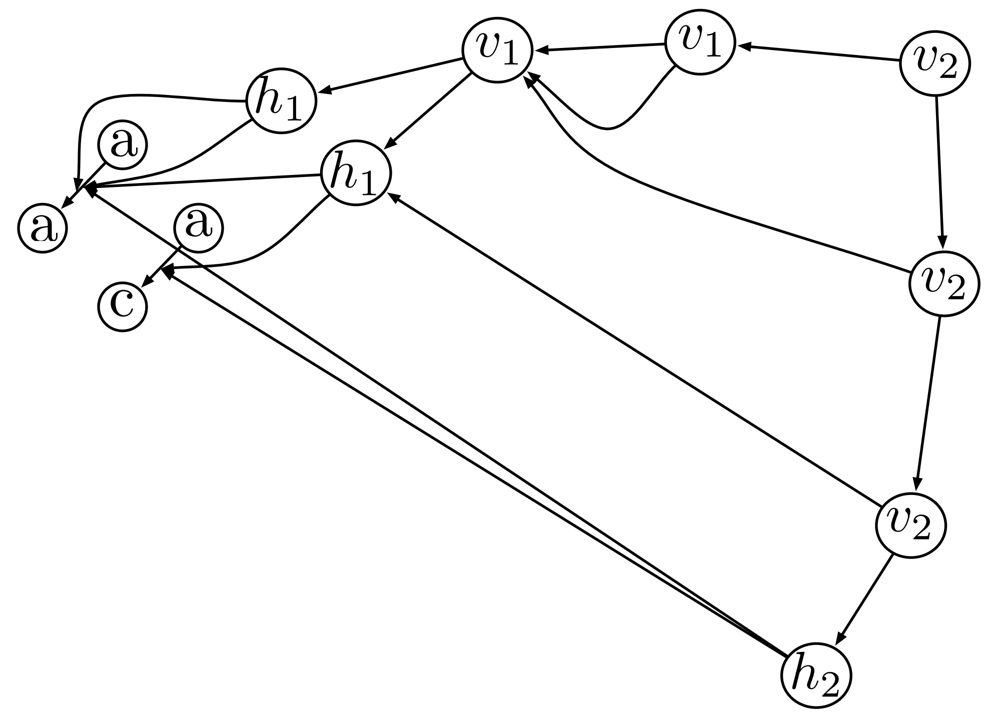
- Identical subtrees in top tree are merged in top DAG.
- => All clusters represented in top DAG are *distinct*.





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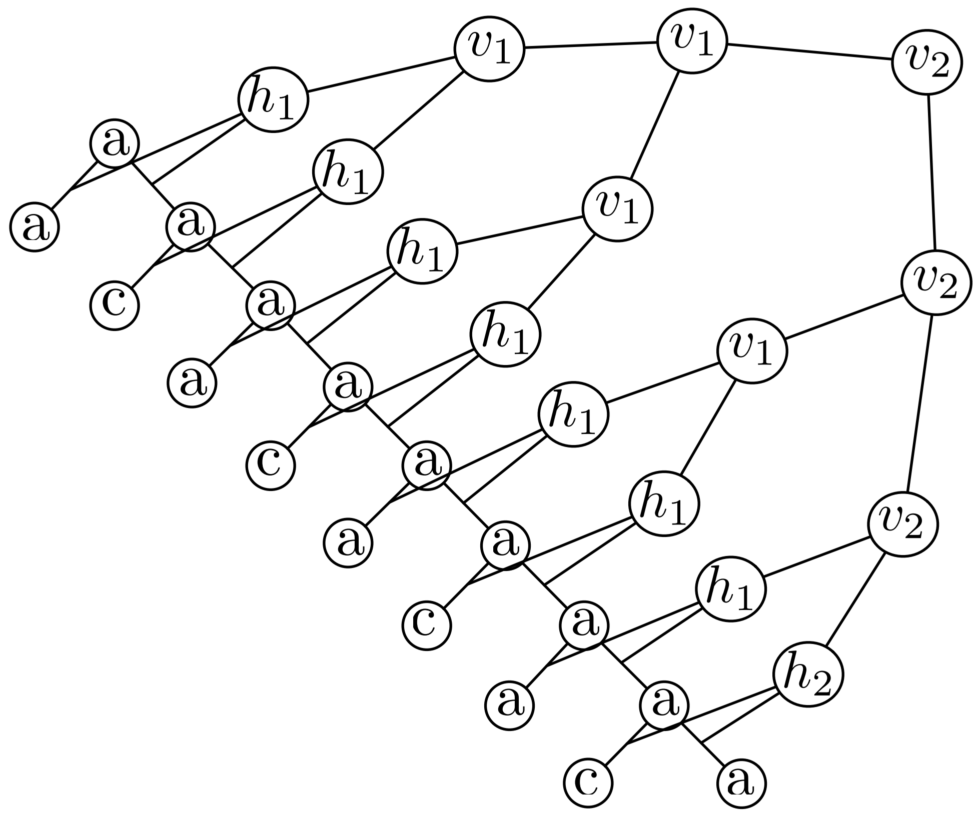
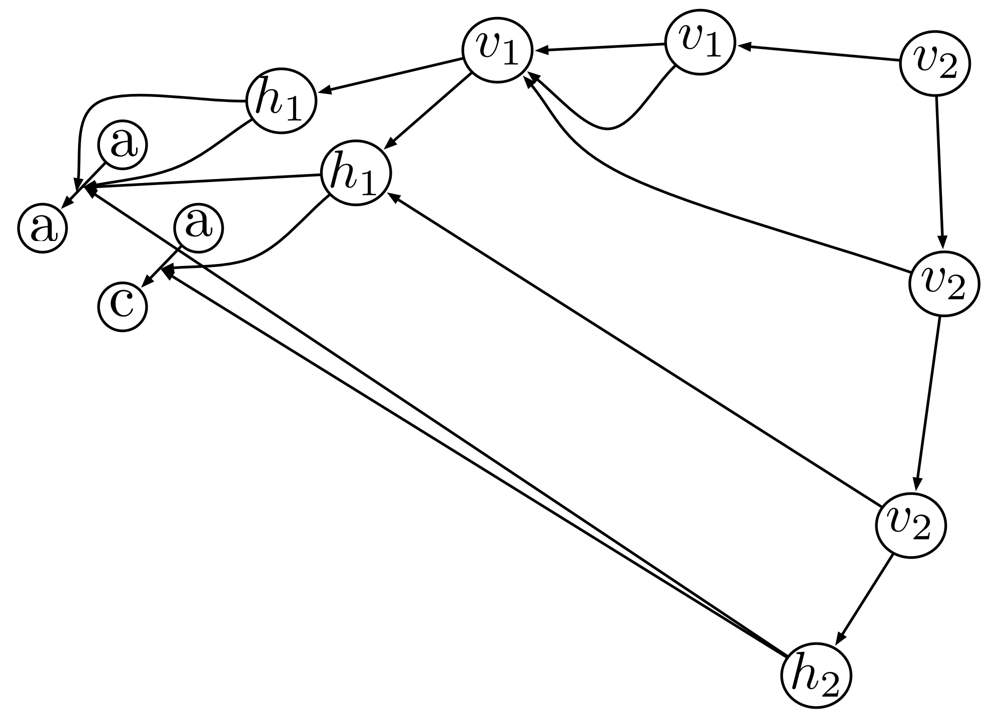
- Identical subtrees in top tree are merged in top DAG.
- => All clusters represented in top DAG are *distinct*.
- => Top tree contains at most $O(N/(\log_\sigma N)^{0.19})$ distinct clusters

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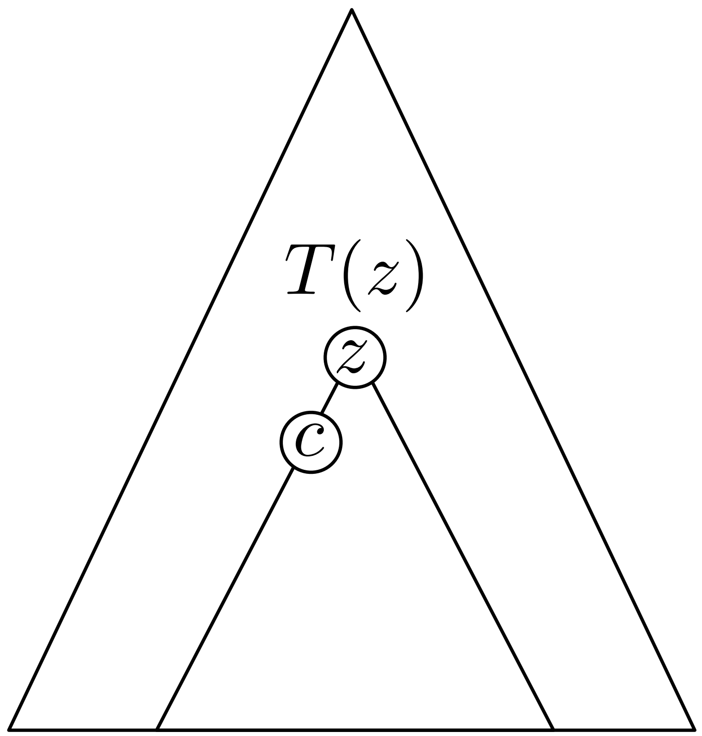
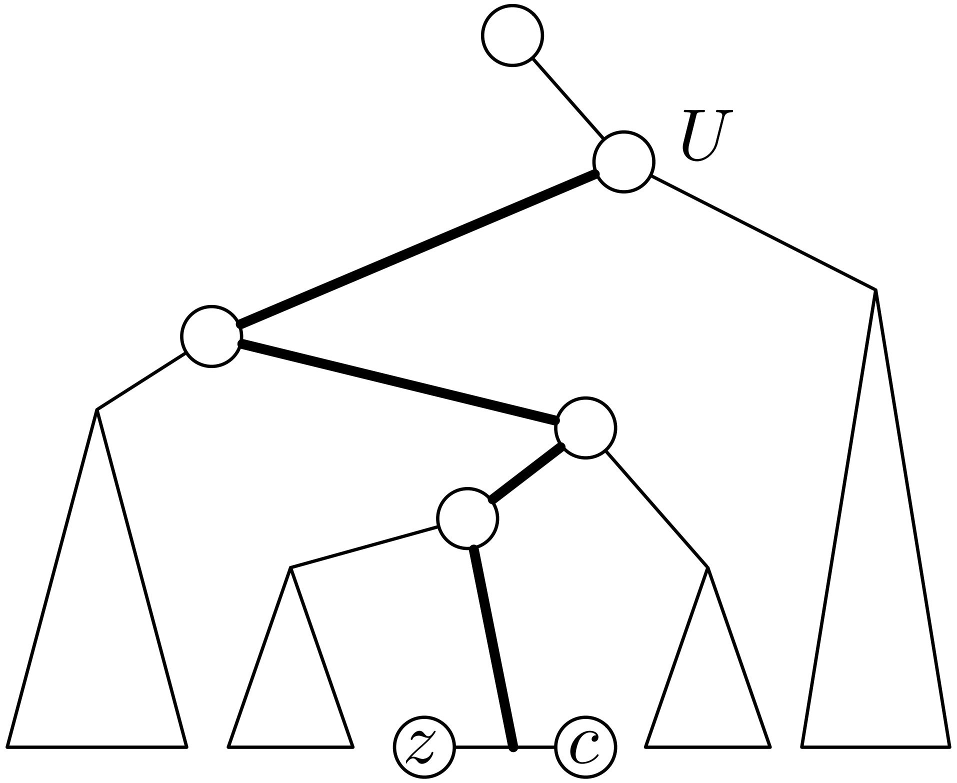
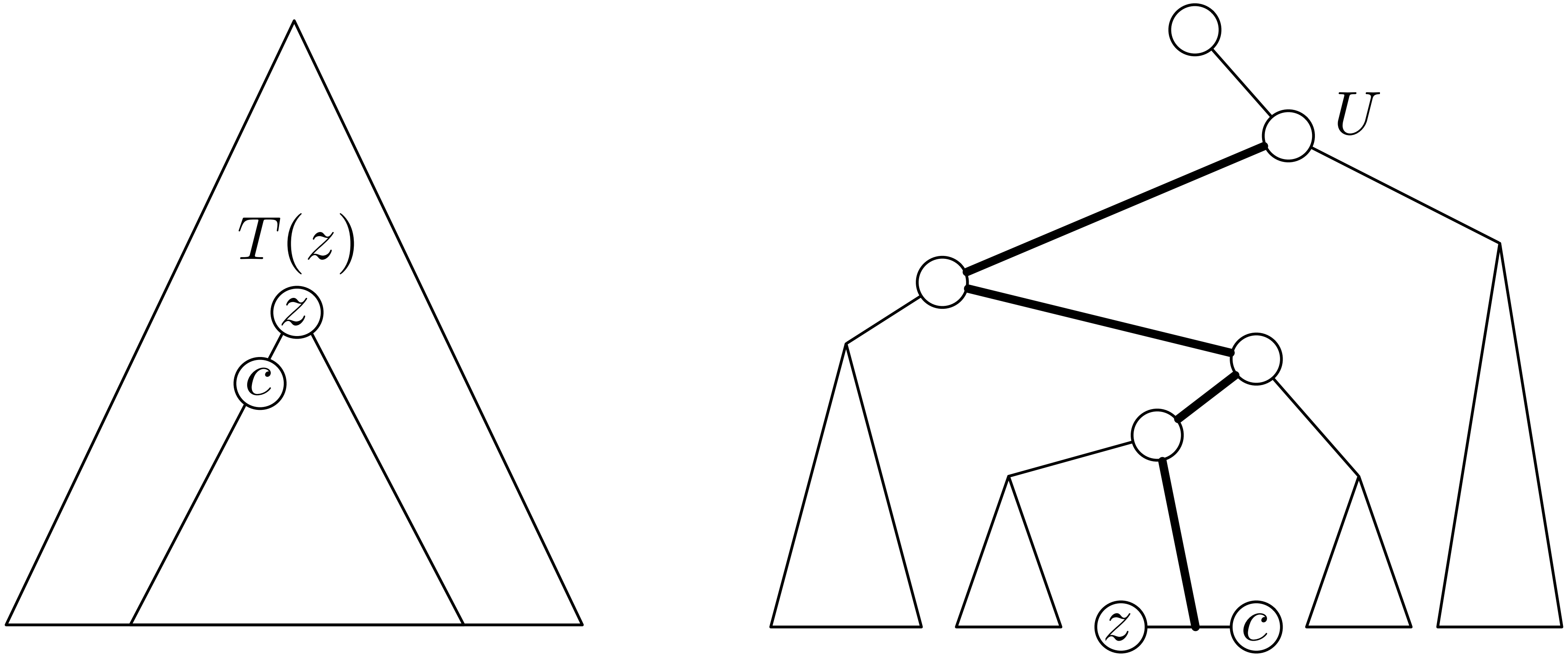
- => Theorem: Top DAG has size at most $O(N/(\log_\sigma N)^{0.19})$

DAG vs. Top DAG

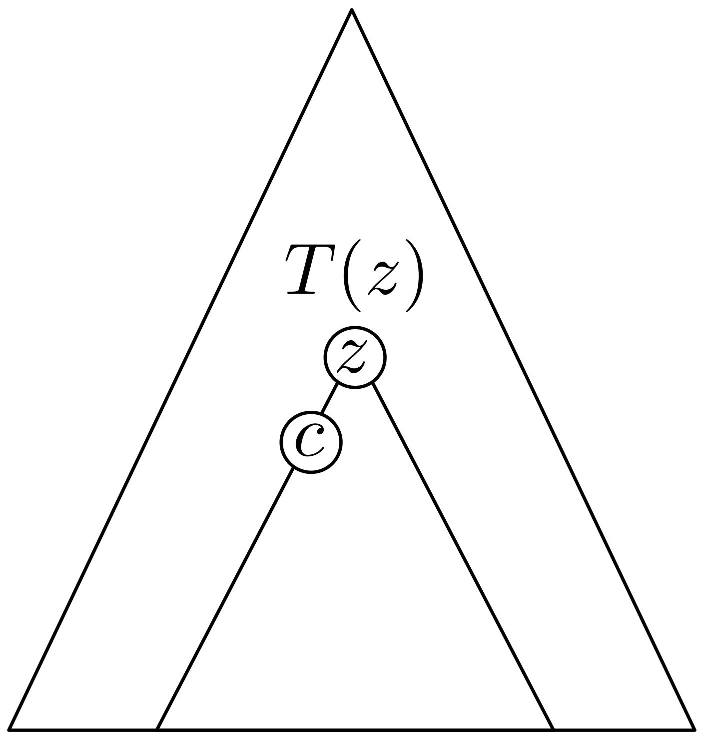
- How good is top DAG compression vs. DAG compression?

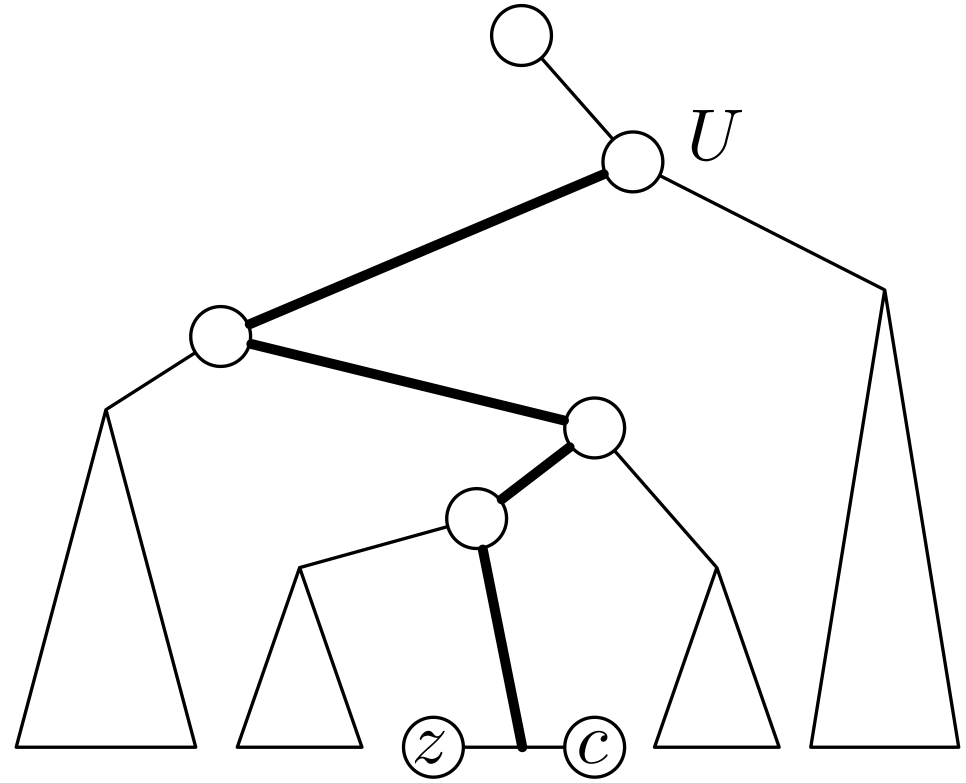

$T(z)$
z
c


U
z
c

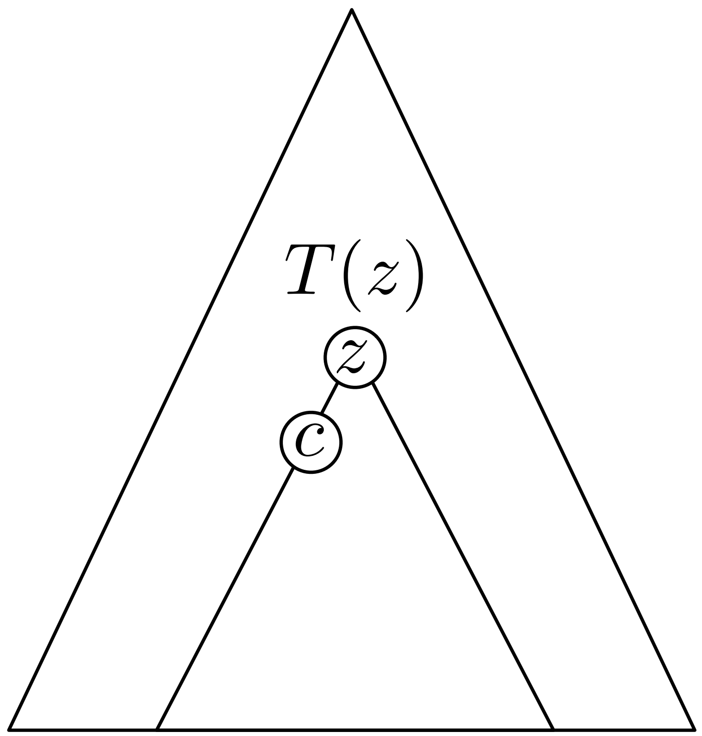


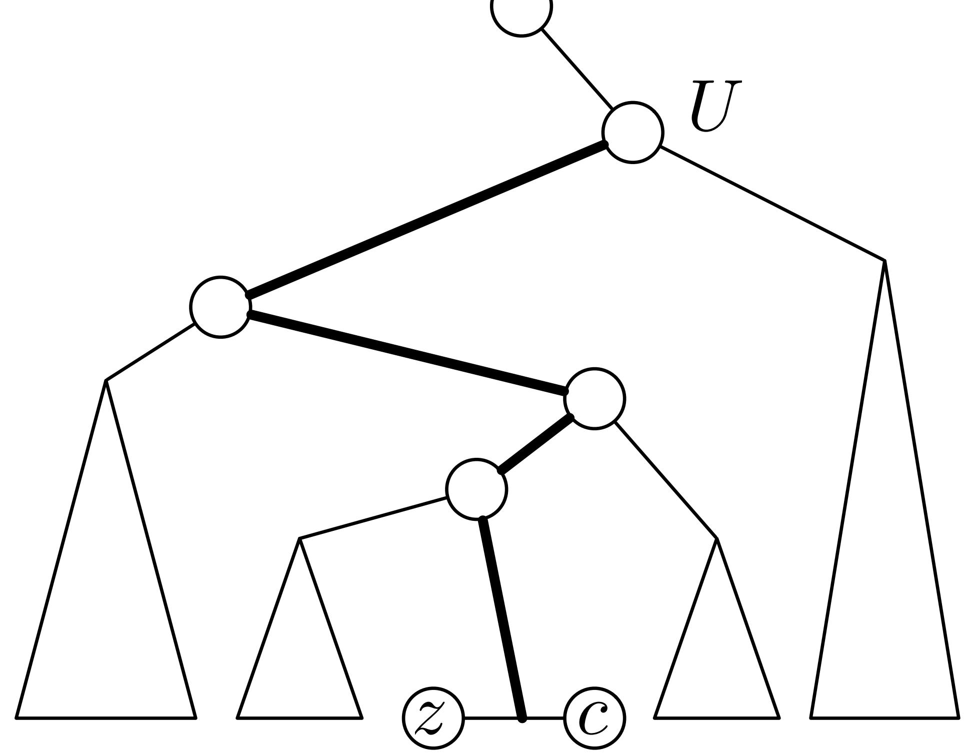
- Consider any subtree T(z) in T. Suppose z has left child c.





- Consider any subtree T(z) in T. Suppose z has left child c.
- T(z) is represented by a set S of O(log N) clusters in top tree (a subset of *off-path* clusters from the smallest cluster U containing T(z) to (z,c)).





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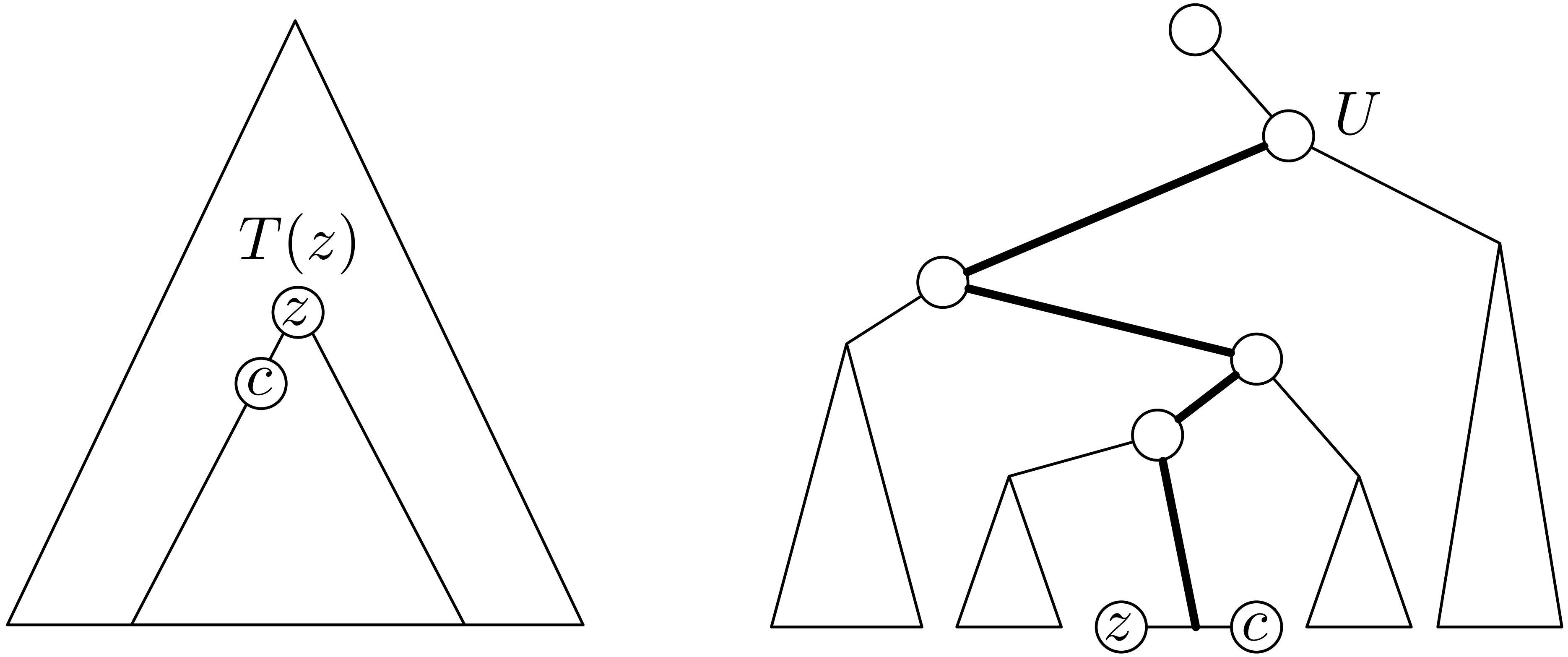
- T(z) is represented by a set S of O(log N) clusters in top tree (a subset of *off-path* clusters from the smallest cluster U containing T(z) to (z,c)).
- Let T(z') be a subtree repeat of T(z). Then T(z') is represented by a set S' of clusters in top tree identical to S.



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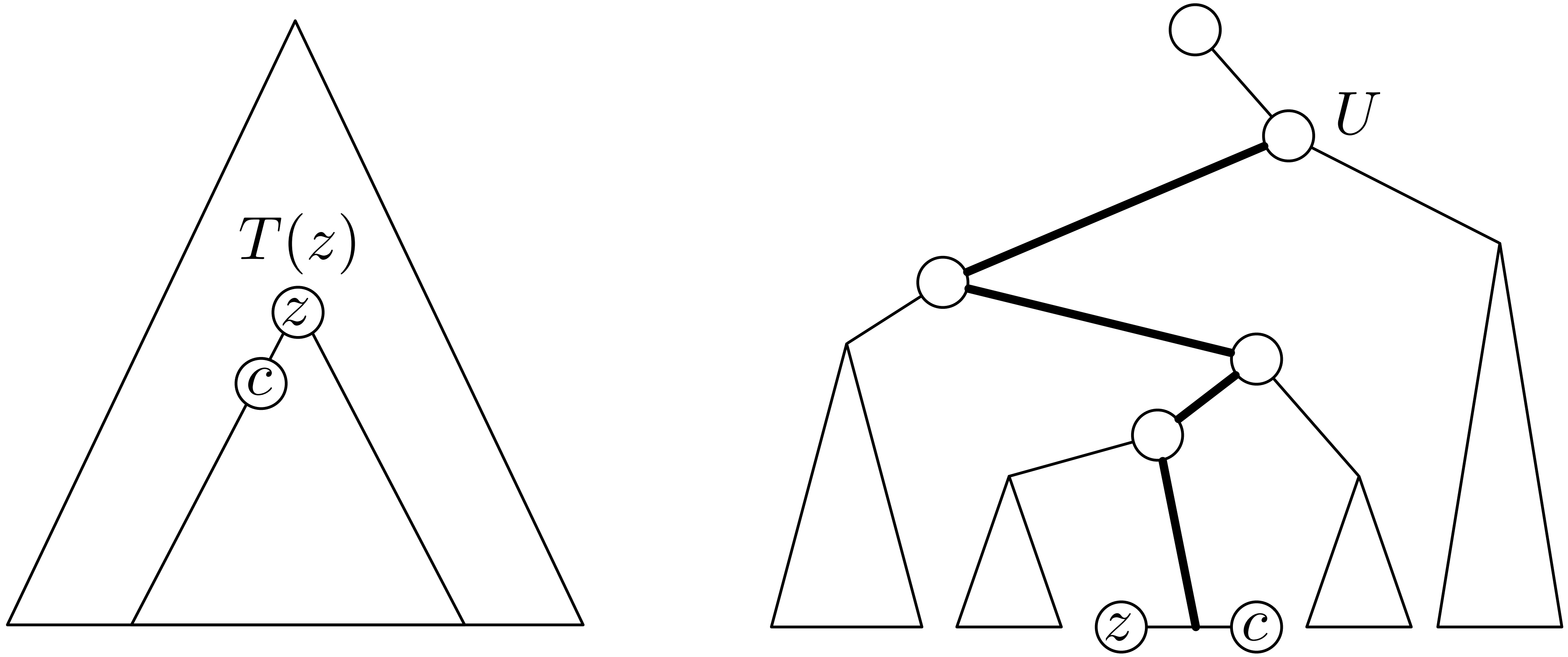
- In DAG T(z') is compressed to a single edge. In top DAG T(z') each of the O(log N) clusters in S' are compressed to an edge.



- Consider any subtree T(z) in T. Suppose z has left child c.
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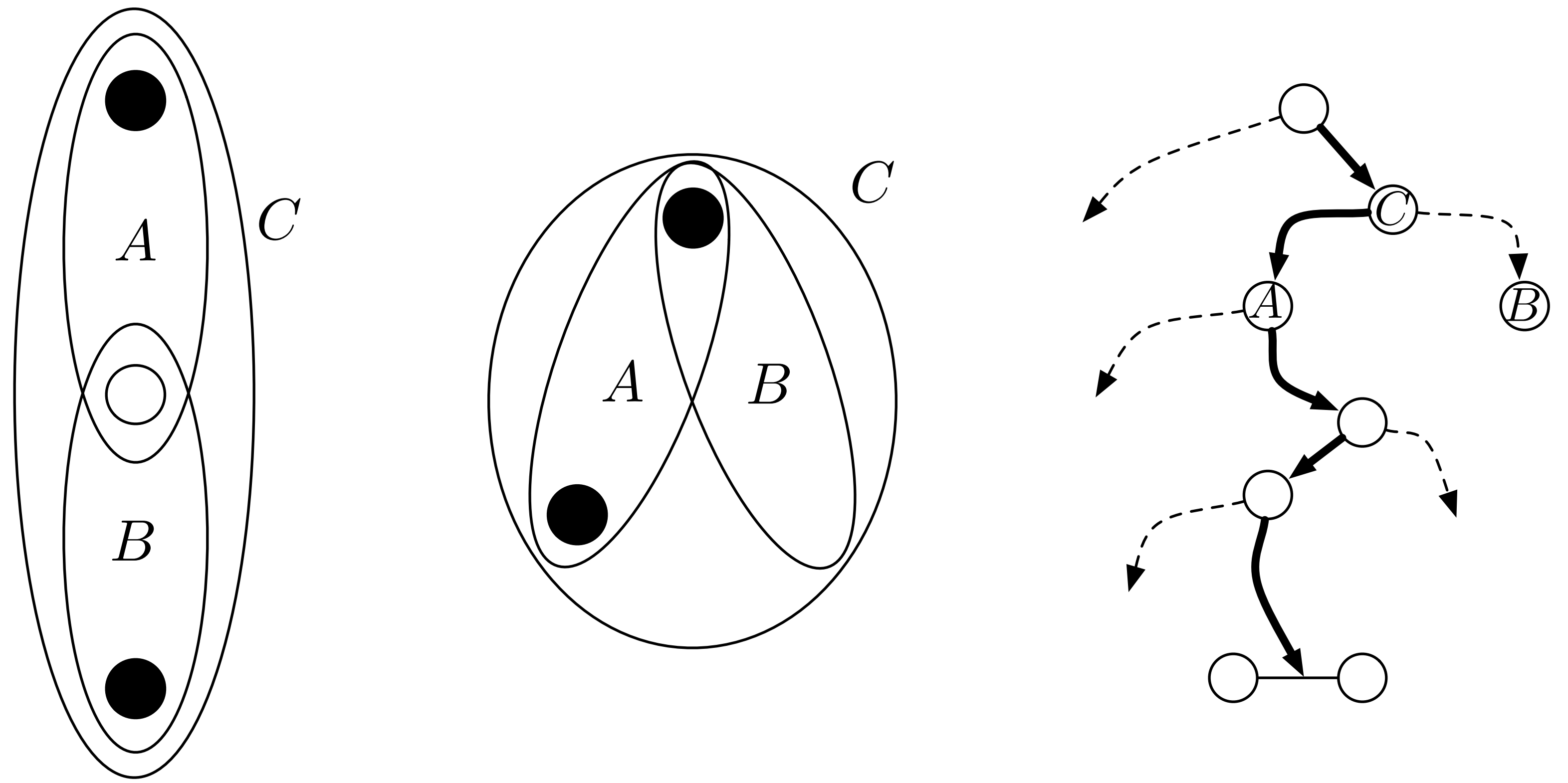
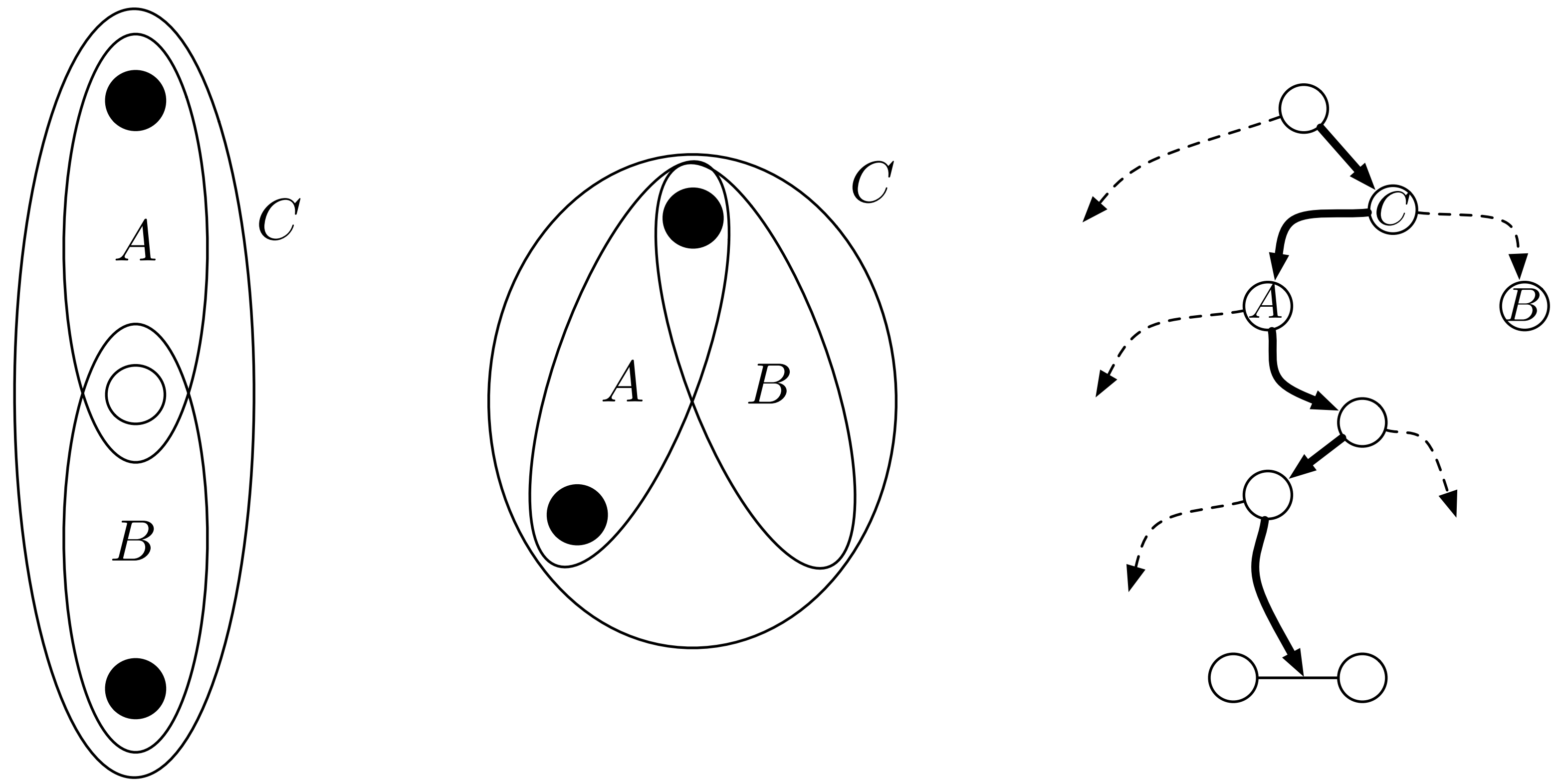
- Let T(z’) be a subtree repeat of T(z). Then T(z’) is represented by a set S’ of clusters in top tree identical to S.
- In DAG T(z’) is compressed to a single edge. In top DAG T(z’) each of the O(log N) clusters in S’ are compressed to an edge.
- => Theorem: The top DAG has size O(D・log N)

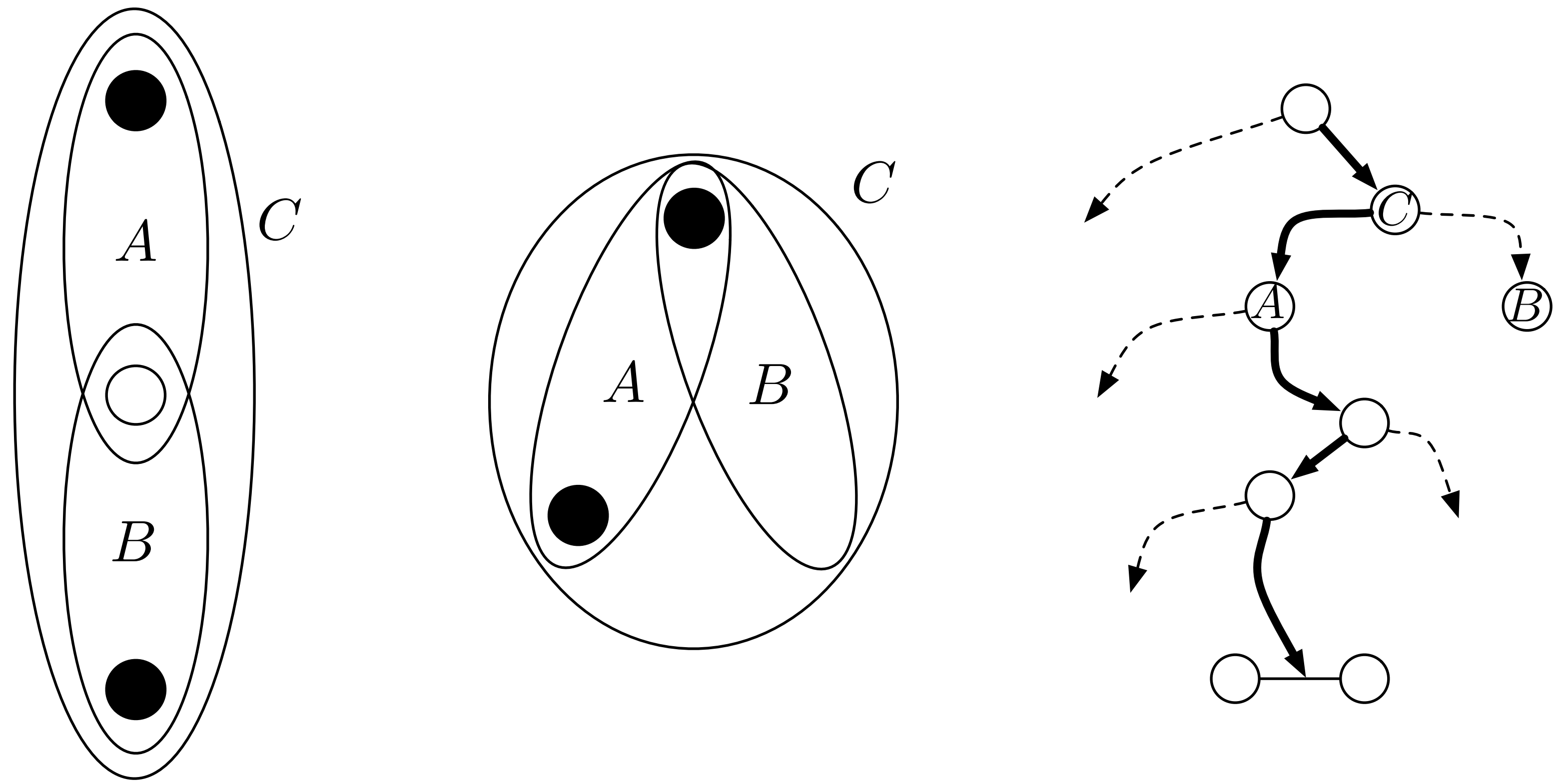
Compressed Navigation

- Identify nodes in T by preorder number.
- Theorem: Using O(n) space we can support the following operations in O(log N) time:
    - Access(x): Return the label associated with node x.
    - Decompress(x): Return the tree T(x).
    - Parent(x): Return the parent of node x.
    - Depth(x): Return the depth of node x.
    - Height(x): Return the height of node x.
    - Size(x): Return the number of nodes in T(x).
    - Firstchild(x): Return the first child of x.
    - NextSibling(x): Return the sibling immediately to the right of x.
    - LevelAncestor(x, i): Return the ancestor of x whose distance from x is i.
    - NCA(x, y): Return the nearest common ancestor of the nodes x and y

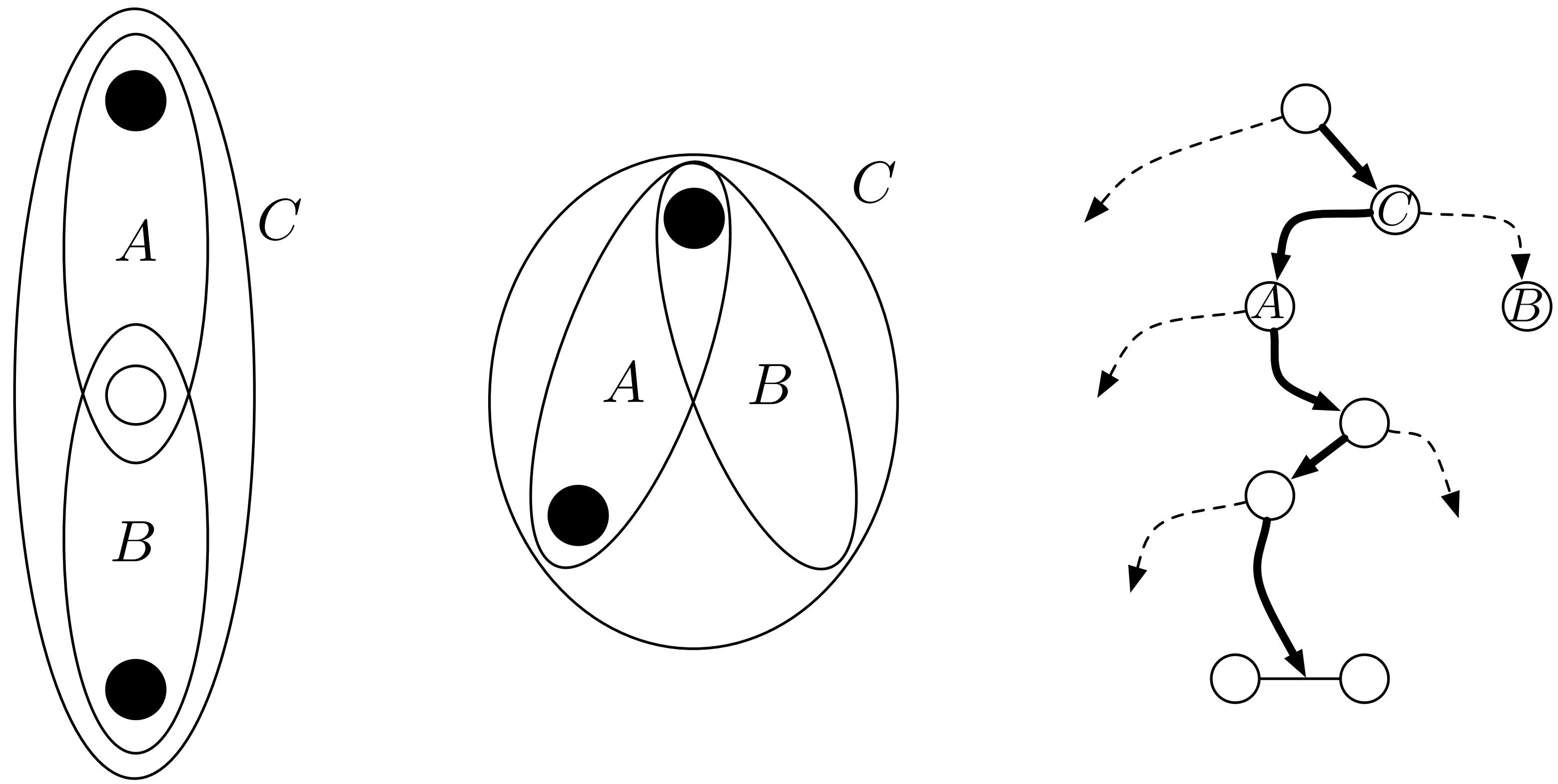

A
B
C
A
B
C
C
A
B



- Store O(1) space information in each node of top DAG (type of merge, height of cluster, size of cluster, ...)



- Store O(1) space information in each node of top DAG (type of merge, height of cluster, size of cluster, ...)
- Implement operations with top down and bottom up recursive searches in top DAG.



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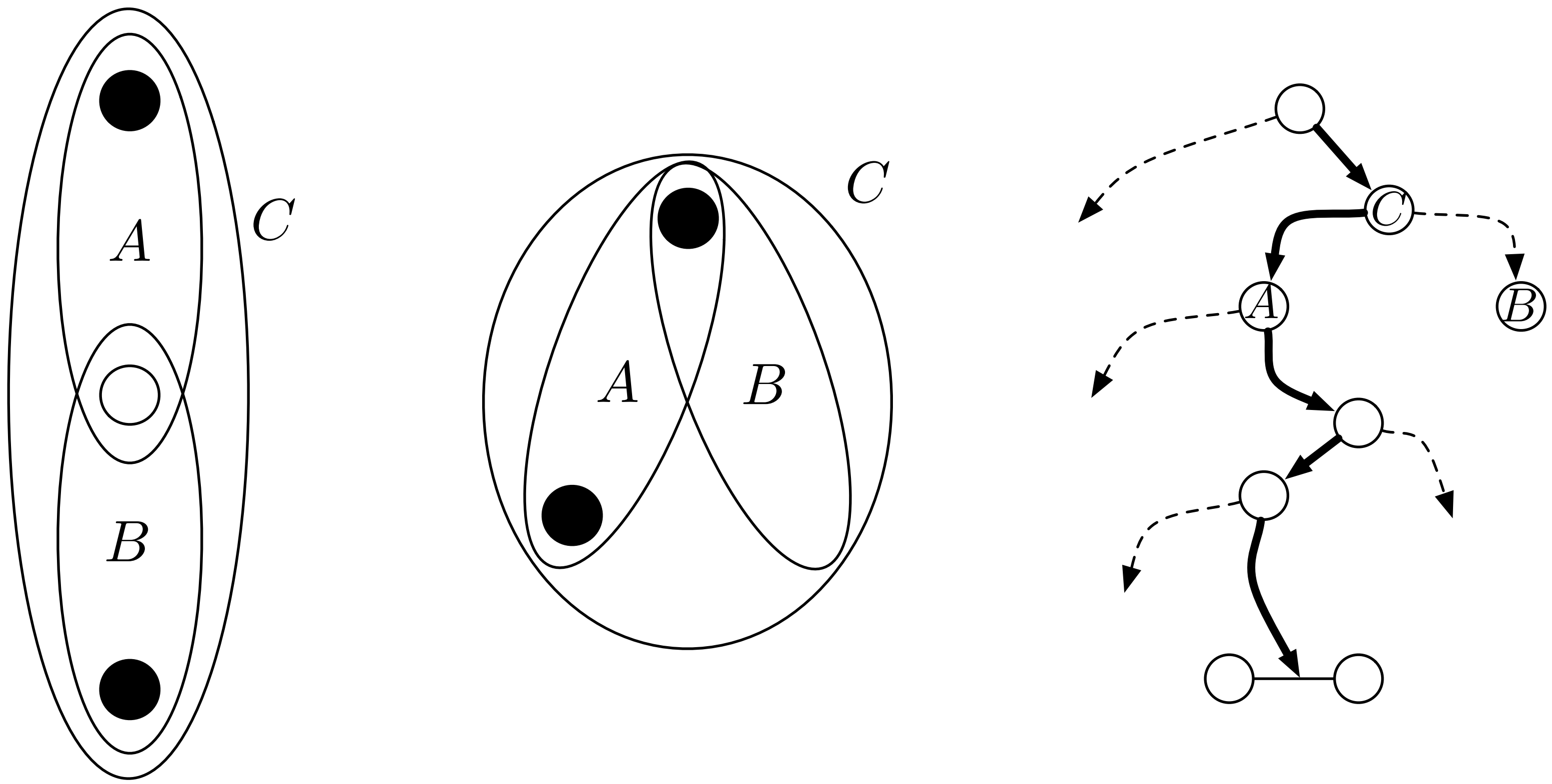
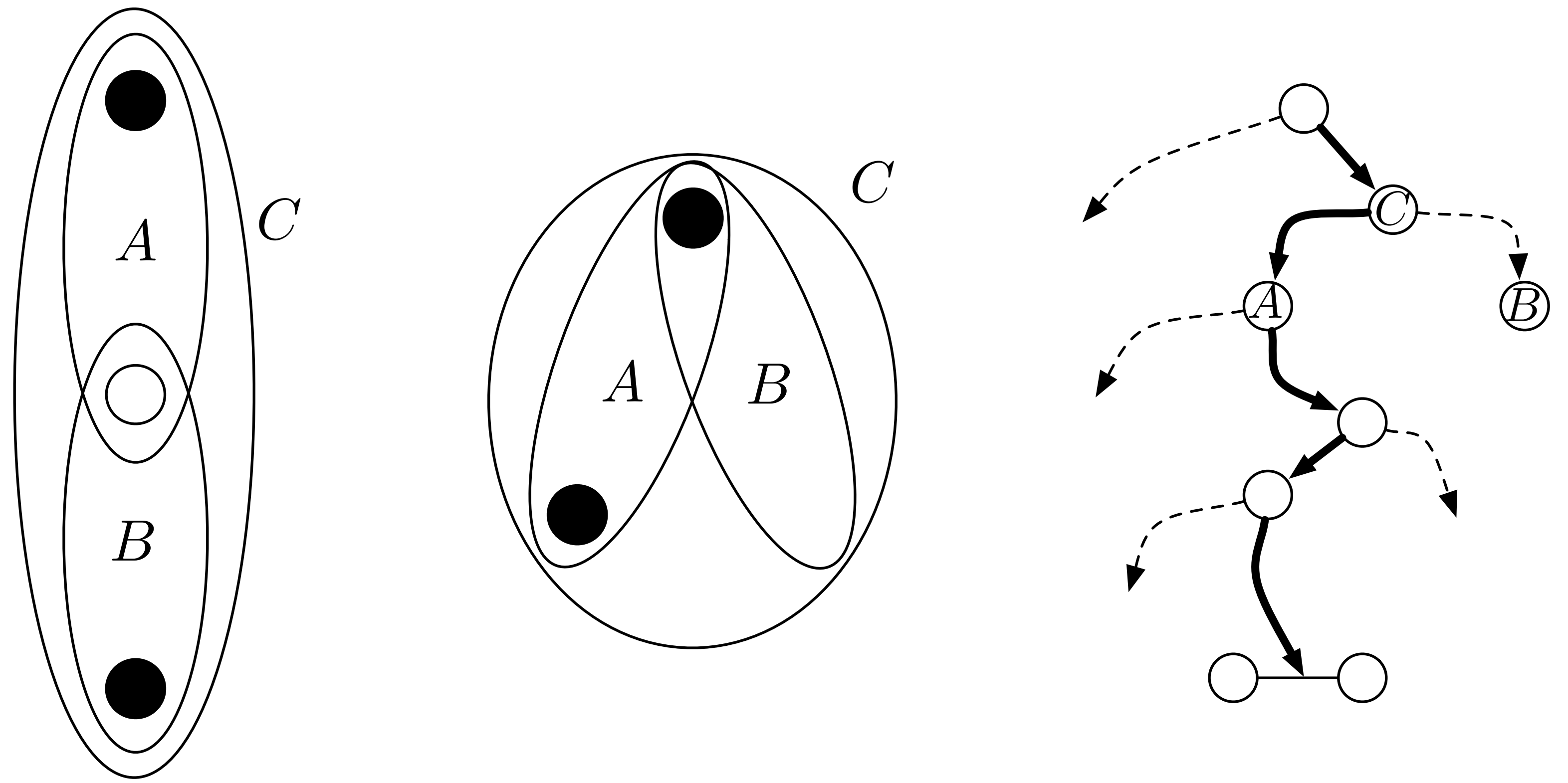
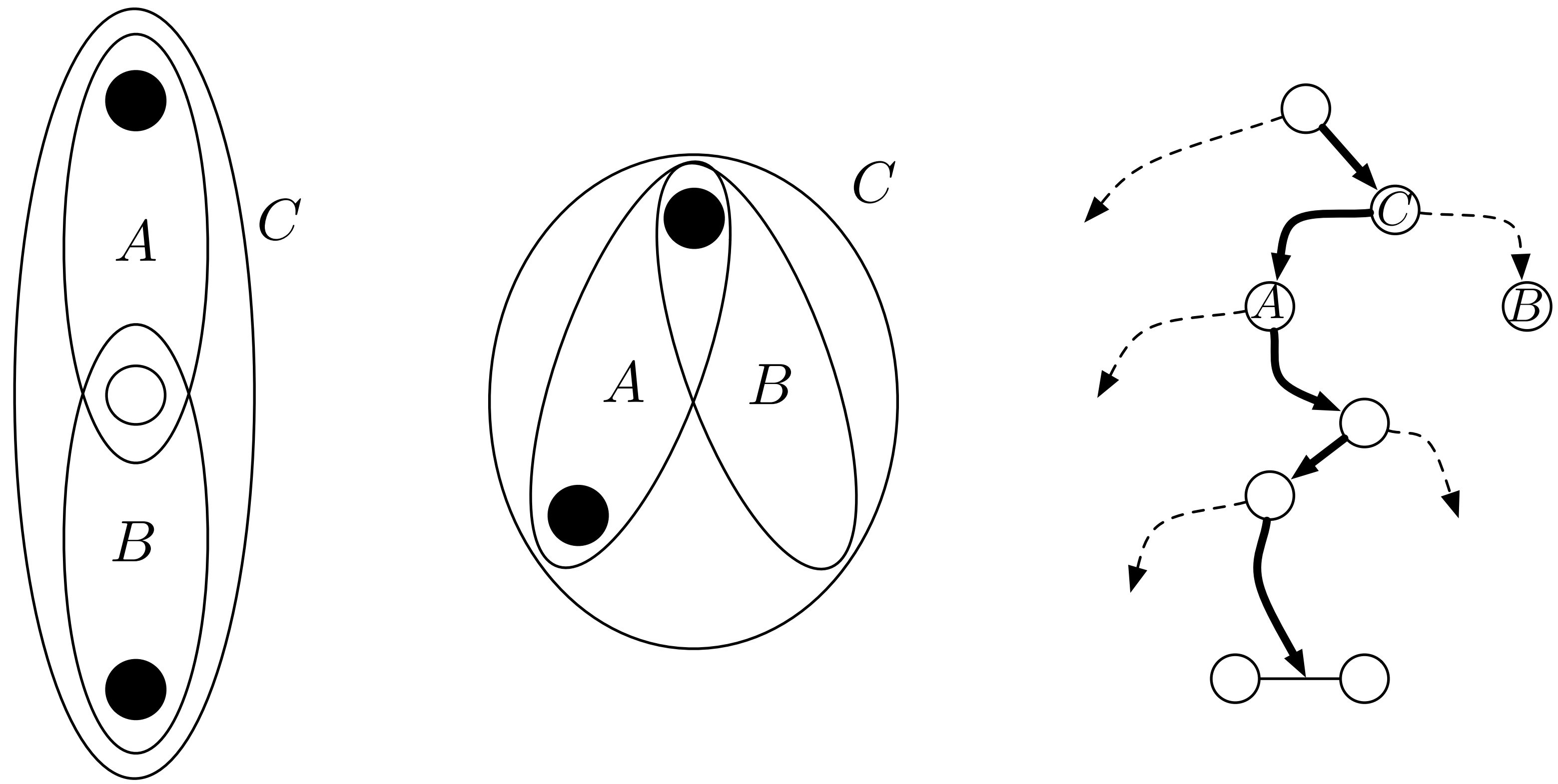
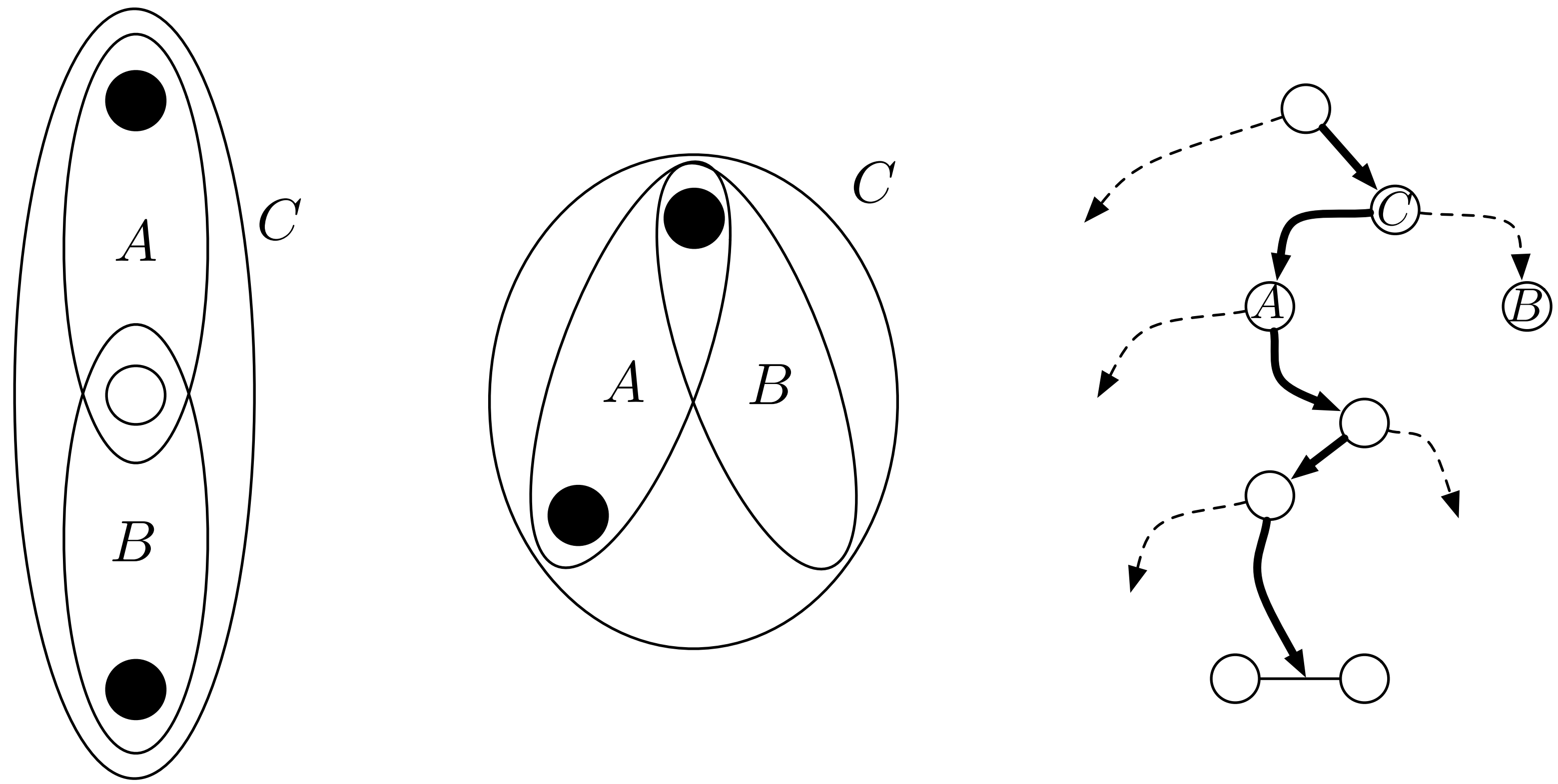


- Store O(1) space information in each node of top DAG (type of merge, height of cluster, size of cluster, ...)
- Implement operations with top down and bottom up recursive searches in top DAG.
- Identify nodes by maintaining *local preorder number* during searches.
- Use constant time in each node => O(log N) time for operation.

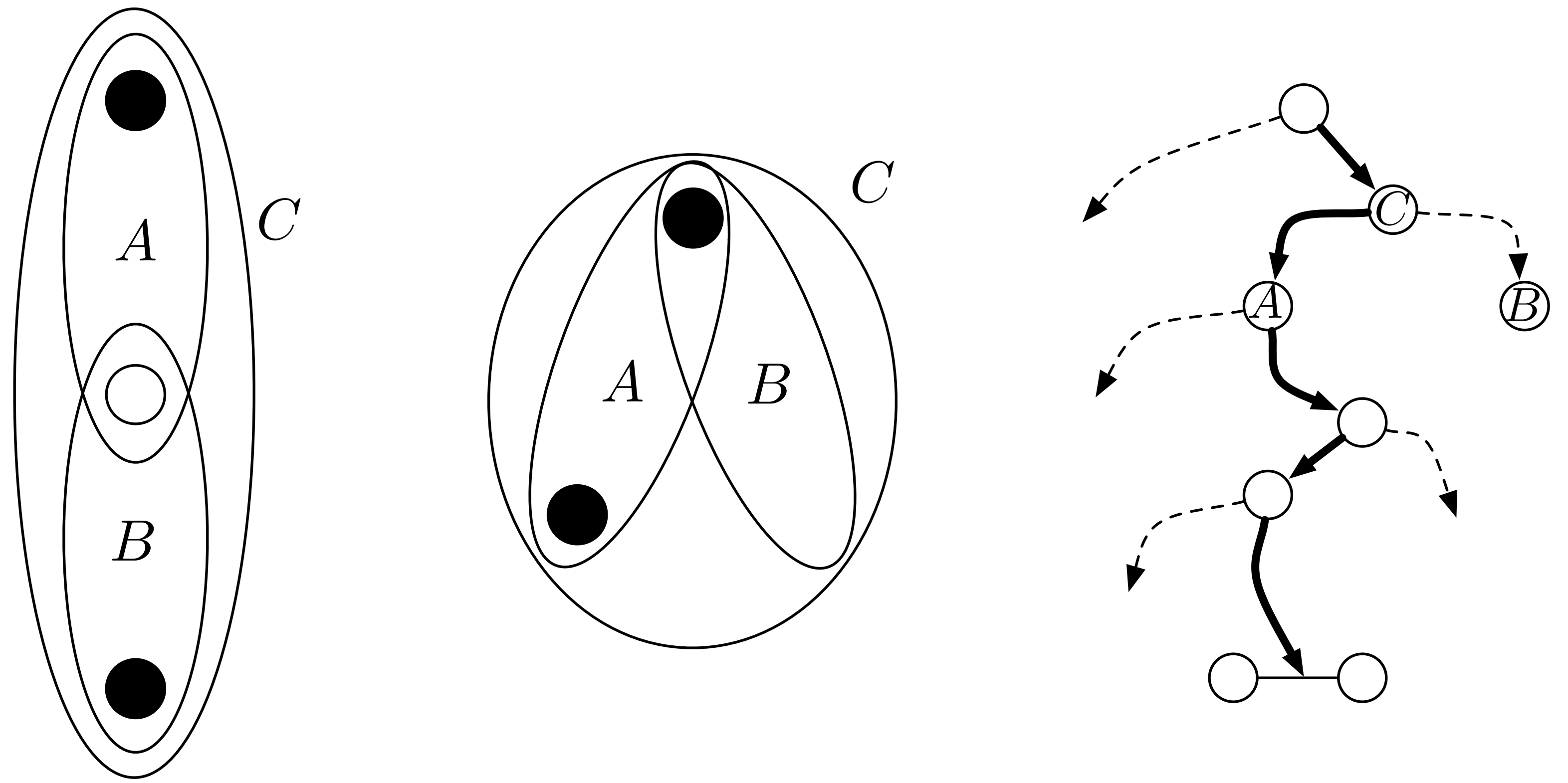

A
B
C
A
B
C
C
A
B



- NCA(x,y):



- NCA(x,y):
    - Top down search for x and y to find smallest cluster C containing x and y.



- NCA(x,y):
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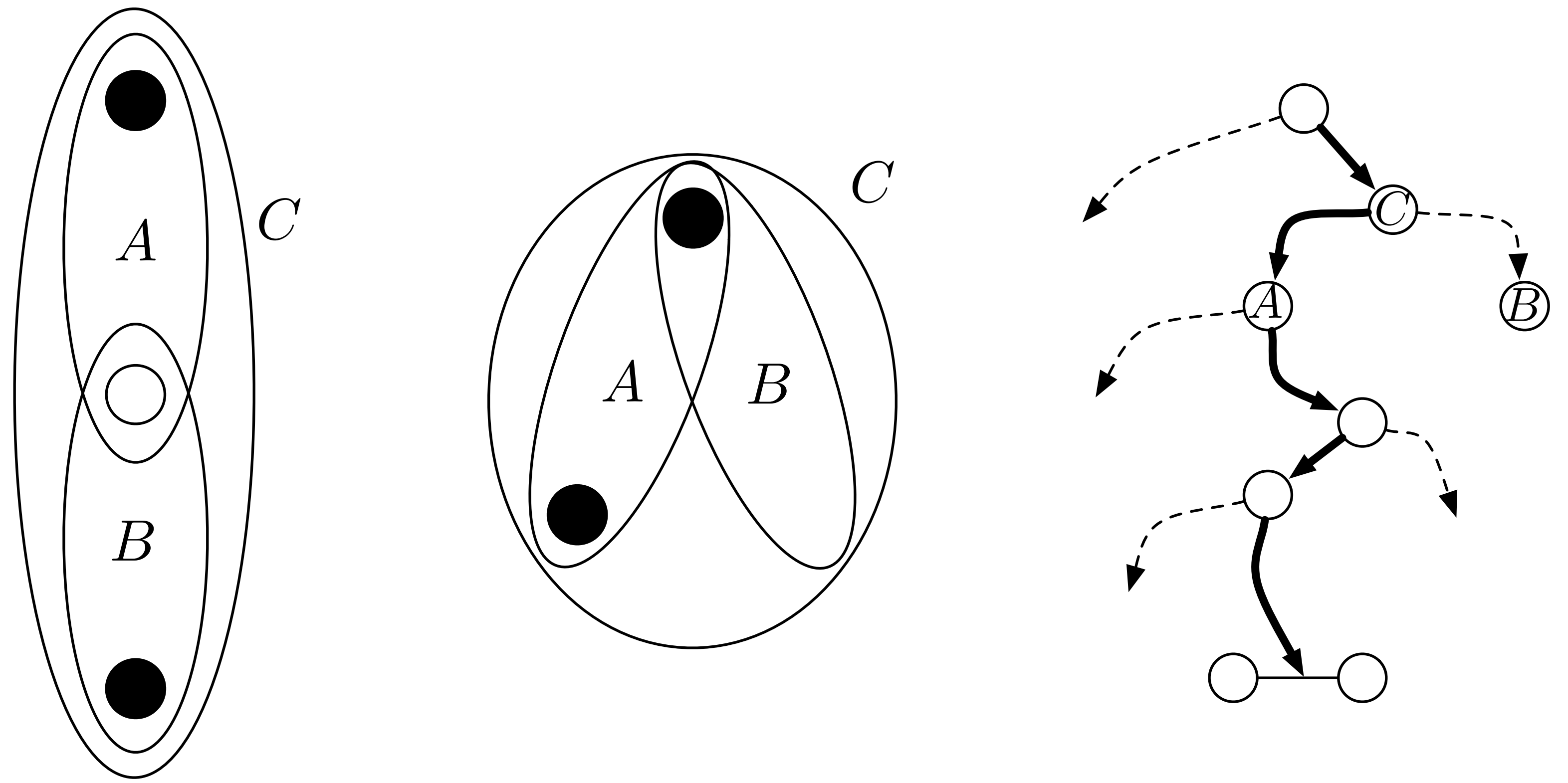
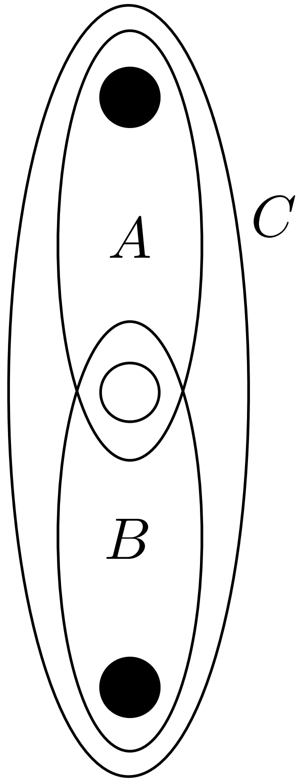
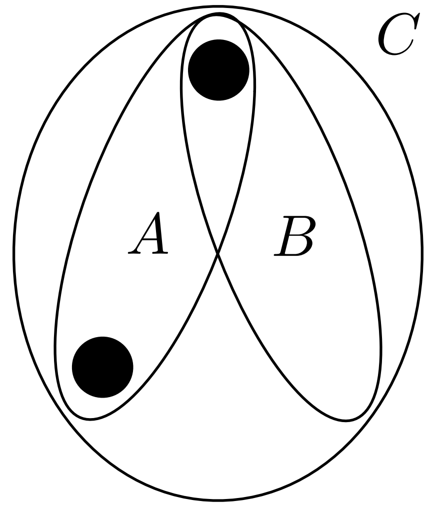
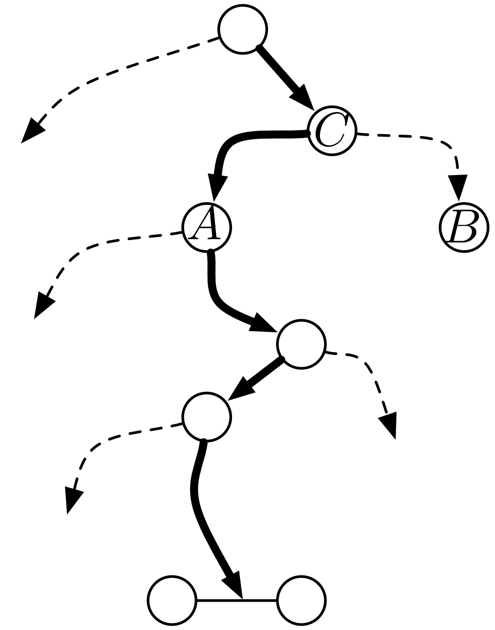
  - Retrieve local preorder number for NCA(x,y) in C.

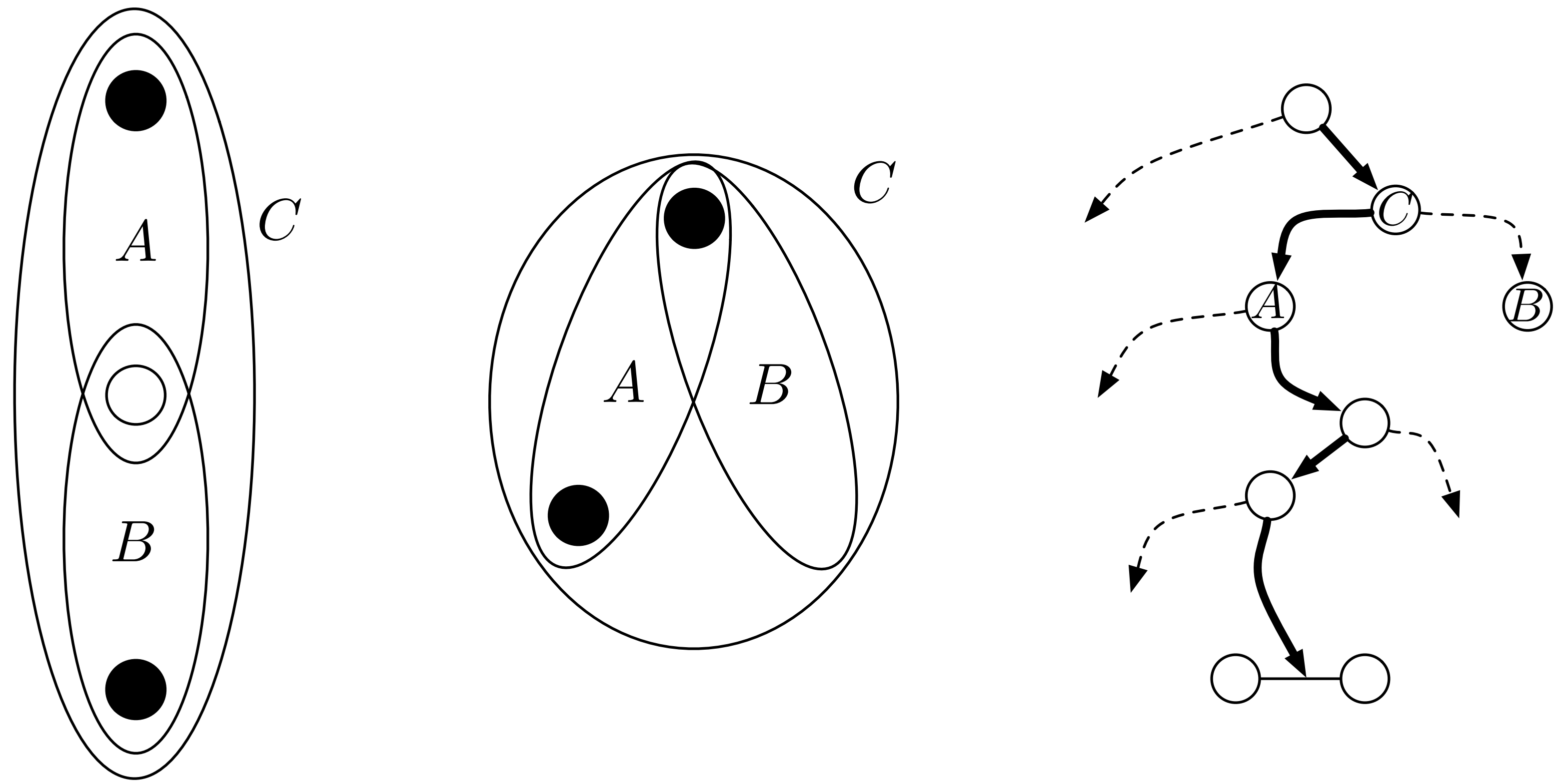


- NCA(x,y):
  - Top down search for x and y to find smallest cluster C containing x and y.
  - Retrieve local preorder number for NCA(x,y) in C.
  - Bottom up search to map local preorder number to global preorder number in T.

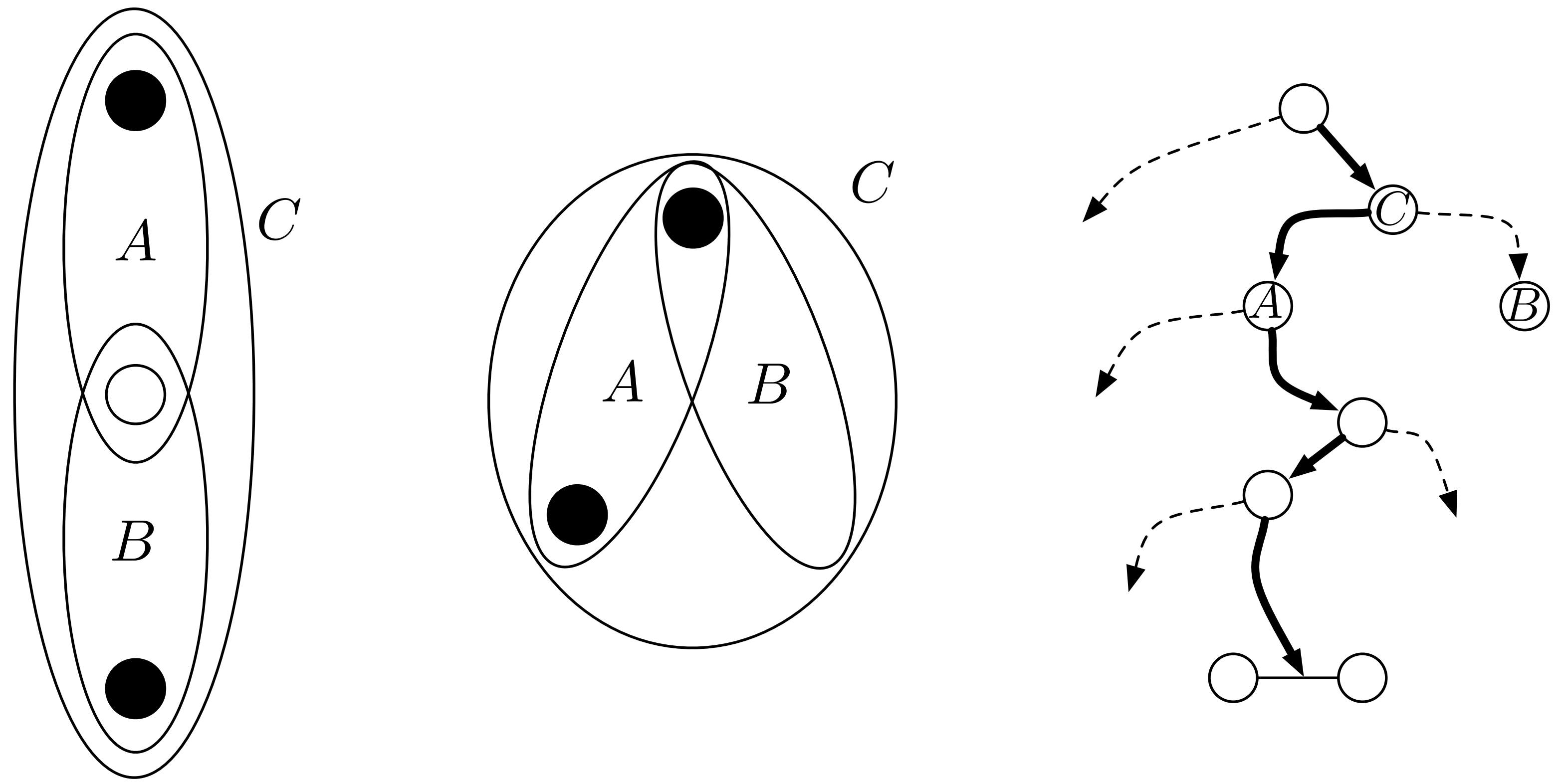

A
B
C


C
A
B

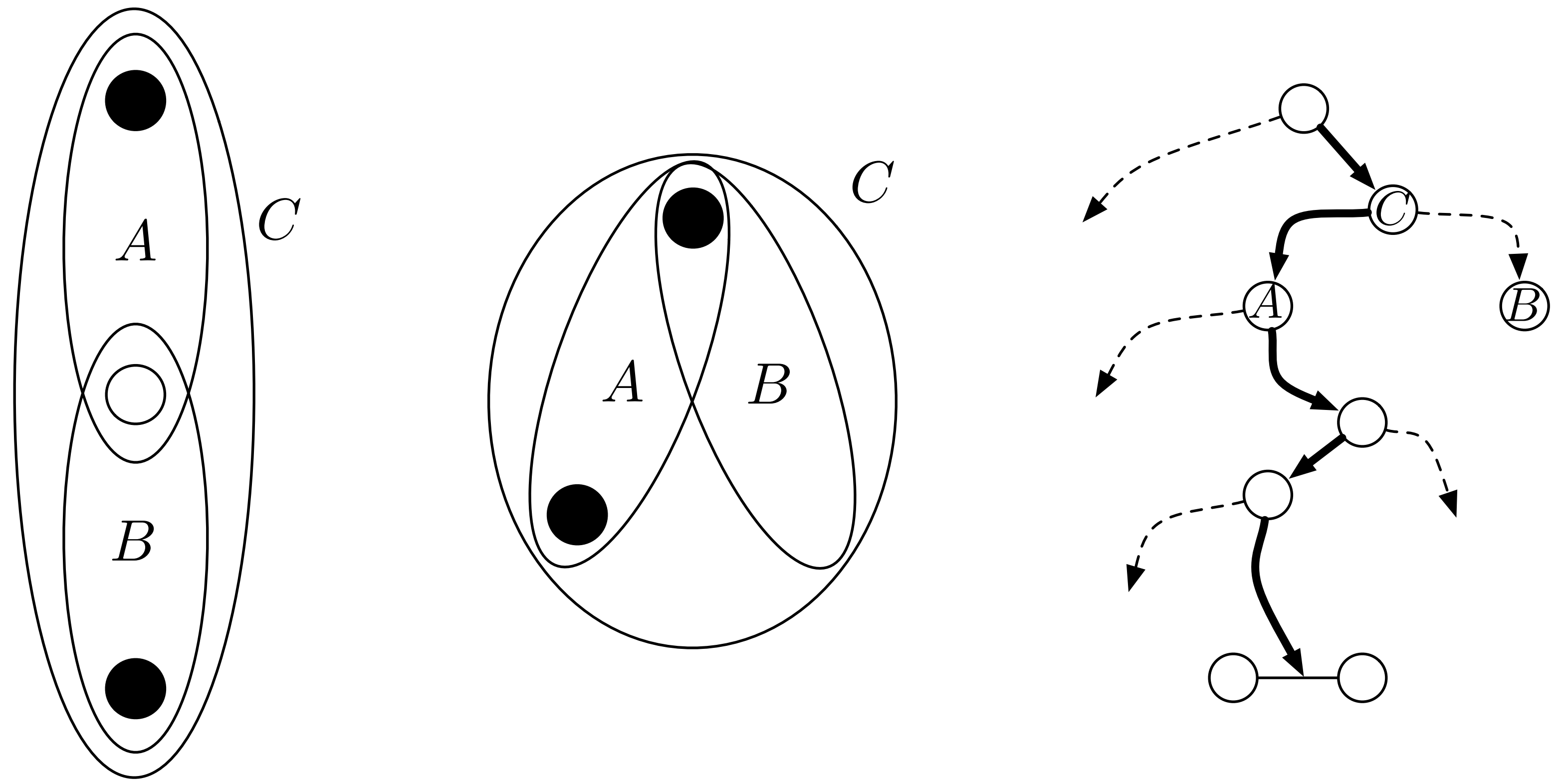

C
A
B



- Size(x)



- Size(x)
    - Top down search for x to find set of off-path cluster representing T(x).



- Size(x)
    - Top down search for x to find set of off-path cluster representing T(x).
    - Return sum of sizes of these cluster.

Summary and Open Problems

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- Top tree compression
  - DAG compression of top tree
  - Compression ratio at least $(\log_\sigma N)^{0.19}$ and never more than a log N factor larger than DAG compression
  - Navigation in O(log N) time.

Summary and Open Problems

- Top tree compression
  - DAG compression of top tree
  - Compression ratio at least $(\log_\sigma N)^{0.19}$ and never more than a log N factor larger than DAG compression
  - Navigation in O(log N) time.
- Open problems
  - Improve $(\log_\sigma N)^{0.19}$ worst case compression ratio for top DAG compression.
  - Compressed pattern matching for trees compressed with repetitions.
  - Practical implementations.