Data Streams, Dyck Languages, and *Detecting Dubious Data Structures*

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Graham Cormode *AT&T Research Labs*
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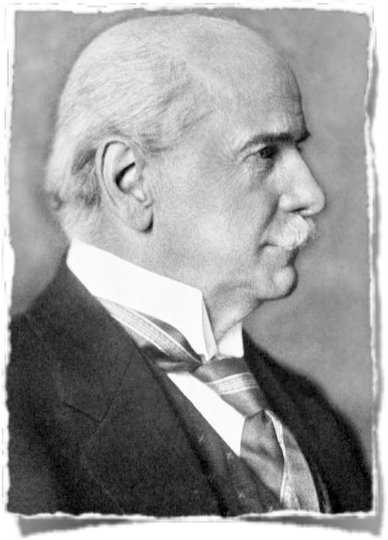
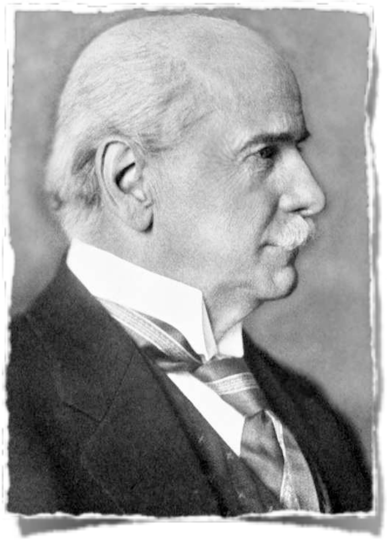
Andrew McGregor *University of Massachusetts, Amherst*





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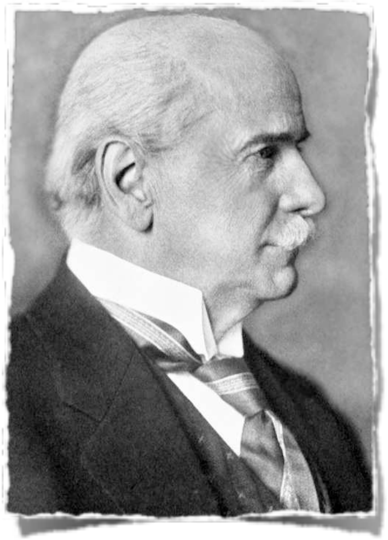

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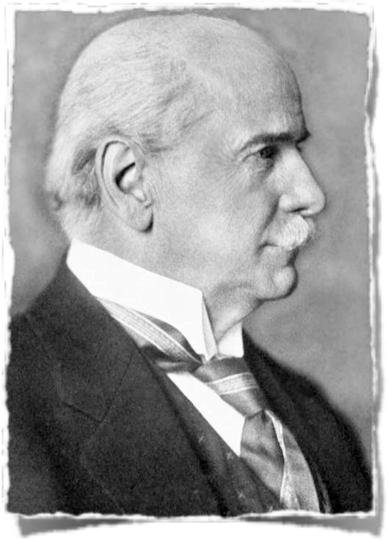

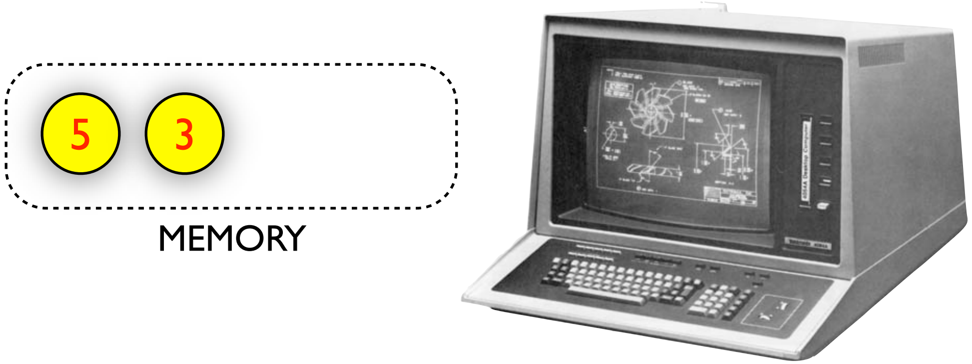

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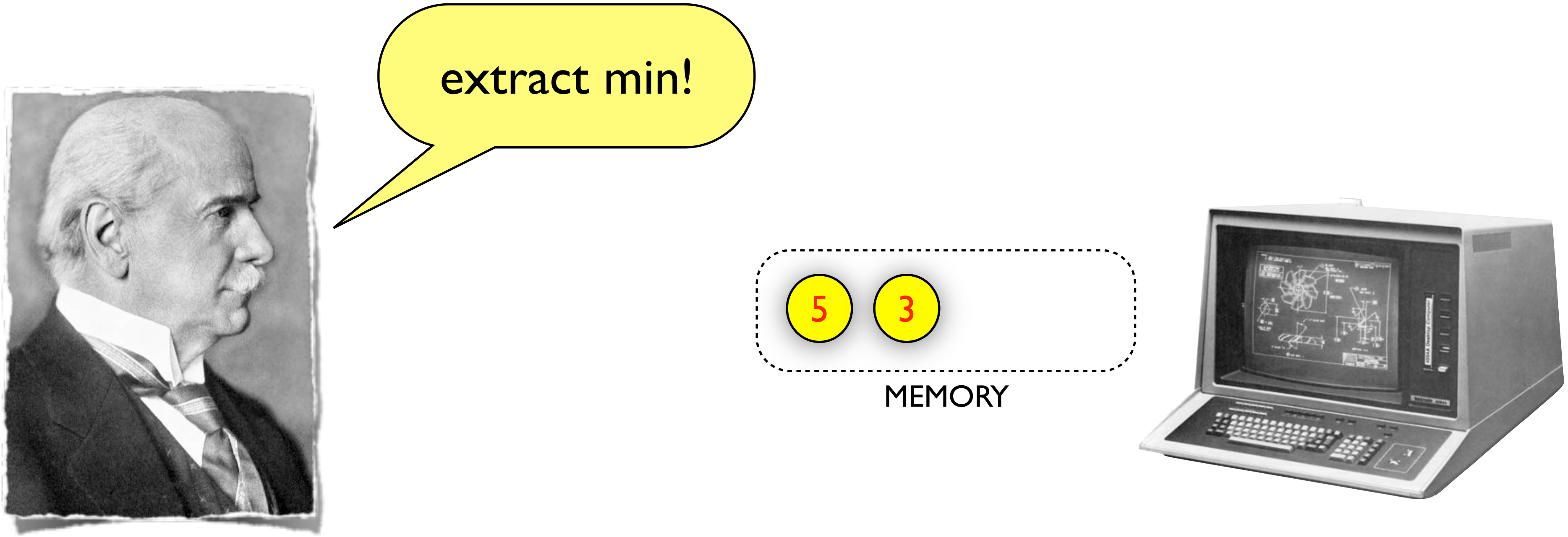

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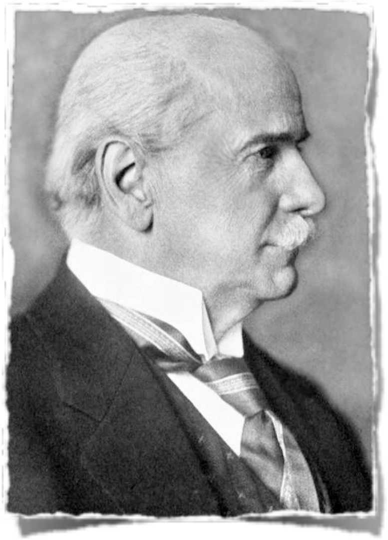

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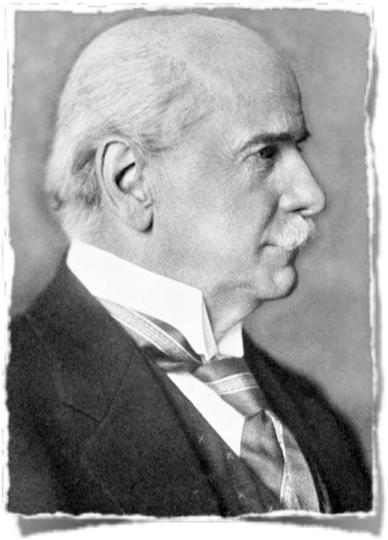

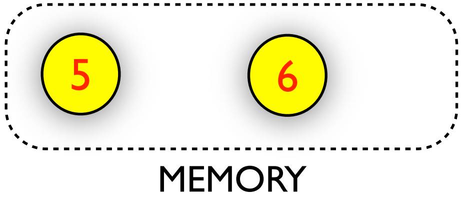

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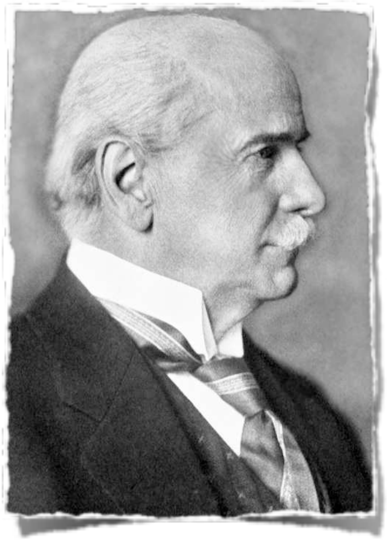

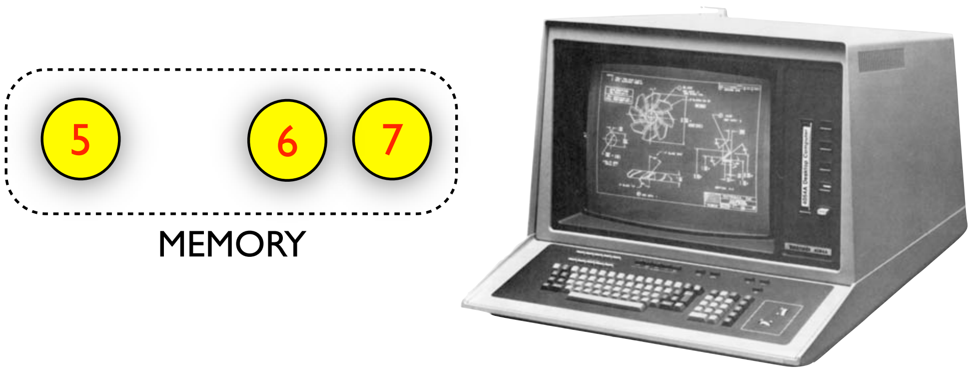

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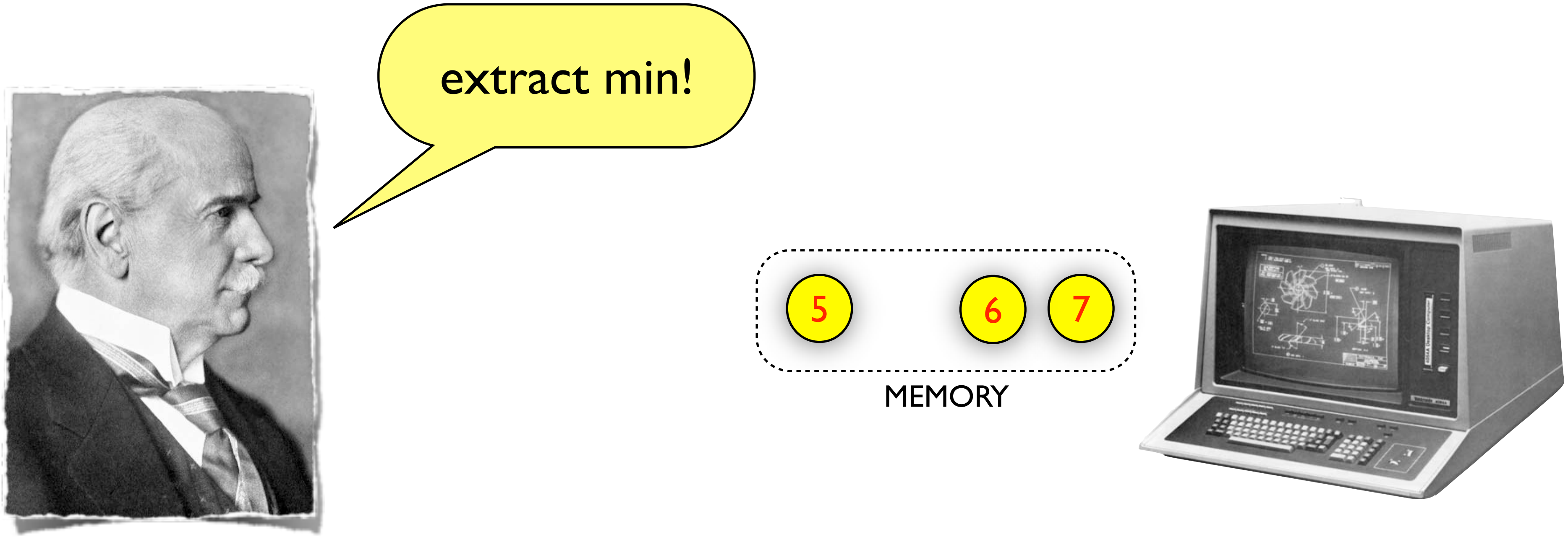

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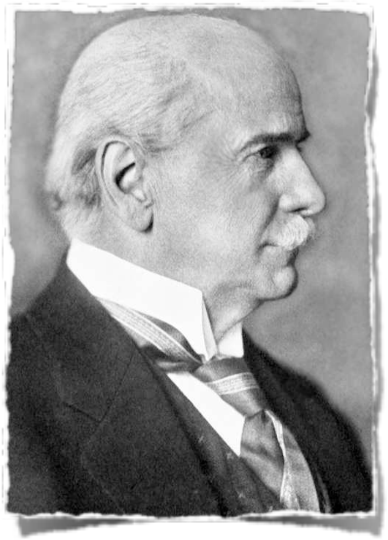

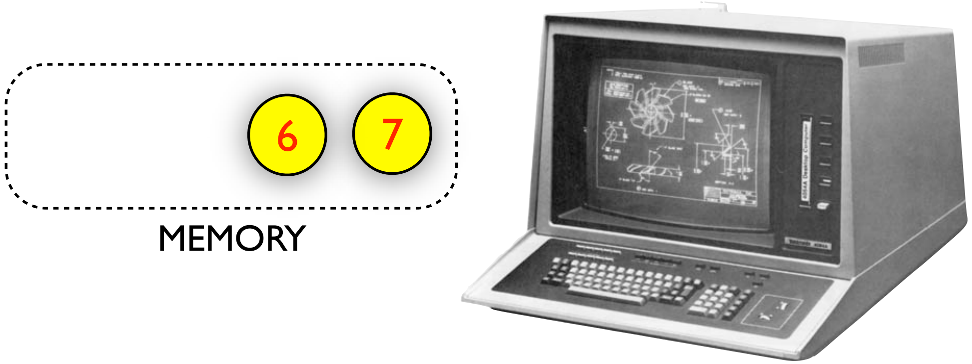

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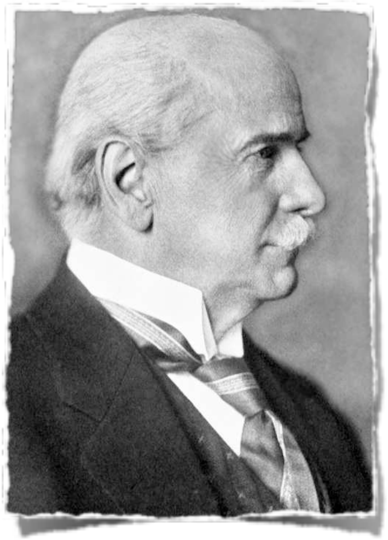

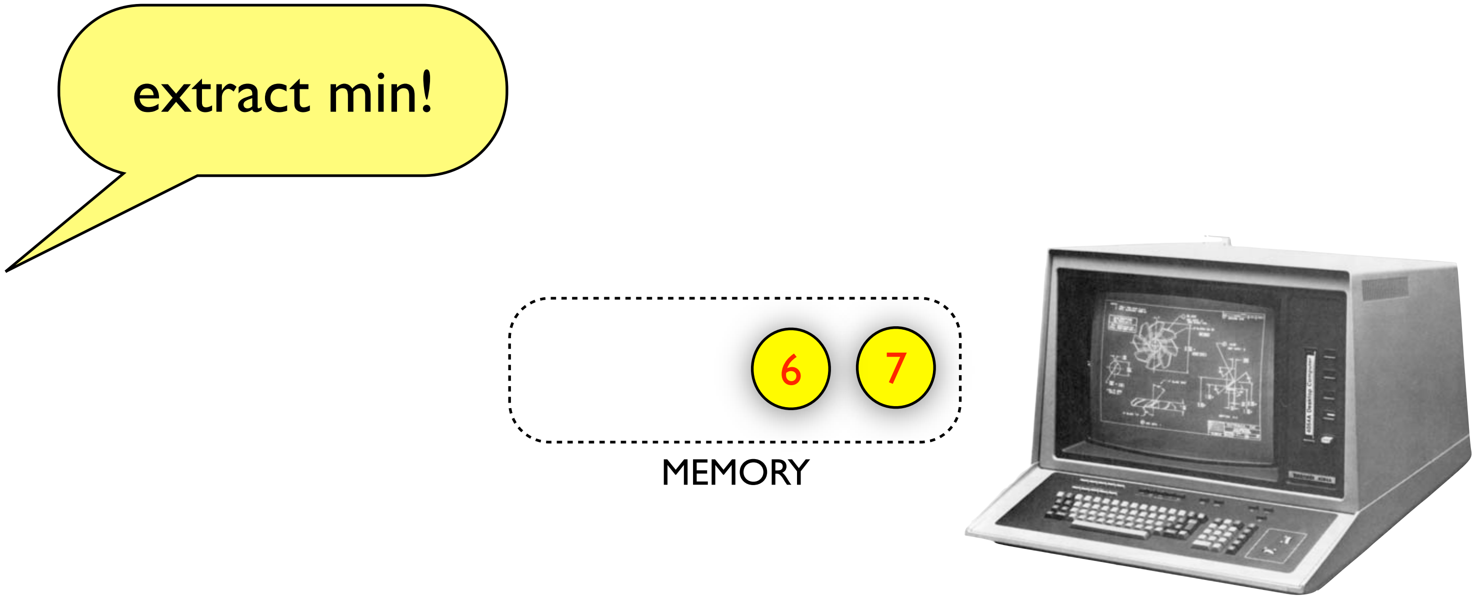

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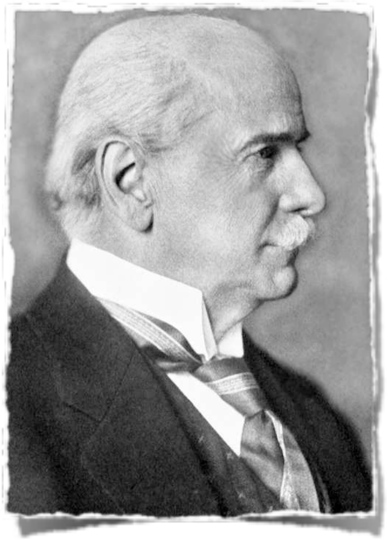

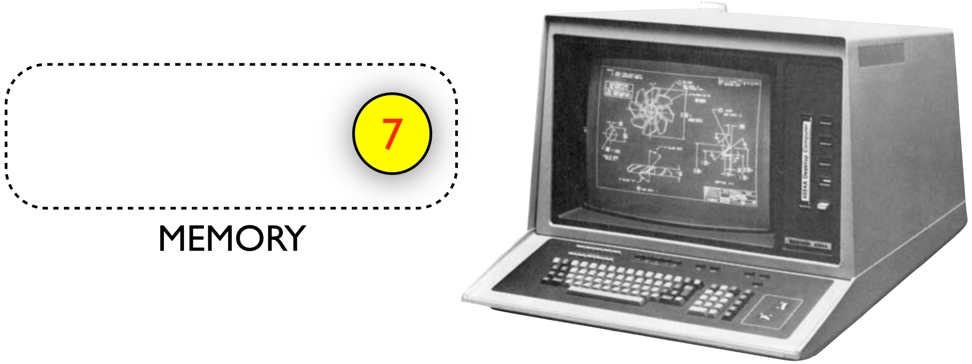

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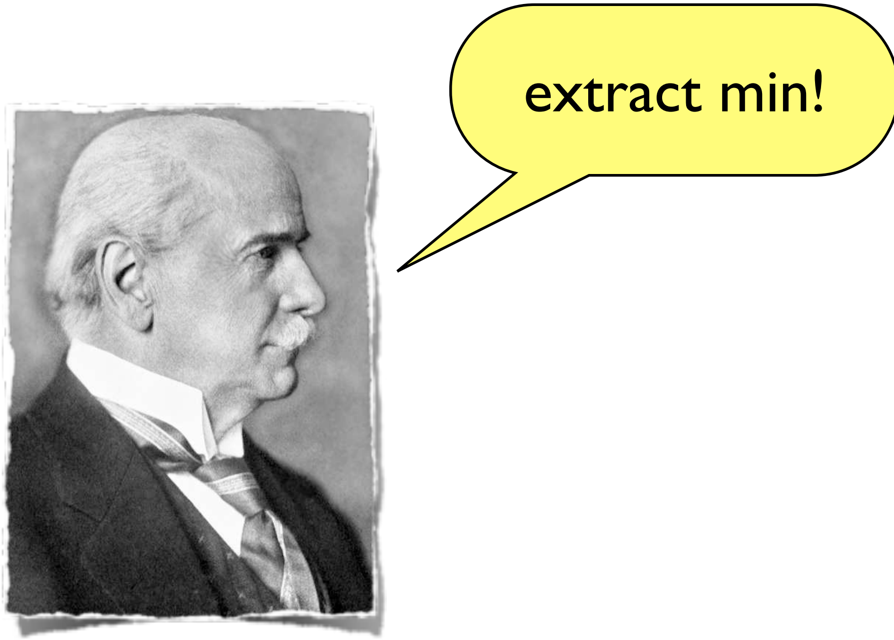

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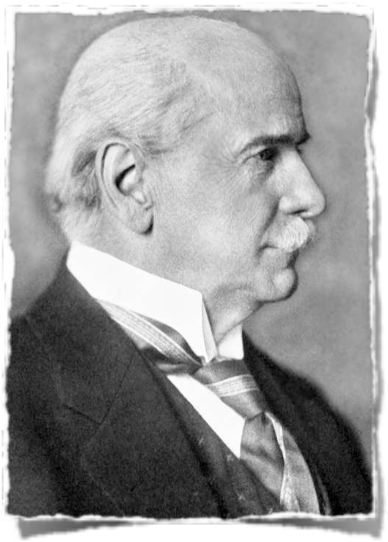

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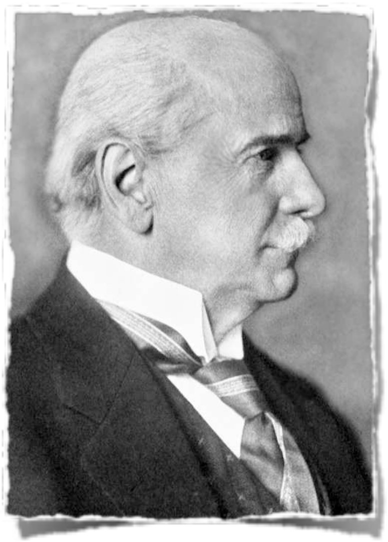

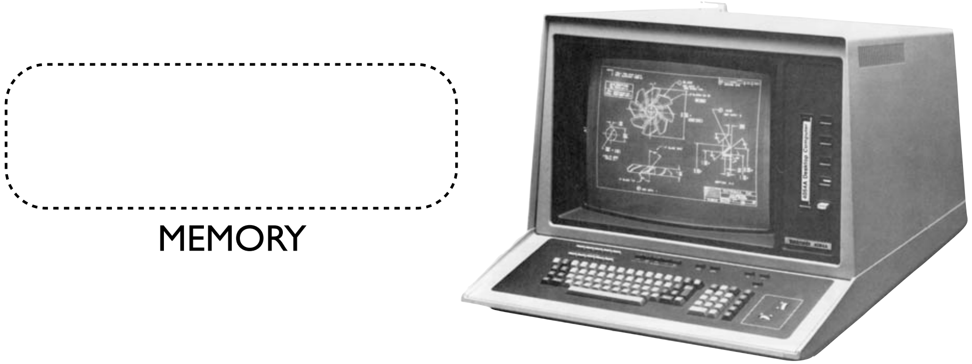

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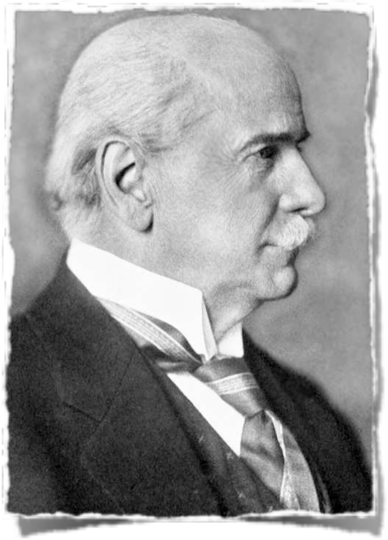

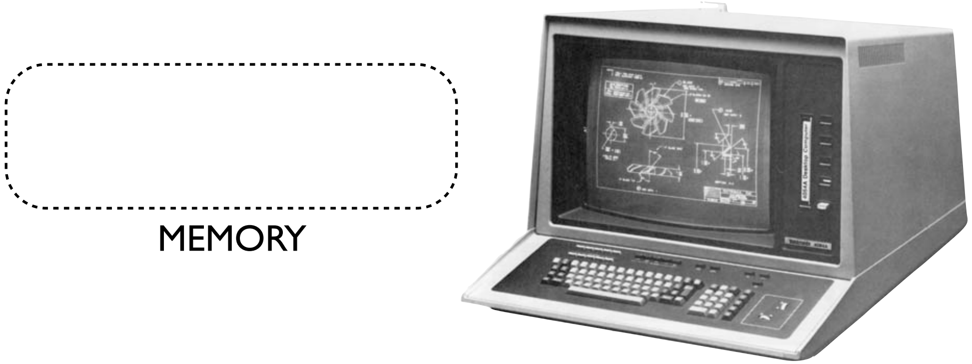

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- *Motivation:* Want to use cheap commodity hardware.

[Blum, Evans, Gemmell, Kannan, Naor '94]

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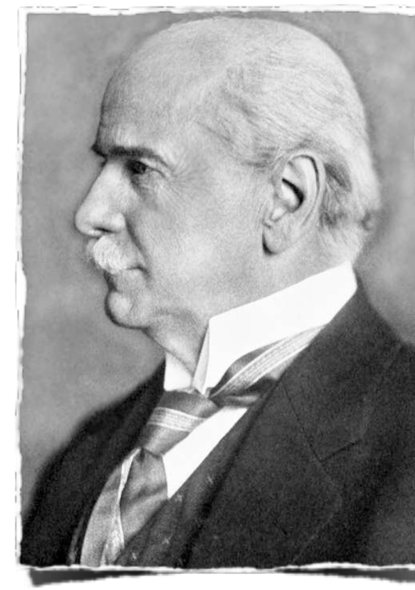
  iii. Explore connections with other problems...

I. Memory Checking

II. Lower Bounds



III. Parenthesis and Passes

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! *For this talk:* Assume inserted elements are distinct and that inserts come before their corresponding extract. I.e., we're trying to identify the following *bad pattern:*

ins(u) ... ext(v) ... ext(u) for some u < v

Epochs and Local Bad Patterns...

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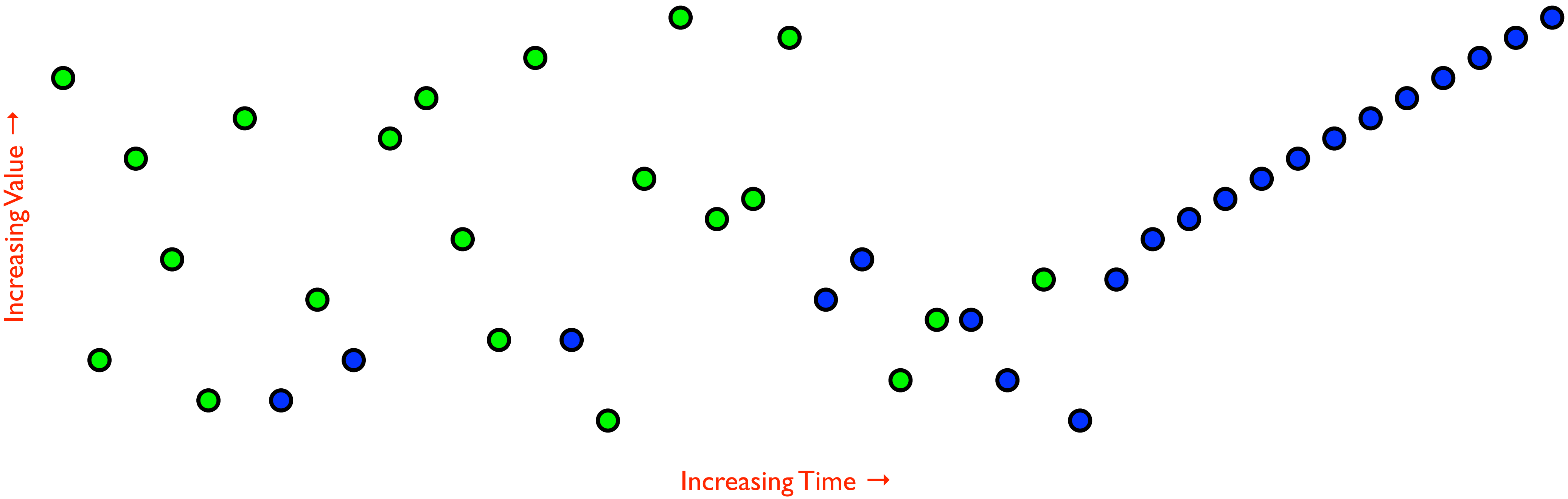

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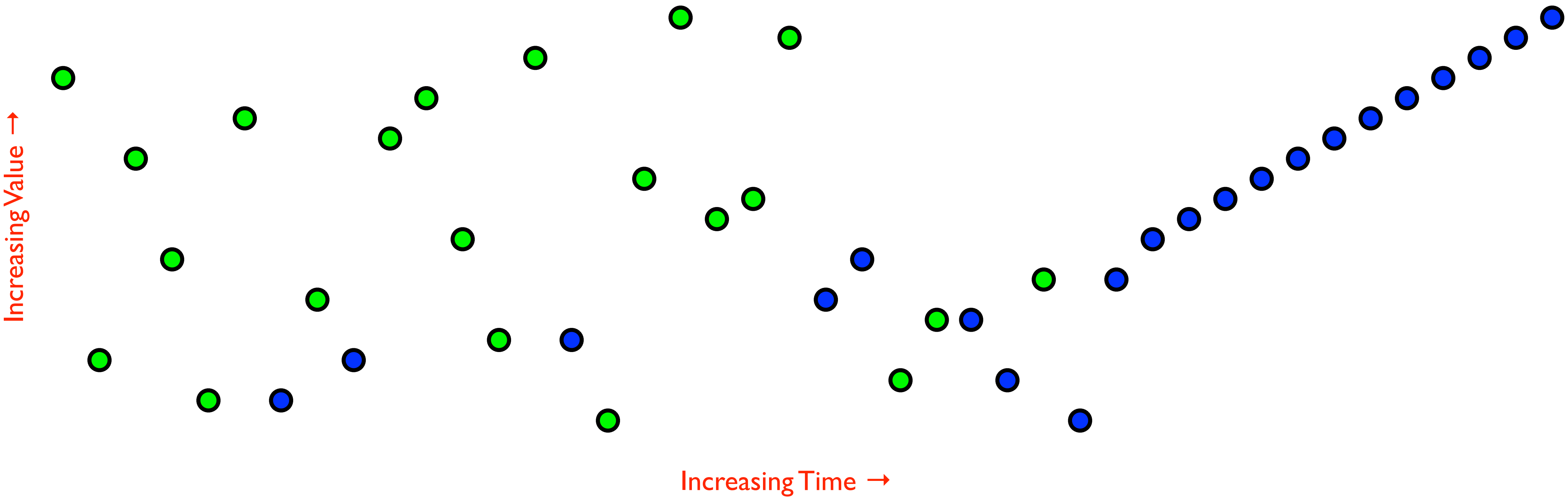

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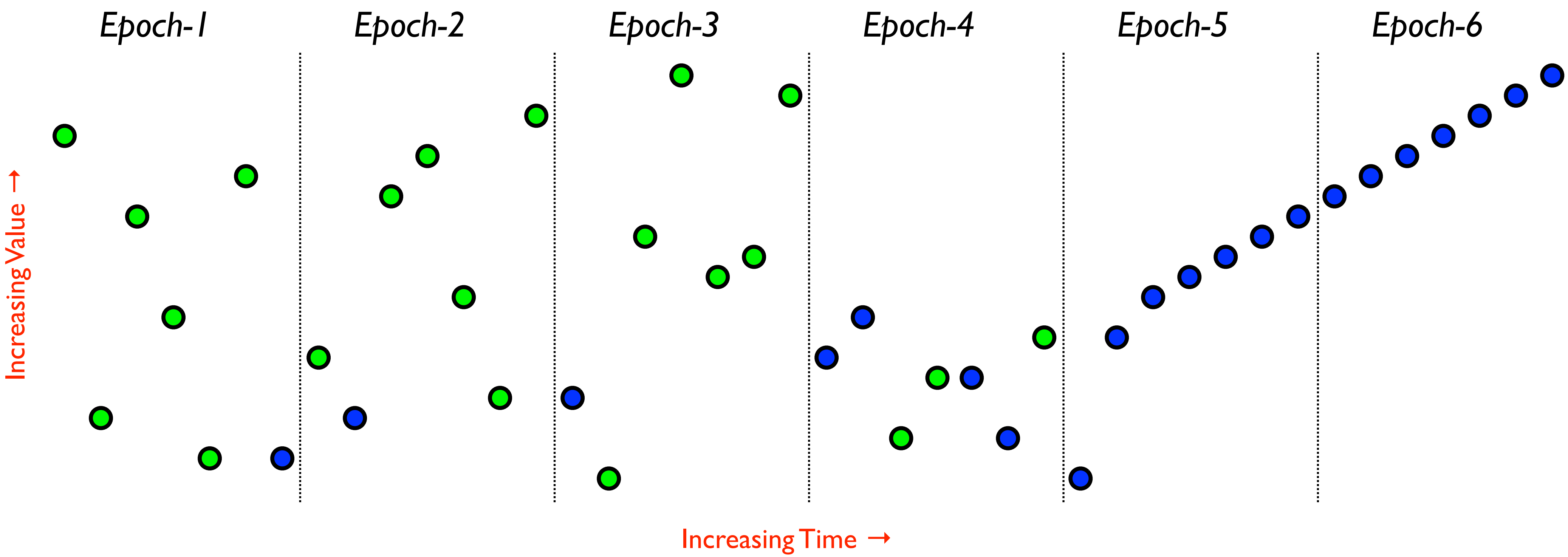

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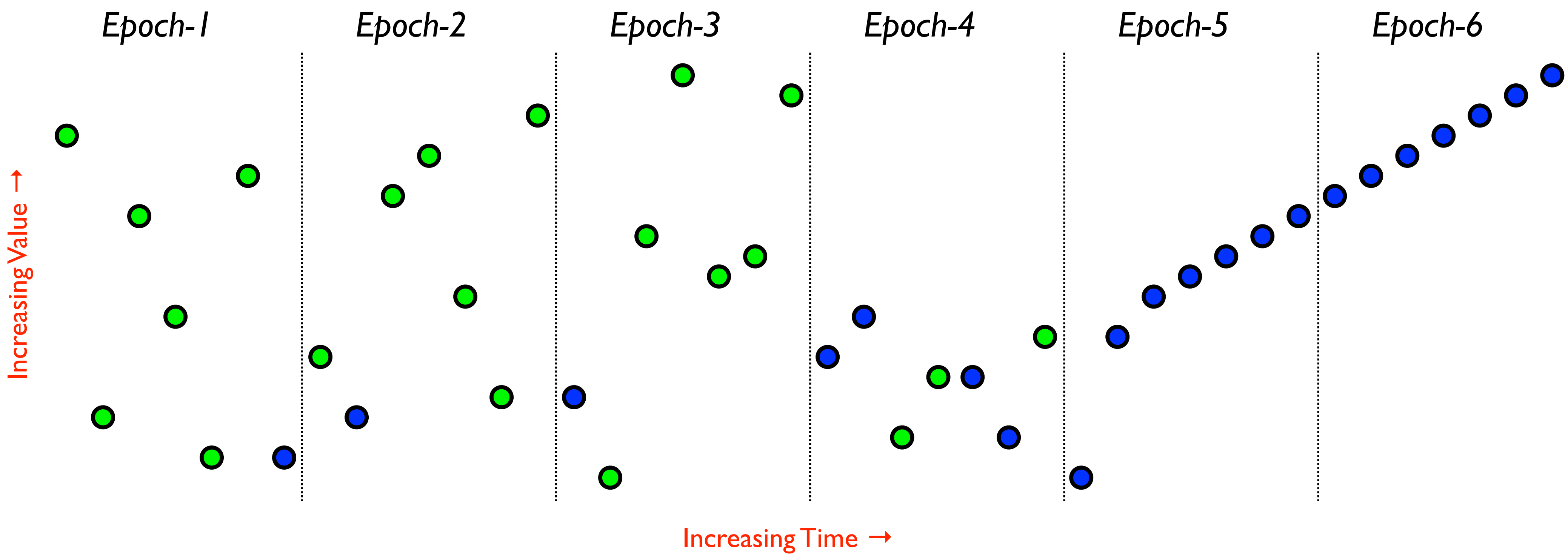

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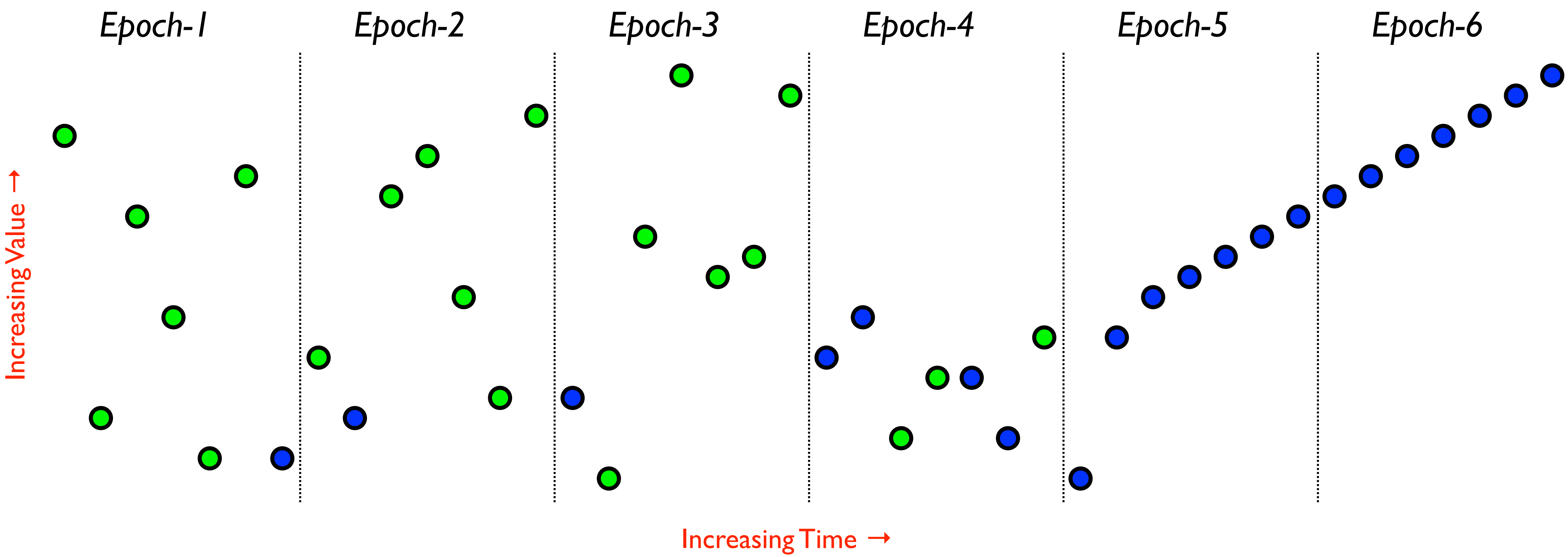

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- Using $O(\sqrt{N})$ space, we can buffer each epoch and check for local bad patterns.

Catching Long-Range Bad Patterns... 1/2

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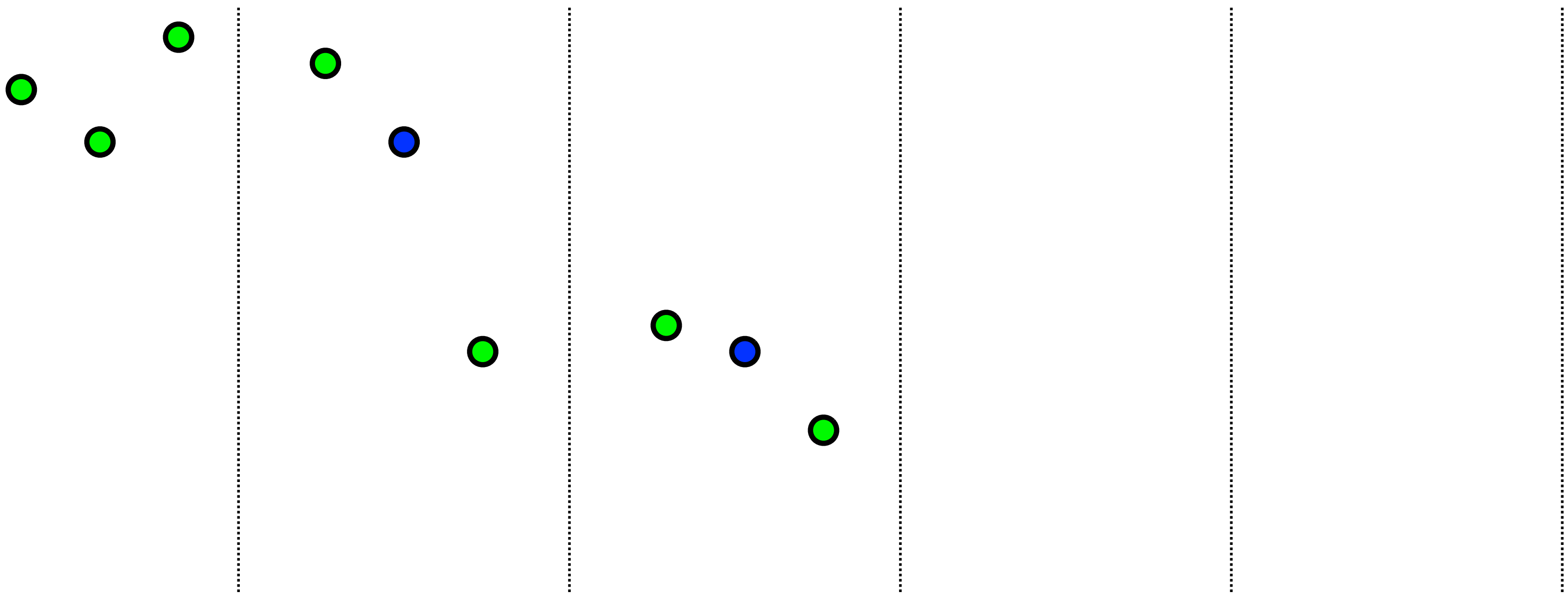

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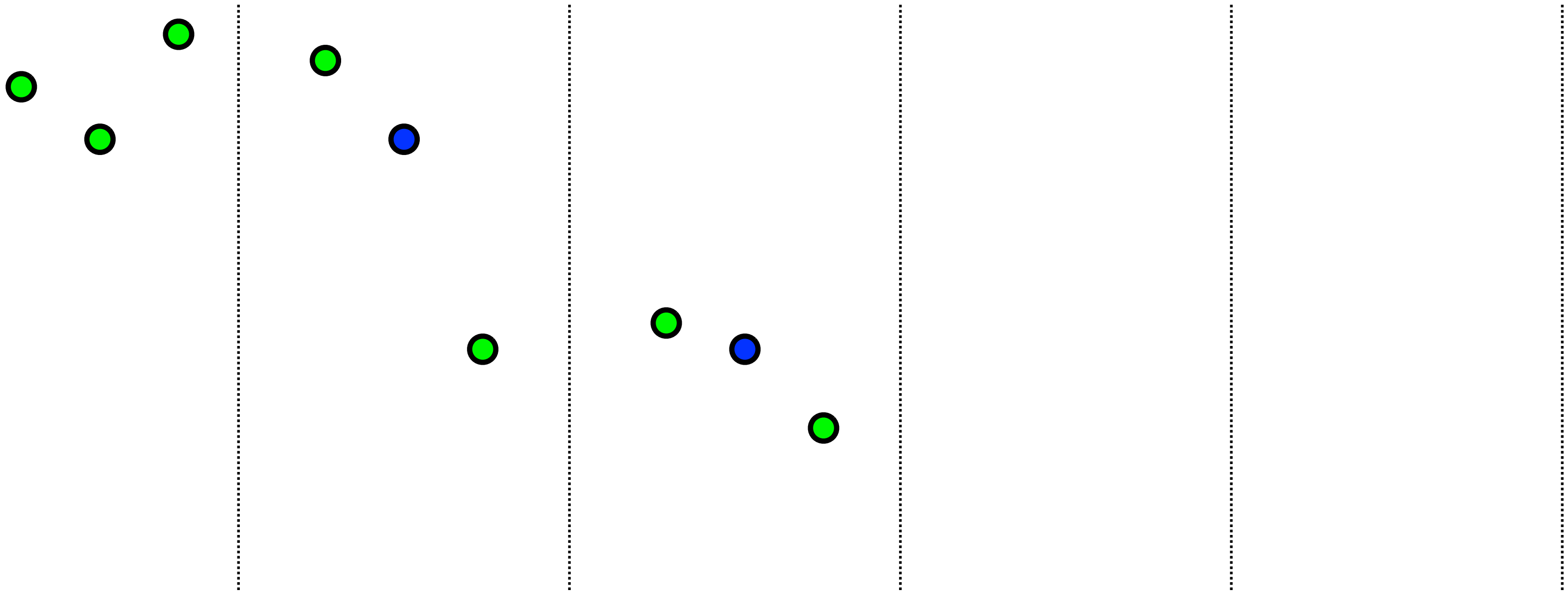

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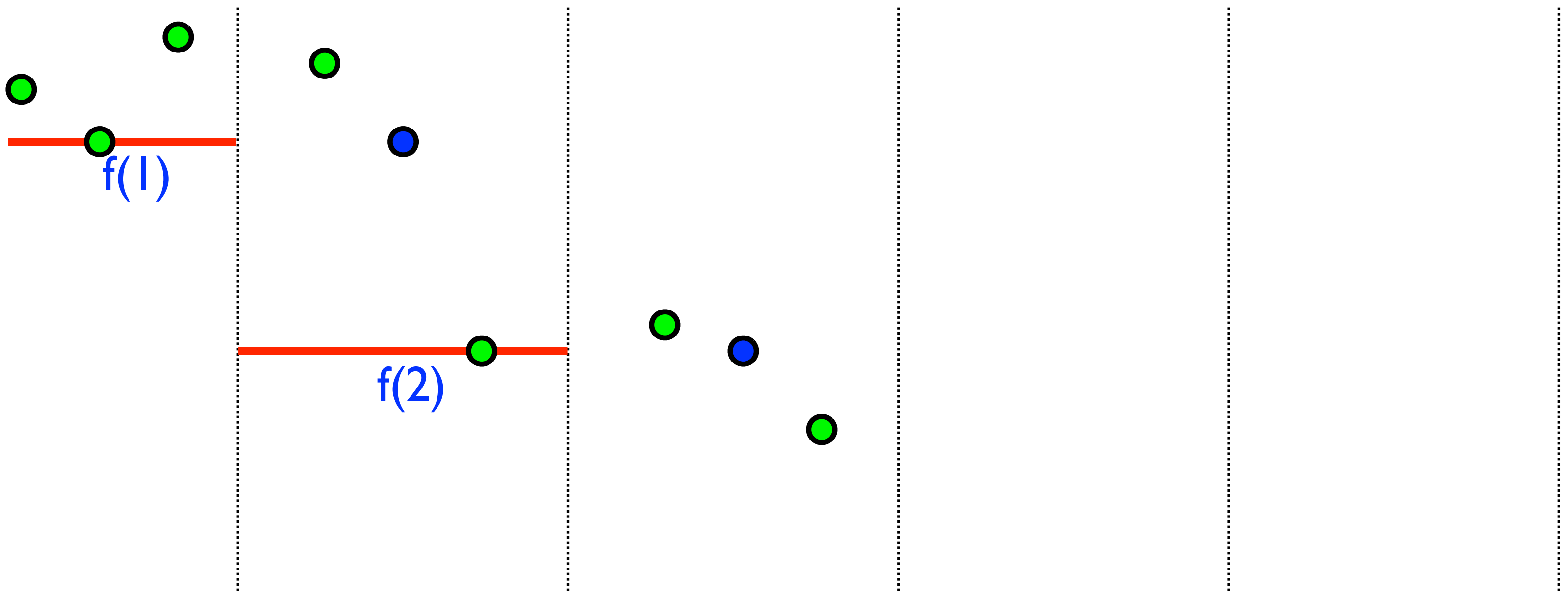

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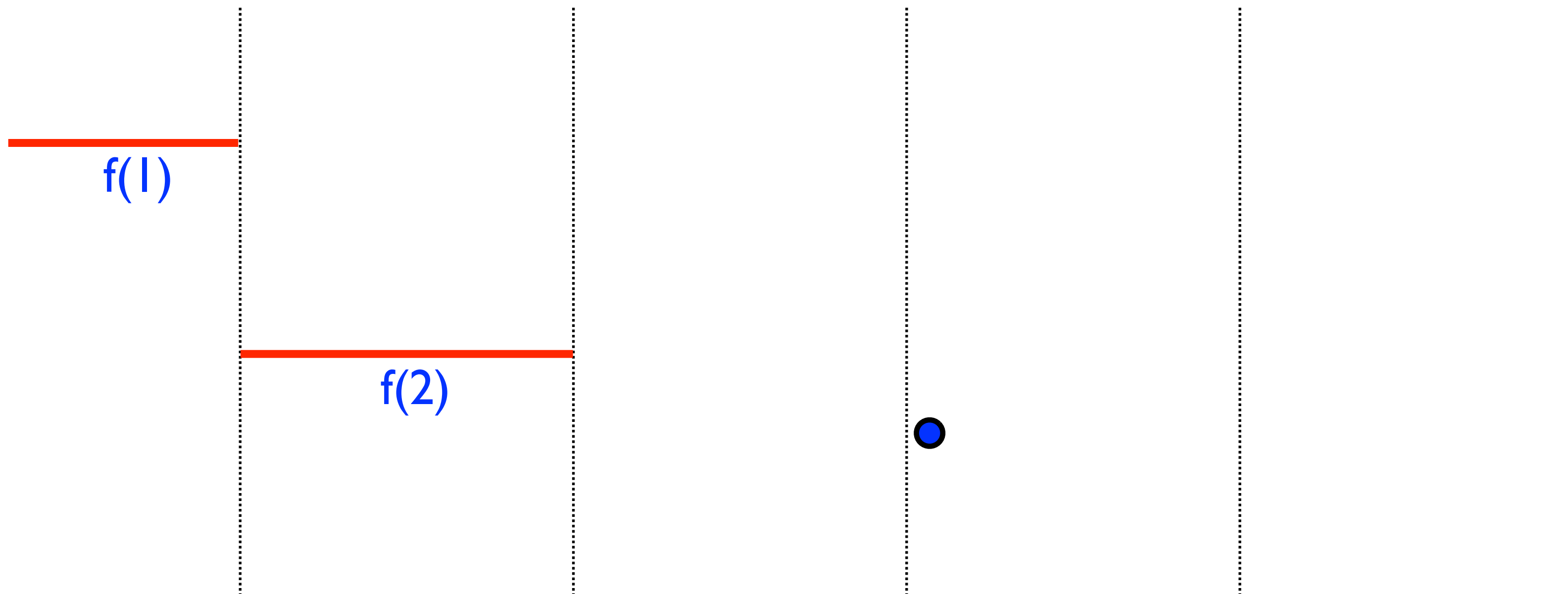

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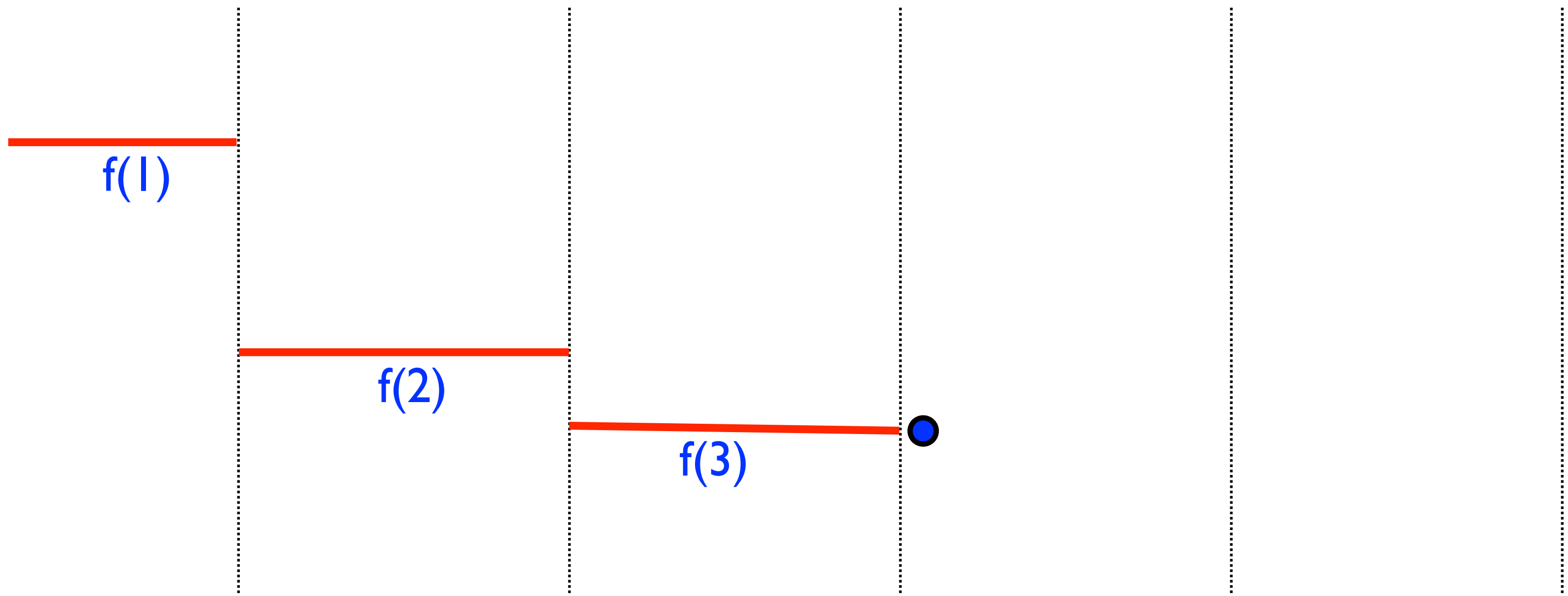

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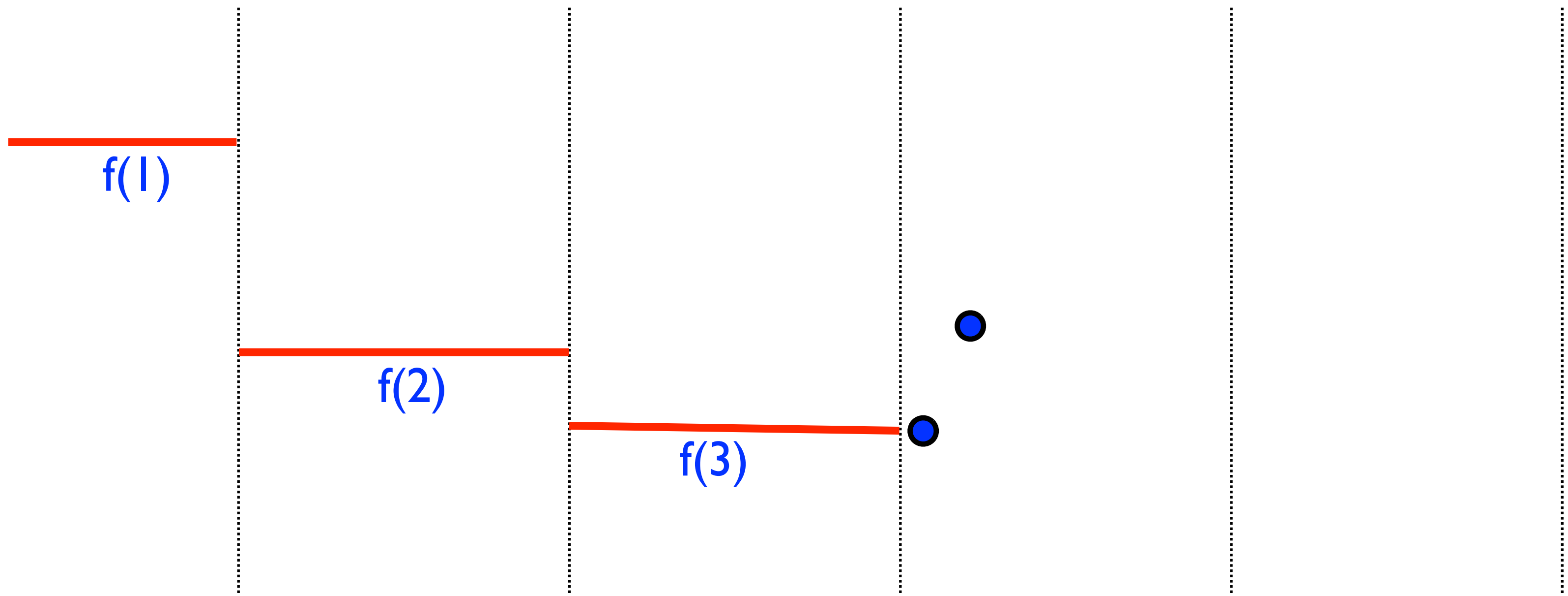

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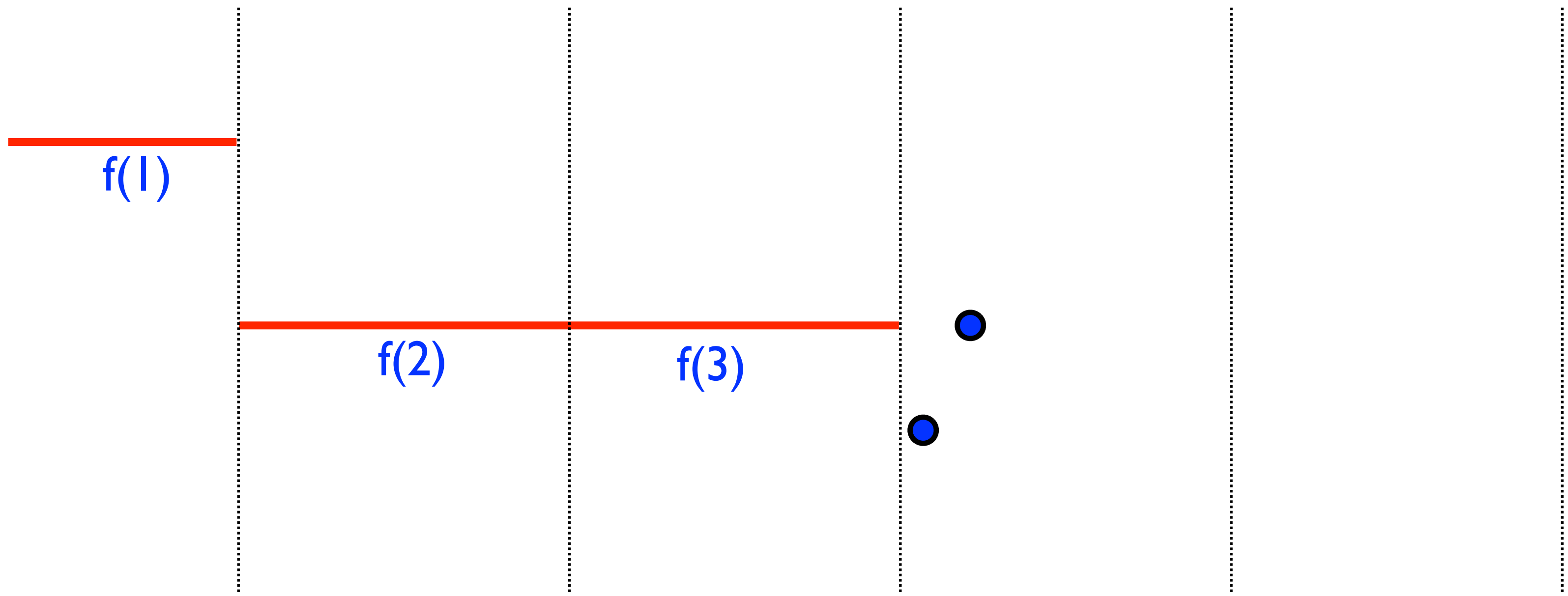

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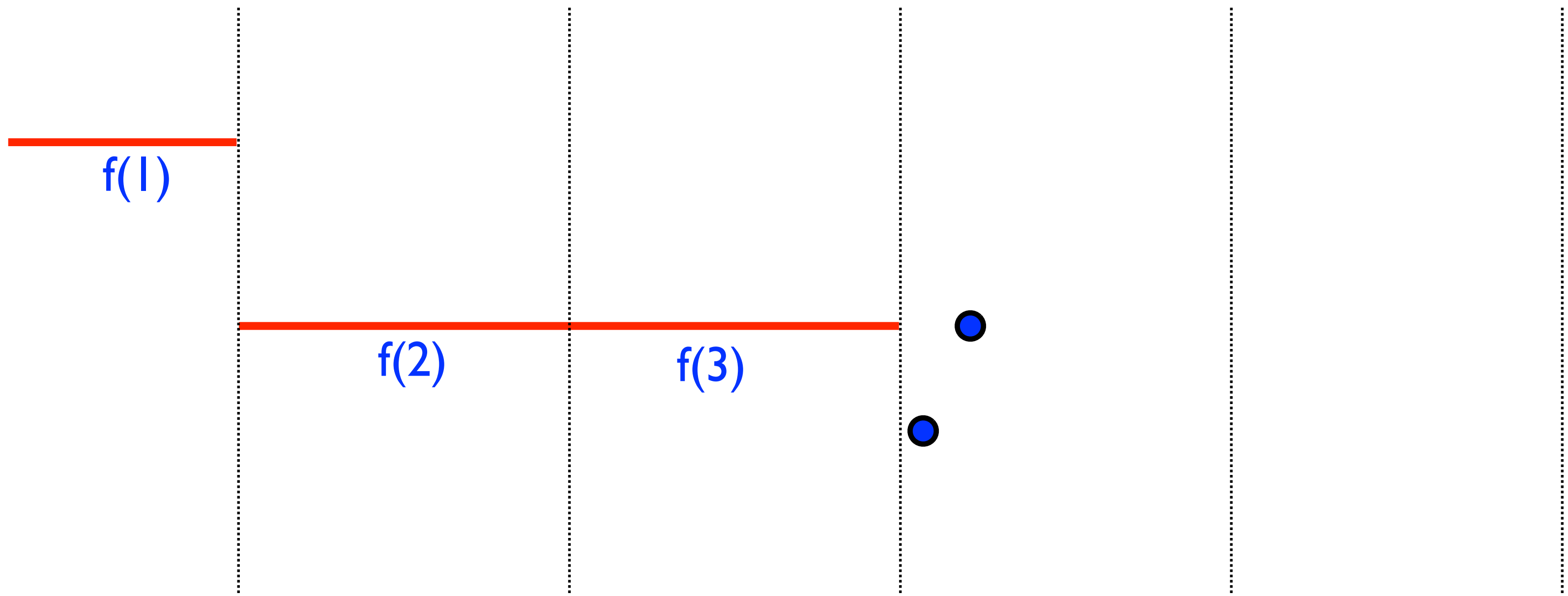

- Maintain the max value extracted between end of i-th epoch and *current time*. Call it f(i).
- *Defn:* Each ins(u) or ext(u) is *adopted* by earliest epoch k with $f(k) \leq u$.

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- *Lemma:* If there are no bad patterns, every ins(u) and ext(u) pair get adopted by the same epoch.
- *Algorithm:* Using fingerprints to check: for each epoch k

  {u : ins(u) adopted by k} = {u : ext(u) adopted by k}.

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- *Extensions:* Sub-linear space streaming recognition of other data structures like stacks, double-ended queues...

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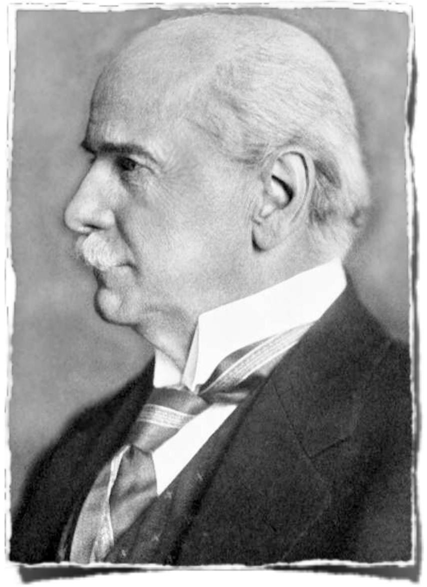
II. Lower Bounds



III. Parenthesis and Passes

II. Lower Bounds

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- Many space lower bounds in data stream model are based on reductions from communication complexity.
- *Augmented Index:* Alice has $x \in \{0,1\}^n$ and Bob has a prefix $y \in \{0,1\}^{k-1}$ of x and $c \in \{0,1\}$. Bob wants to check if $c = x_k$.
- *Thm:* Any 1/3-error, one-way protocol from Alice to Bob for AI_n requires $\Omega(n)$ bits sent. [Miltersen et al. JCSS '98]

Multi-player Augmented Index

 

Multi-player Augmented Index

x^1 y^1, c^1 x^2 y^2, c^2 x^3 y^3, c^3

- We now have 2m players $A_1, \ldots, A_m, B_1, \ldots, B_m$ where each A_i and B_i have an instance (x^i, y^i, c^i) of AI_n

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- *Thm:* Any 1/3-error protocol has a $\Omega(\min m,n)$ bit message.
- *Corollary:* Any algorithm for PQ requires $\Omega(\sqrt{N})$ space.

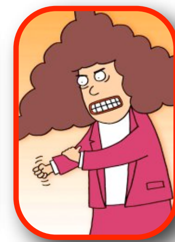
Reduction from multi-player AI to PQ…


0010


0, 0


0100

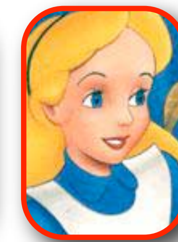

01,1


1010


,1


1010

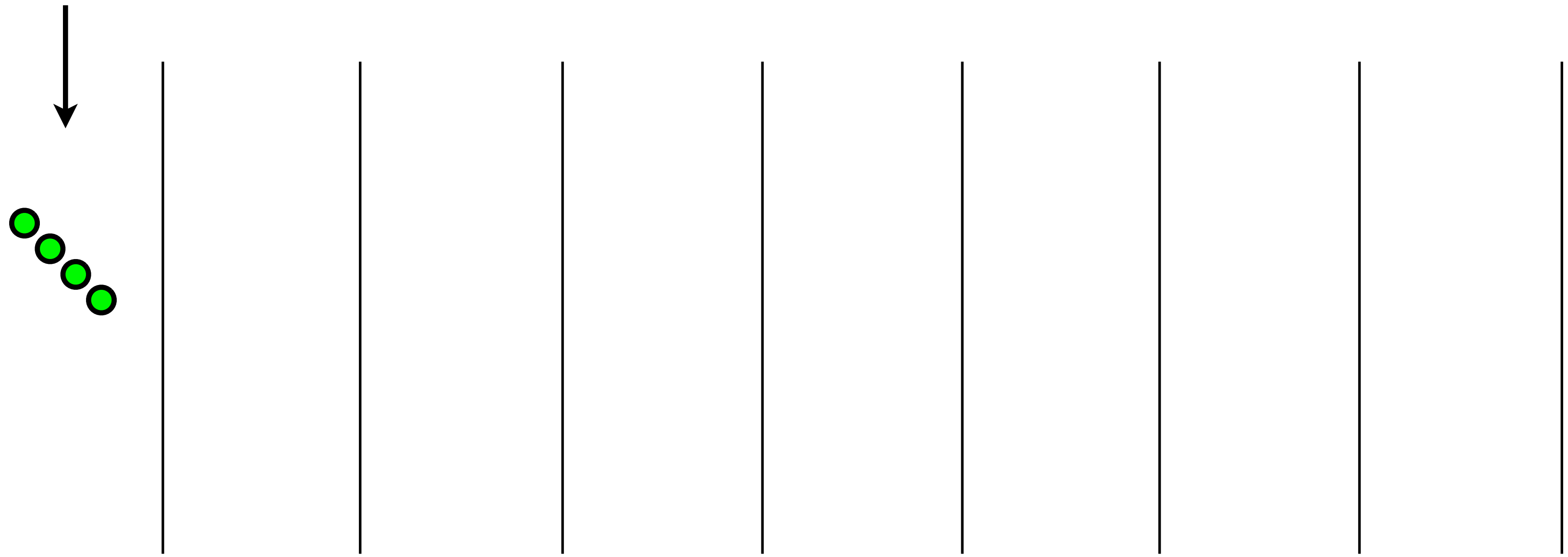

0100


0010

“Ascension Problem” [Magniez, Mathieu, Nayak ’10]

Reduction from multi-player AI to PQ...

0010 | 0, 0 | 0100 | 01,1 | 1010 | ,1 | 1010 | 0100 | 0010



ins(12.0)
ins(11.1)
ins(10.0)
ins(9.0)

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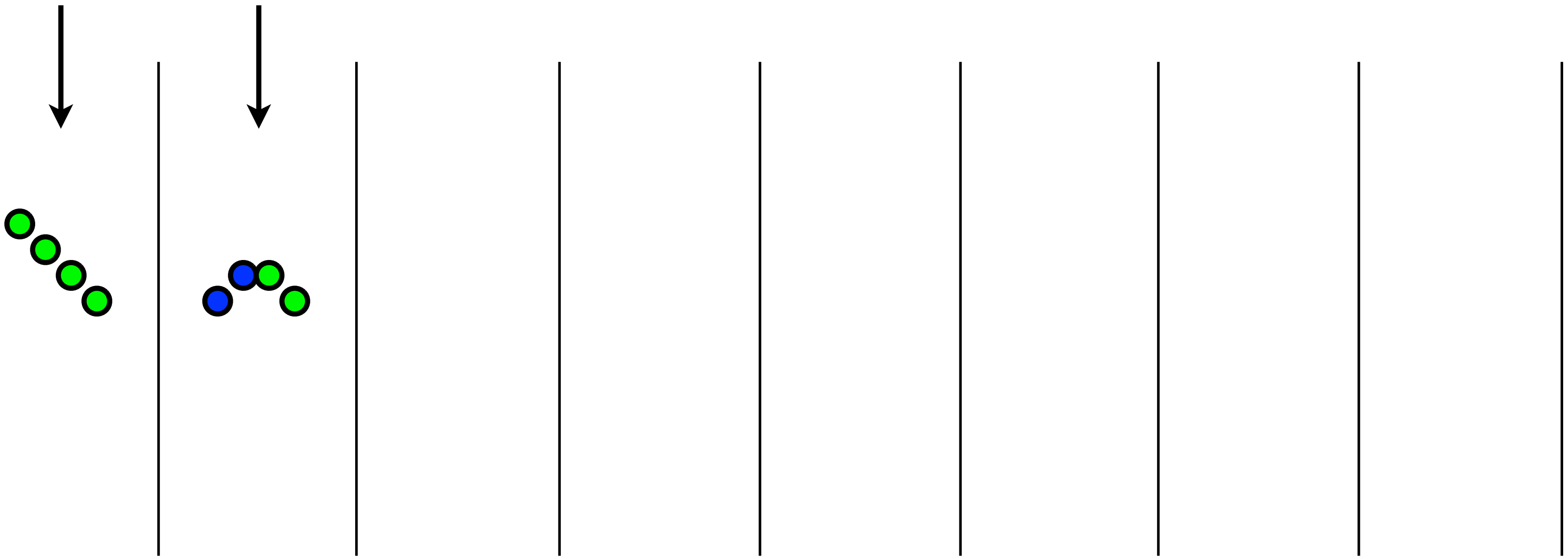
0010   0, 0   0100   01,1   1010   ,1   1010   0100   0010



ins(12.0)	ext(9.0)
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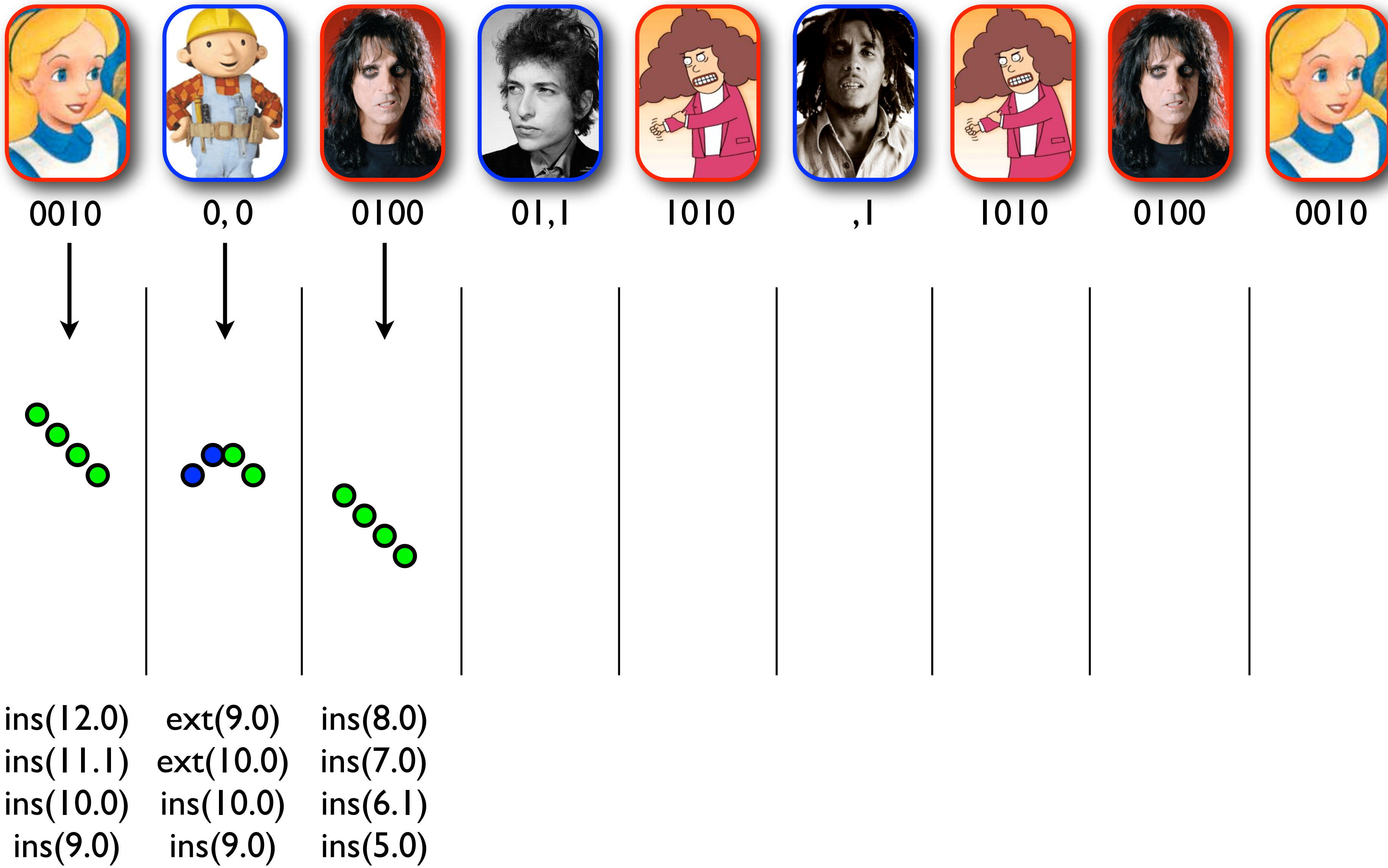

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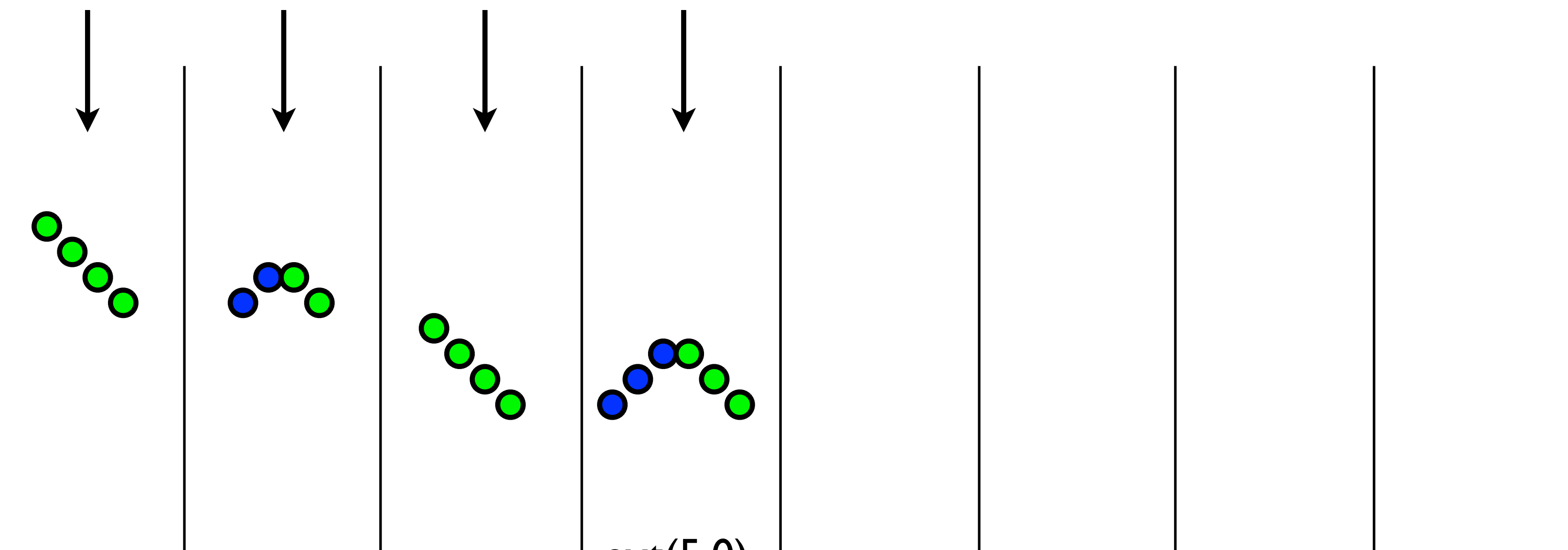
0010	0, 0	0100	01,1	1010	,1	1010	0100	0010
			ext(5.0)					
ins(12.0)	ext(9.0)	ins(8.0)	ext(6.1)					
ins(11.1)	ext(10.0)	ins(7.0)	ext(7.1)					
ins(10.0)	ins(10.0)	ins(6.1)	ins(7.1)					
ins(9.0)	ins(9.0)	ins(5.0)	ins(6.1)					
			ins(5.0)					



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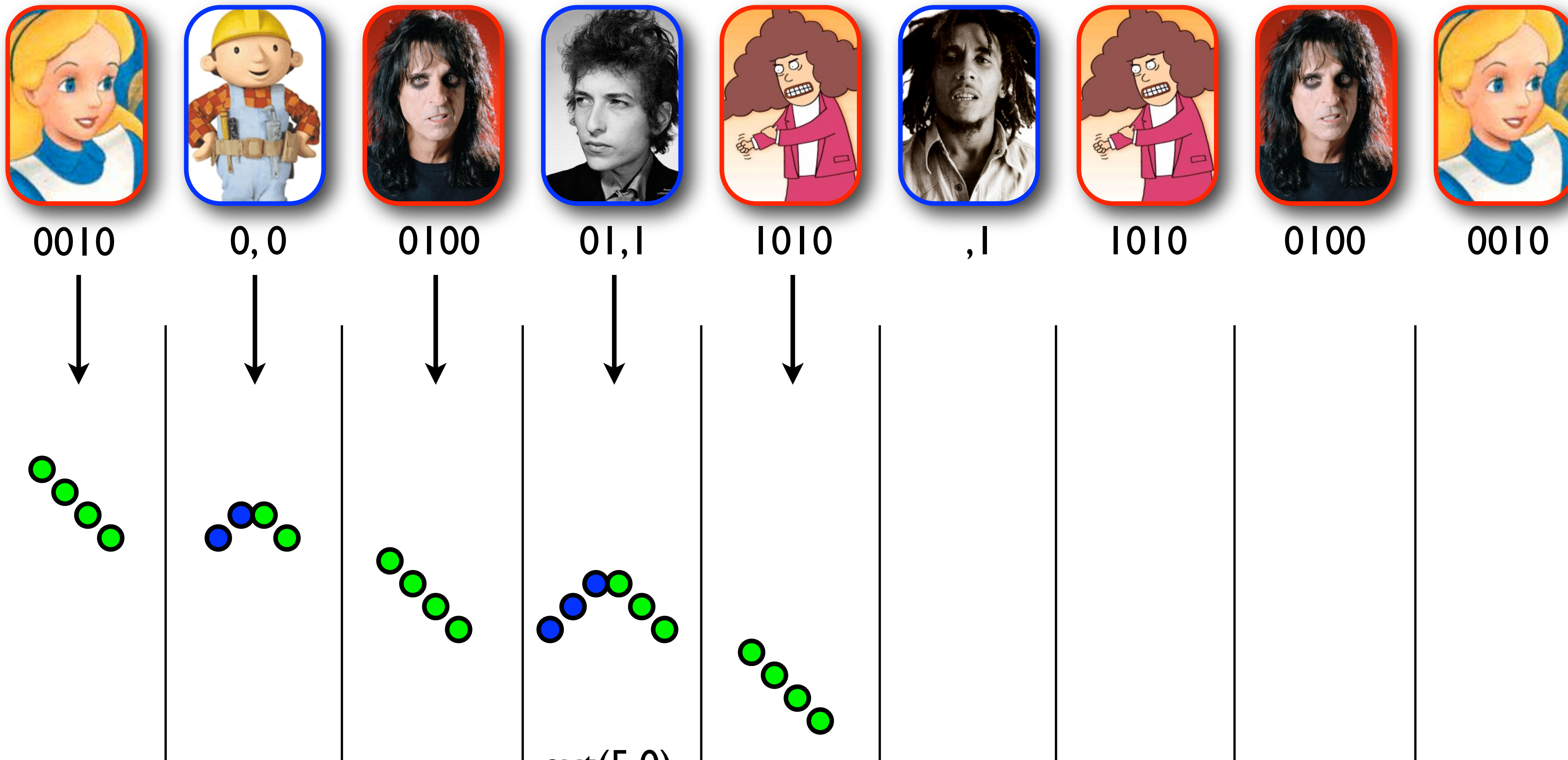

			ext(5.0)	
ins(12.0)	ext(9.0)	ins(8.0)	ext(6.1)	ins(4.0)
ins(11.1)	ext(10.0)	ins(7.0)	ext(7.1)	ins(3.1)
ins(10.0)	ins(10.0)	ins(6.1)	ins(7.1)	ins(2.0)
ins(9.0)	ins(9.0)	ins(5.0)	ins(6.1)	ins(1.1)
			ins(5.0)	

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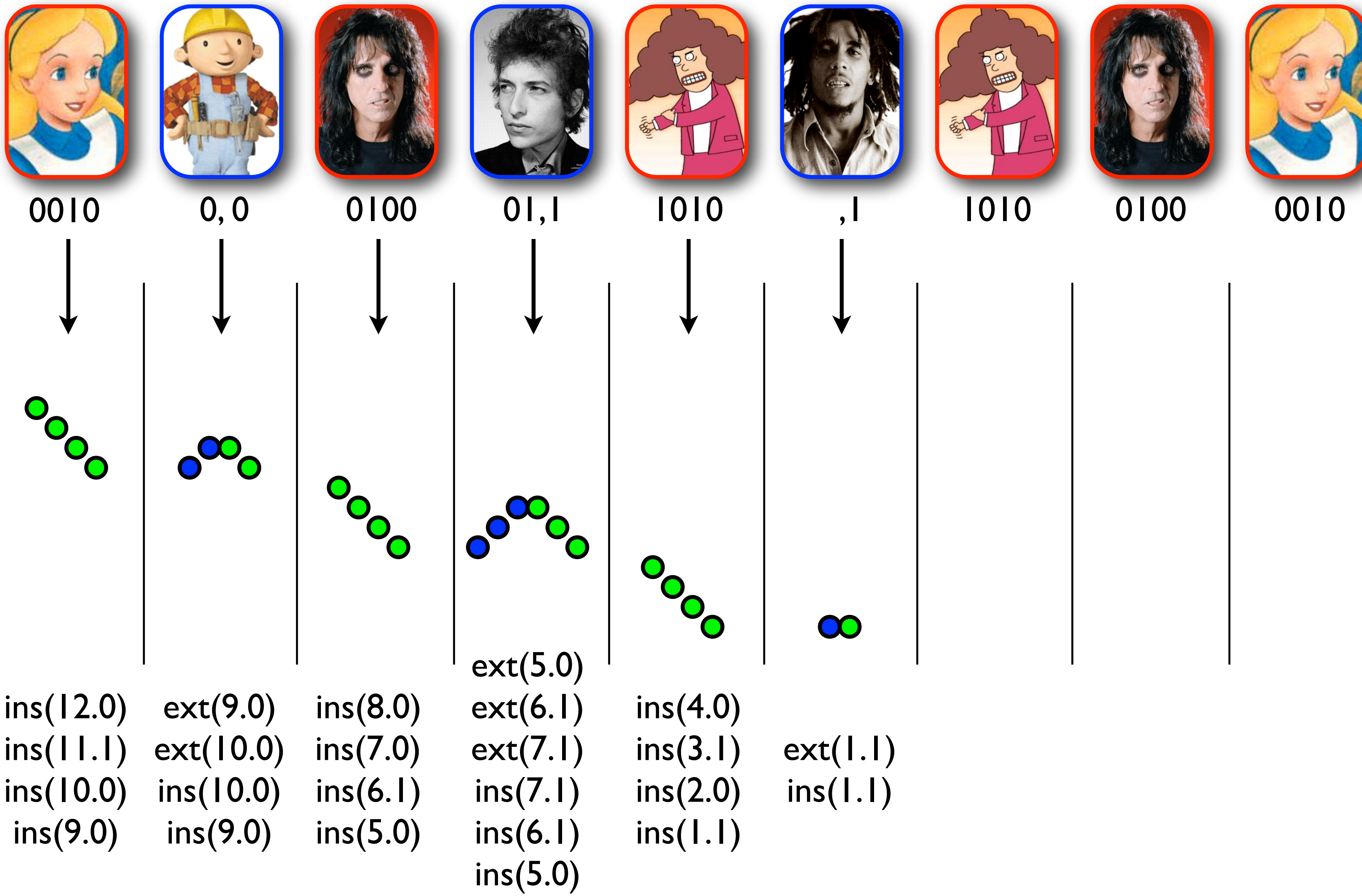

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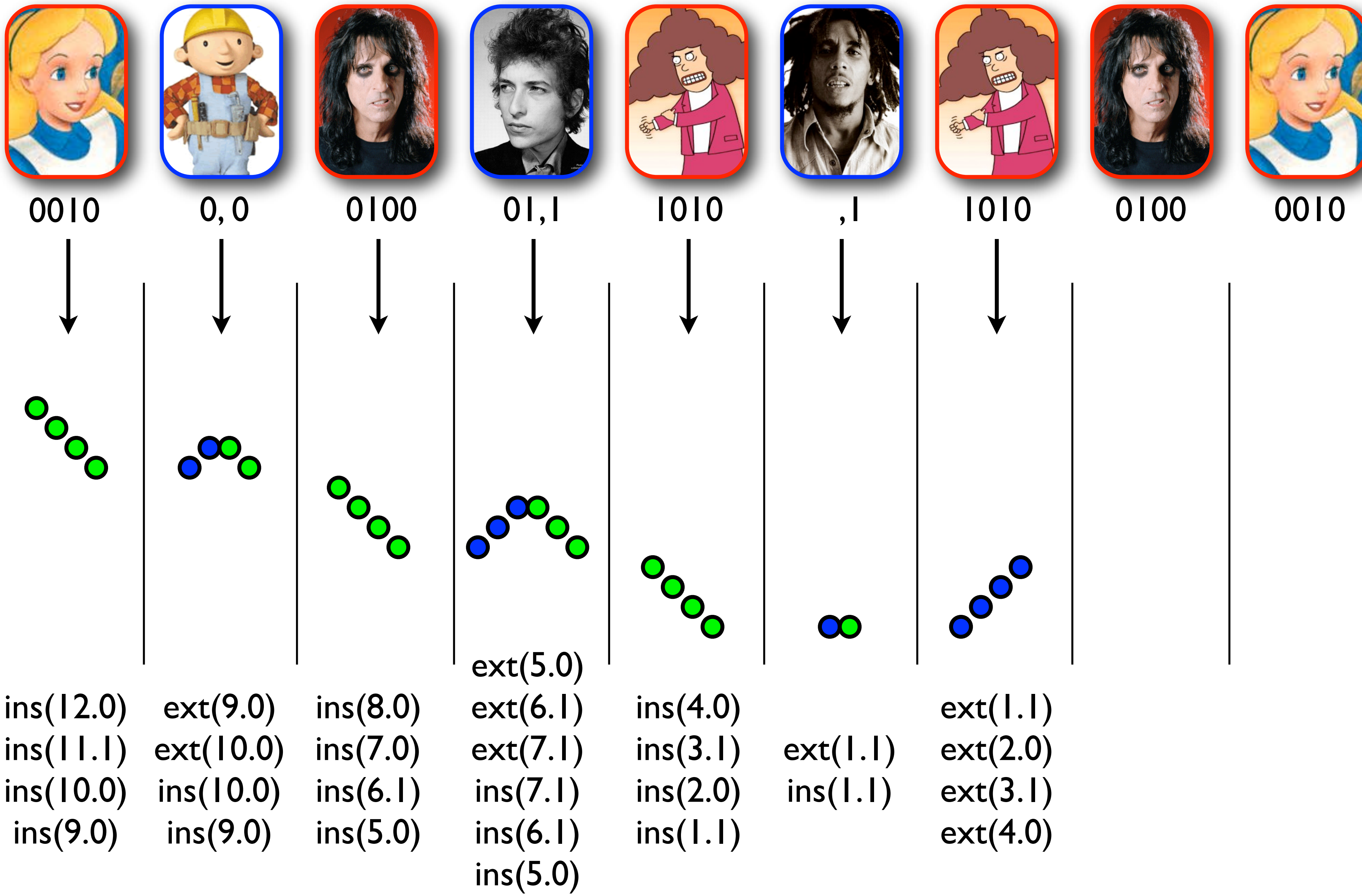

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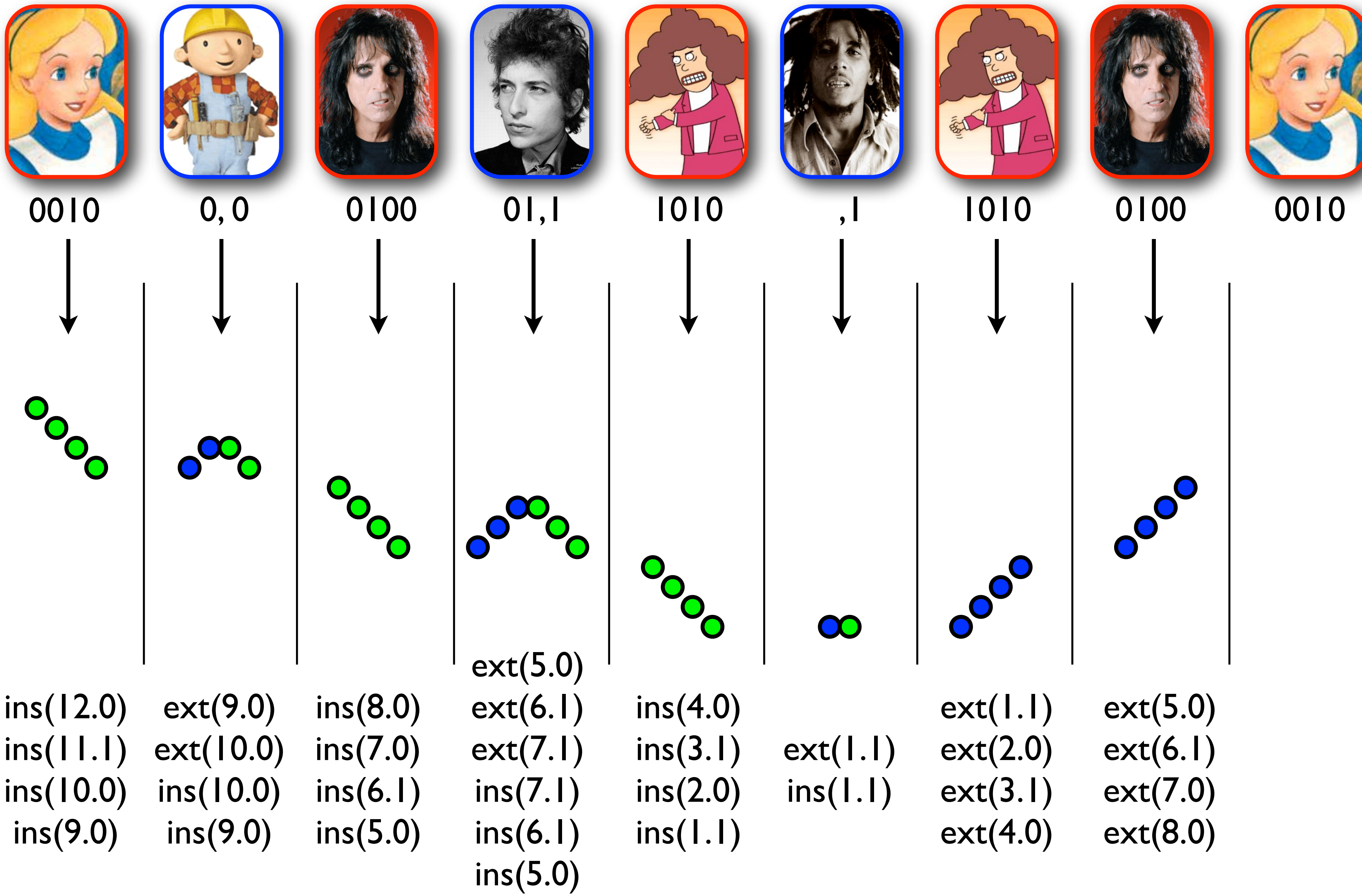

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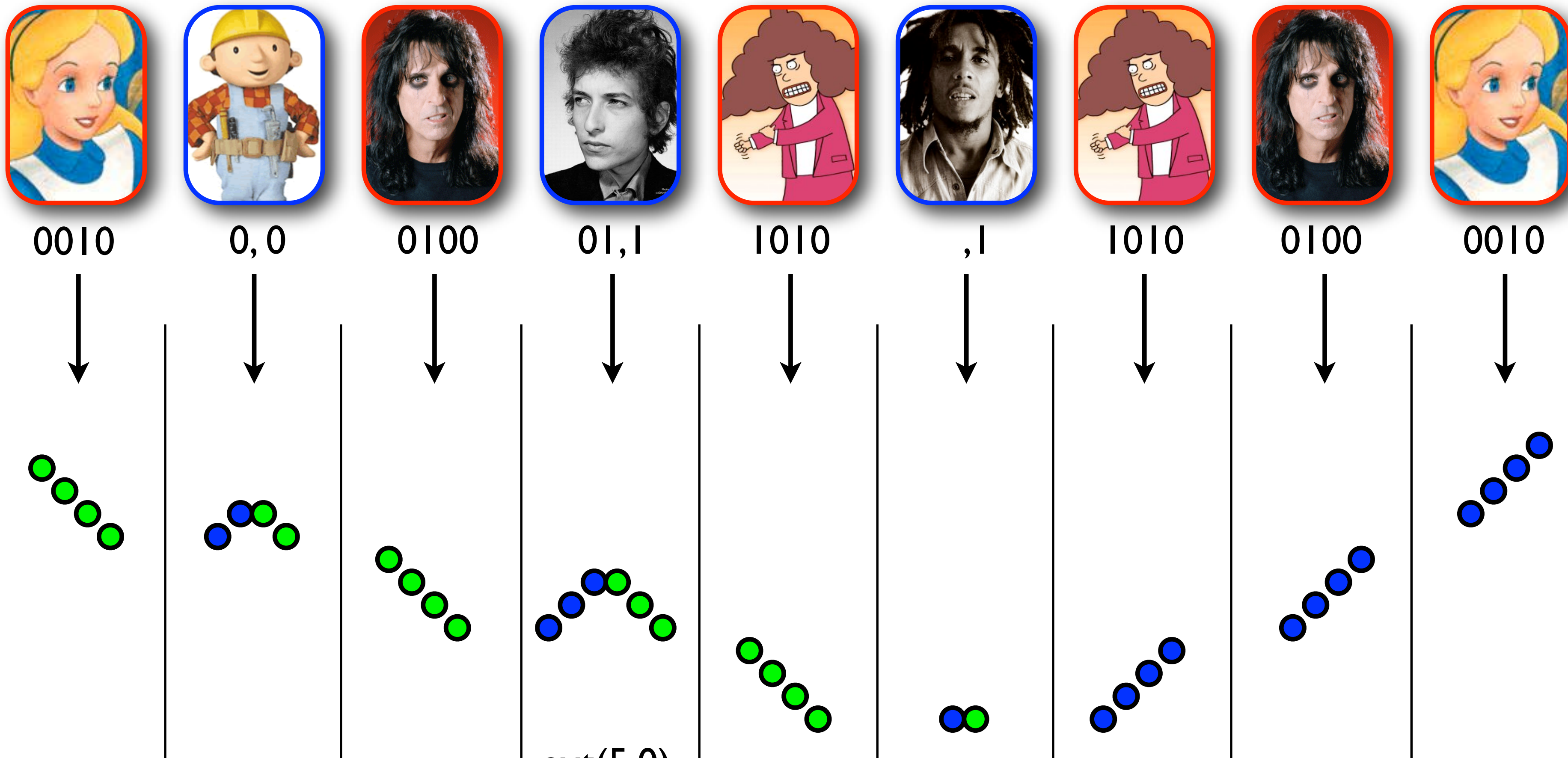

			ext(5.0)					
ins(12.0)	ext(9.0)	ins(8.0)	ext(6.1)	ins(4.0)		ext(1.1)	ext(5.0)	ext(9.0)
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ins(9.0)	ins(9.0)	ins(5.0)	ins(6.1)	ins(1.1)		ext(4.0)	ext(8.0)	ext(12.0)
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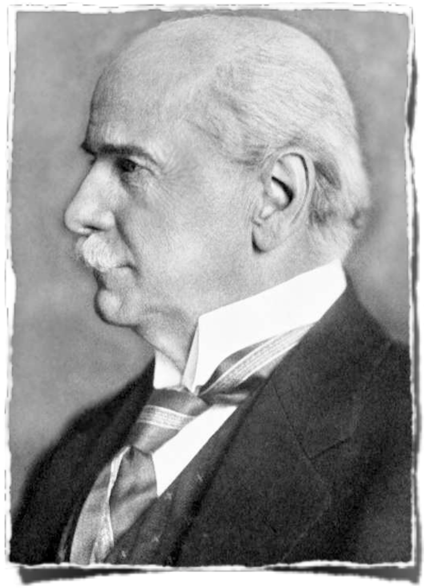
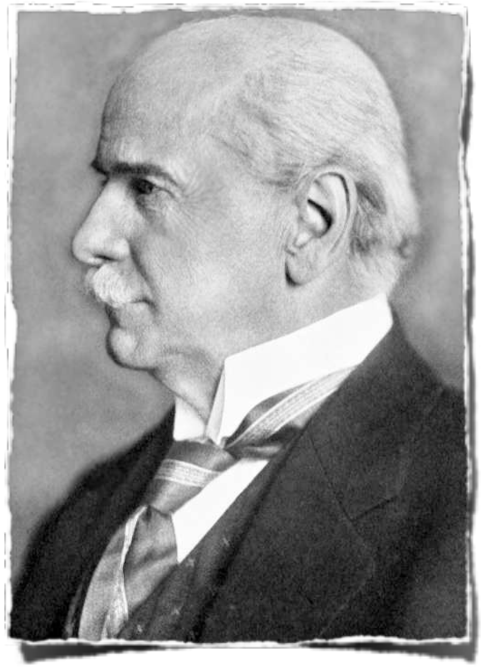
  3. Therefore, $s = \Omega(\min m,n)$ for length mn sequence.

I. Memory Checking

II. Lower Bounds



III. Parenthesis and Passes



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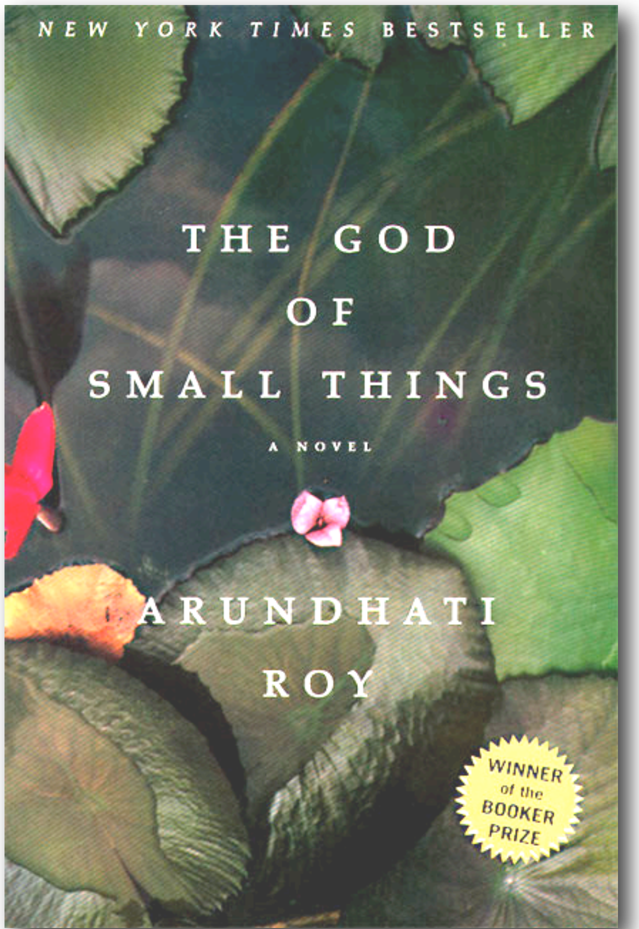
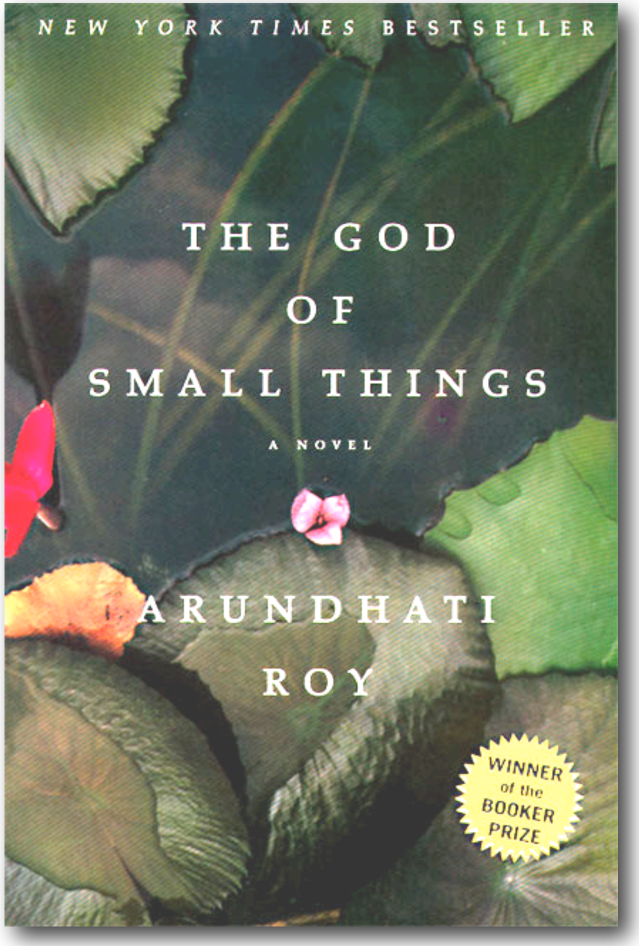

NEW YORK TIMES BESTSELLER
THE GOD OF SMALL THINGS
A NOVEL
ARUNDHATI ROY
WINNER of the BOOKER PRIZE



Fictional Quote:

"After Ammu died (after the last time she came back to Ayemenem (she had been swollen with cortisone and a rattle in her chest that sounded like a faraway man shouting), Rahel drifted."

DYCK Language Problem

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- DYCK_2 is the set of strings of properly nested brackets when there are two different types of brackets:

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"How useful is reading backwards? Do we also get a space saving if you can take multiple forward passes?"

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- *Example 1:* Necessary and sufficient space to find the median of n values $\Theta(n^{1/p})$.
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"Reading backwards can be very helpful!"

? *Open Problem:* Stream complexity of recognizing other languages and examples of backwards phenomena?

Summary

Memory Checking: Sub-linear space recognition of various data-structure transcript languages!

Theory of Stream Computation: Forward and backward pass better than many forward passes!

Further Work: Annotations, stream language recognition, ...

Thanks!

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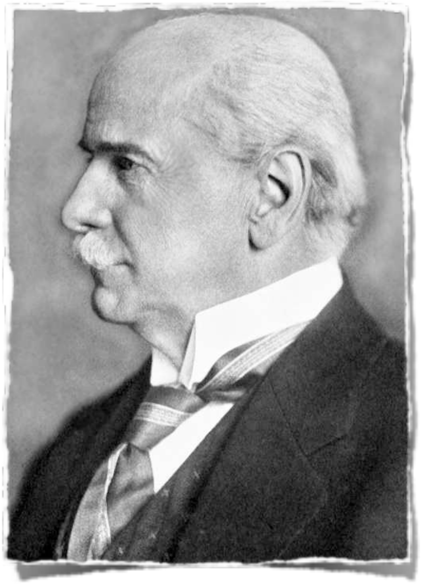

III. Parenthesis and Passes

IV. Augmented Index Bound

Information Complexity

[Chakrabarti, Shi, Wirth, Yao '01]

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- ***Entropy and Mutual Information:***

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- *Information cost method:* Consider mutual information between random input for a communication problem and the communication transcript:

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- Can restrict to partial transcript and subsets of input: useful for proving direct-sum arguments.

Information Complexity of AI_n

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- ***Defn:*** Let P be a protocol for AI_n using public random string R. Let T be the transcript and (X, K, C)~ξ. Define

$$
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\mathrm{icost}_{\xi}^{A}(\mathrm{P}) &= I(T : X \mid K, C, R) \\
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- *Thm:* Let P be a randomized protocol for AI_n with error 1/3 under the uniform distribution μ. Then,

$$\mathrm{icost}_{\mu_0}^{A}(P) = \Omega(n) \quad \text{or} \quad \mathrm{icost}_{\mu_0}^{B}(P) = \Omega(1)$$

where μ_0 is μ conditioned on $X_K=C$.

MULTI-AI$_{m,n}$ versus AI$_n$

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- ***Defn:*** Let Q be a protocol for MULTI-$AI_{m,n}$ using public random string R. Let T be transcript and $(X^i,K^i,C^i)_{i\in[m]}\sim\xi$.

$$\mathrm{icost}_\xi(Q) = I(T_m : K^1, C^1, \ldots, K^m, C^m \mid X^1, \ldots, X^m, R)$$

where T_m is the set of messages sent by B_m.

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- *Thm (Direct Sum):* If there exists a p-round, s-bit, ε-error protocol Q for MULTI-AI$_{m,n}$ then there exists a p-round, ε-error randomized protocol P for AI$_n$ where

i. Alice sends at most ps bits

ii. $m \cdot \mathrm{icost}^B_{\mu_0}(P) \leq \mathrm{icost}_{\mu_0^{\otimes m}}(Q)$

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- *Thm:* Any p-round, s-bit, 1/3-error protocol Q for MULTI-$AI_{m,n}$ requires $ps=\Omega(\min m,n)$.

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i. By direct sum theorem, there exists ε-error, p-pass protocol P for AI$_n$ such that:

$$
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p \cdot s &\geq \mathrm{icost}_{\mu_0^{\otimes m}}(Q) \geq m \cdot \mathrm{icost}^{B}_{\mu_0}(P) \\
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\end{aligned}
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$$p \cdot s \geq \mathrm{icost}^A_{\mu_0}(P)$$

  ii. By information complexity of AI_n

$$\max(m \cdot \mathrm{icost}^B_{\mu_0}(P), \mathrm{icost}^A_{\mu_0}(P)) = \Omega(\min(m, n))$$

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